AC 2010-1674: THE DEVELOPMENT OF A Q-MATRIX FOR THE CONCEPT ASSESSMENT TOOL FOR STATICS

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The Development of a Q-Matrix for the Concept Assessment Tool for Statics

Abstract

A concept inventory (CI) is a multiple-choice instrument designed to evaluate whether a person has an accurate, working knowledge of a specific set of concepts. An important role of CI's is to provide instructors with clues about the pre-conceptions their students hold which may be actively interfering with learning. Only a few engineering CI's have been able to be applied successfully in instructional settings, due in part to statistical analysis techniques that are typically applied to the instrument, which measure the item performance data of the CI's. However, these strategies do not measure students' cognitive abilities. To begin filling this gap, a study was conducted to determine the applicability of a new statistical method called the Fusion Model to the Concept Assessment Tool for Statics (CATS) among engineering students from various universities. A four-phase procedure was developed to apply the Fusion Model to CATS. Each phase had a specific objective that was tied to a primary research question. This paper focuses on phase 1 – the generation of a Q-matrix that relates a set of cognitive attributes to specific CATS questions. The process used in this phase of the study consisted of analyzing the items in CATS and determining the cognitive attributes required for students to choose the correct answer. These attributes were identified based on Minstrell's framework – facets of understanding. Results from this study led to the development of a Q-matrix in which 13 attributes were identified among the 27 items. Six of those attributes were identified and expected to be more problematic. Identification of these attributes provide an additional diagnostic information to instructors because instructors will know not only which concepts the students find more difficult, but also what specific attributes contribute to making the concept difficult. With this added information, instruction can be targeted to those specific attributes or concepts.

Introduction

For the past several years, programs within the National Science Foundation (NSF) have provided funding to Science, Technology, Engineering, and Math (STEM) educators to encourage the creation of various discipline specific concept inventories[1]. A concept inventory (CI) is a multiple-choice instrument designed to evaluate whether a person has an accurate, working knowledge of a specific set of concepts[2]. An important role of CI's is to provide instructors with clues about the pre-conceptions their students hold which may be actively interfering with learning. Similarly, the Concept Assessment Tool for Statics (CATS) has been developed to diagnose students' conceptual understanding of fundamental statics concepts[3]. The multiple choice questions in CATS were developed according to specific statics concepts. Similarly the distractors (wrong choices) were designed based on students' common errors when solving static problem. Engineering statics is a pivotal course in fields related to mechanical and civil engineering. It is the branch of mechanics concerned with the analysis of loads (force, torque/moment) on physical systems in static equilibrium[4]. Specifically, statics concepts are used in the analysis of structures such as those in architectural and structural engineering.

It is argued that to date, the work done in developing and implementing STEM CI's has more focused on the development of the instrument than on its formative application in instructional settings[5]. Ultimately, the goal of a CI is to be applied as a diagnostic tool that would directly inform instructors' understanding of students' cognitive capabilities in a way that might guide instruction of concepts on specific domains. To accomplish this, the Fusion Model – a diagnostic measurement model – has been developed to determine students' diagnostic profiles relative to a set of cognitive attributes selected for measurement and reporting [6]. The goal of the model is to assert whether or not students have mastered a set of cognitive attributes on the basis of observable responses.

In the Fusion Model, the cognitive attributes are related to the tasks through a Q-matrix – a binary representation of underlying cognitive attributes required for successful performance on a set of tasks. The entries in the matrix, 0 or 1, indicate which cognitive attributes are required for each task. These entries together with the examinee's latent class uniquely determine the examinee's probability for correctly performing each task. Therefore, the successful application of the Fusion Model requires the identification of the cognitive attributes expected for students to master a specific domain.

This paper focuses on the identification of statics cognitive attributes (SCA) according to students' responses to CATS. These SCA will be used to generate a Q-matrix that will be used to apply the Fusion Model to CATS and ultimately, to generate patterns of errors and expected mastery profiles that will therefore make possible the application of CATS for diagnostic assessment. Finally, to complete this task, the following research questions guided this study: **(1) What are the statics cognitive attributes (SCA) required for each of the items in CATS, and (2) How can the process of determining cognitive attributes be generalized to work for other concept inventories?**

Theoretical Framework

Successful conceptual change research has focused on changing students' naïve ideas in ways that can lead students to correct understanding of science concepts[7]. There are different theoretical views of conceptual change processes. Specifically, Minstrell's framework – facets of students' thinking – argues that assessment should be able to provide teachers with a diagnosis of students' thinking based on their conceptions[8]. In general, the process applied by Minstrell consisted of four steps (refer to Figure 1): (1) identify the learning target of a specific course – concept clusters; (2) identify particular conceptions of a specific phenomena – facets; (3) organize the facets within each cluster – facet clusters, (4) rank facet clusters with respect to how problematic they are according to a combination of students' difficulties, teachers' perceptions, and classroom observations. This methodology was used to determine the SCA required for the Q-matrix.

According to Minstrell[8], facets are particular conceptions that could be either naïve or limited in their consideration of a specific phenomena. Facets can be considered as an approximate understanding of some concept. Similarly, facet clusters include not only the learning goals, but also students' reasoning, conceptual and procedural difficulties. Each one builds up on ideas students have toward the targets of learning. For the purpose of this study, we are going to refer to facet as cognitive attributes (CA).

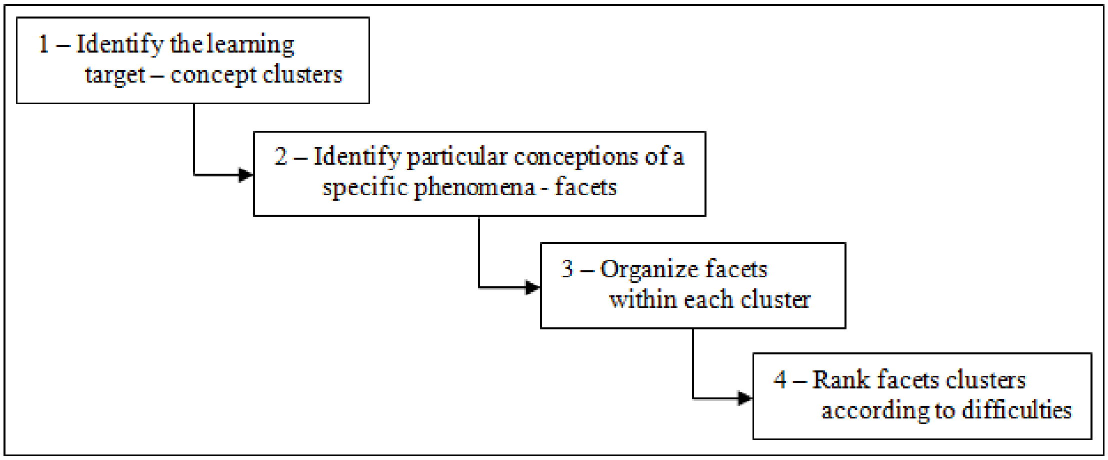

Figure 1 – Minstrell's framework - Facets of Students' Thinking: The Process

Methods

Instrument

CATS is a concept inventory designed as a diagnostic instrument for statics-related concepts. The questions of the instrument have the intention of detecting errors reflecting on incorrect concepts, instead of errors in mathematical analysis[4]. Results for unidimensional reliability (KR-20 alphas) have fluctuated between 0.70 and 0.90, which is highly desirable for CI's. The instrument consists of 27 questions that test nine different concepts (refer to Appendix 1). Each of the questions in CATS reflects on a specific concept but also for each question four of the alternatives were established based on the common errors students hold. These errors could affect the examinees' decisions for choosing their answers, therefore, they were also considered when identifying the SCA.

Participants

The data collected correspond to engineering students of about 22 different institutions from the 2006-07 academic year. The sample provided by Dr. Steif consisted of 1,354 participants from which 1,087 (80%) were male and 267 (20%) were female. In general, students were either sophomores and/or juniors enrolled primarily in Statics, but also in two follow-on courses: dynamics and mechanics of materials.

Methodology and Results

The methodology applied to this study was guided by Minstrell's theoretical framework explained in the previous section, which consisted of four steps. An explanation of the process and corresponding findings is presented below.

1. Identify the learning target of a specific course – concept clusters

Dr. Steif has done extensive work with respect to statics concepts. He identified four concept clusters to be important and difficult for students to understand once completed a static course (refer to Appendix 2). These concepts were identified by evaluating various engineering curricula and statics textbooks, and also from interviews with experienced faculty[9]. Other CI's have used other strategies to determine the cluster concepts such as Delphi process and focus groups among others[5].

2. Identify particular conceptions of a specific phenomena – cognitive attributes

The process used to identify SCA consisted of analyzing each of the 27 items of CATS and determine the cognitive attributes – Minstrell's facets of understanding – required for students to choose the correct answer. On the latest version of CATS, there are three questions (items) for each of the nine concepts tested on the instrument. Appendix 1 presents a description of the concepts addressed on CATS. Fortunately, Dr. Steif has also identified eleven common errors students make when solving statics problems (refer to Appendix 3). He argues that these errors reflect known conceptual errors exhibited by students[3]. Therefore, the identification of SCA was aided by the nine concepts tested on CATS (Appendix 1) but also by the eleven most common errors students make (Appendix 3).

Since each of the questions in CATS reflects on a specific concept and also for each question four of the alternatives were established based on the common errors students hold (distractors). As mentioned previously, the distractors might influence a student's decision, therefore, they were also considered when identifying the SCA. Item 1 from CATS was selected to explain the process applied (refer to Figure 2). This item refers to statics' cluster concept #3: drawing forces on separate bodies (refer to Appendix 2). This concept requires for students to identify forces acting on a subset of a system of bodies.

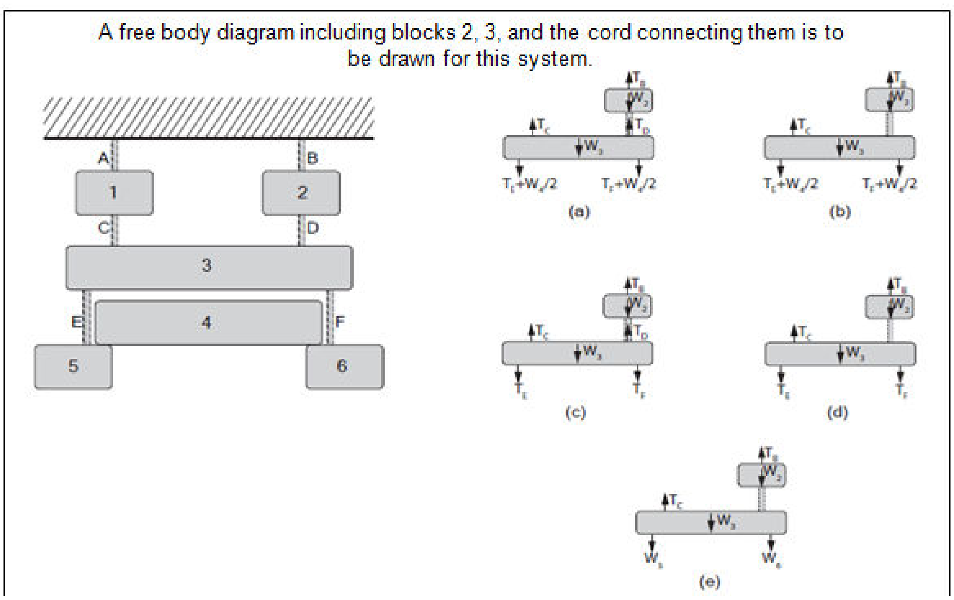

Figure 2 – Sample problem: Item #1 from CATS (with permission of Dr. Steif)

Furthermore, the designer of CATS established that this item encompasses three common errors from the list of conceptual errors presented in Appendix 3. These errors include:

- Error 1 - leave a force off the free body diagram when it should be acting.
- Error 4 - drawing a force as acting on the body in the FBD, even though that force is exerted by a part which is also included in the FBD.
- Error 5 - drawing a force as acting on the body of the FBD, even though that force does not act directly on the body.

Based on this information, the item was then analyzed to determine the requirements for students to select the appropriate FBD from the alternatives provided. For each of the instrument items, two questions were answered: (1) what concepts does an examinee require for choosing the correct answer, and (2) what might cause an examinee to choose one of the distractors?

The answer to the first question for item 1 is very simple. In this type of question, examinees should know how to represent the elements asked in the problem. For this question, only blocks 2 and 3 and the rope connecting them should appear in the diagram. Also, an examinee should indicate the weight of only the blocks included on the diagram (blocks 2 and 3) and the tension on the ropes that connect to blocks 2 and 3 only. A lack of understanding from any of the concepts just explained will cause an examinee to choose the incorrect answer.

Therefore, for an examinee to answer correctly item 1, he/she should have an understanding of the representation of weights. This cognitive attribute is directly related to errors 4 and 5 and will be referred to as weight on block (attribute 4). A misconception of this attribute will cause an examinee to select alternatives a, b, and e. The second cognitive attribute for this item will be referred to as tension in ropes (attribute 6) and is directly related to the ability of an examinee to represent this type of force (tension) on a FBD. This attribute is directly related with error 1. A misconception of this cognitive attribute will lead an examinee to choose alternative c. Thus, the correct answer for this item is alternative d. Therefore, item 1 requires the understanding of attributes 4 (weight on blocks) and 6 (tension on ropes).

To validate the results of this analysis, other experts in Statics followed the same process. They were asked to answer the two questions presented above for each of the items. A total of three groups of questions (9 items) from the inventory were assigned arbitrarily to each one. Two of the volunteers are engineering faculty from another university with over 20 years of experience teaching Statics. The third volunteer has over 10 years of experience as a structural engineer. Their analysis was revised and compared to check if there were differences. Very few discrepancies were found and discussed until 100% agreement was reached. The discussion led to the generation of a set of cognitive attributes, represented in Appendix 4.

Once the SCA were identified for all 27 items, they were discussed with Dr. Steif until consensus was reached10. In general, disagreements arose from either misinterpretation of what was asked on an item, or what was needed to choose any of the alternatives. This conversation led to the development of a new representation of items and SCA which is presented in Appendix 5. Once agreement was reached according to Dr. Steif's experience, not only as a developer but also as a user of CATS with his courses; the next step consisted on naming each of the attributes accordingly for ease of reference. Figure 3 presents the representation of the SCA used to develop the initial Q-matrix.

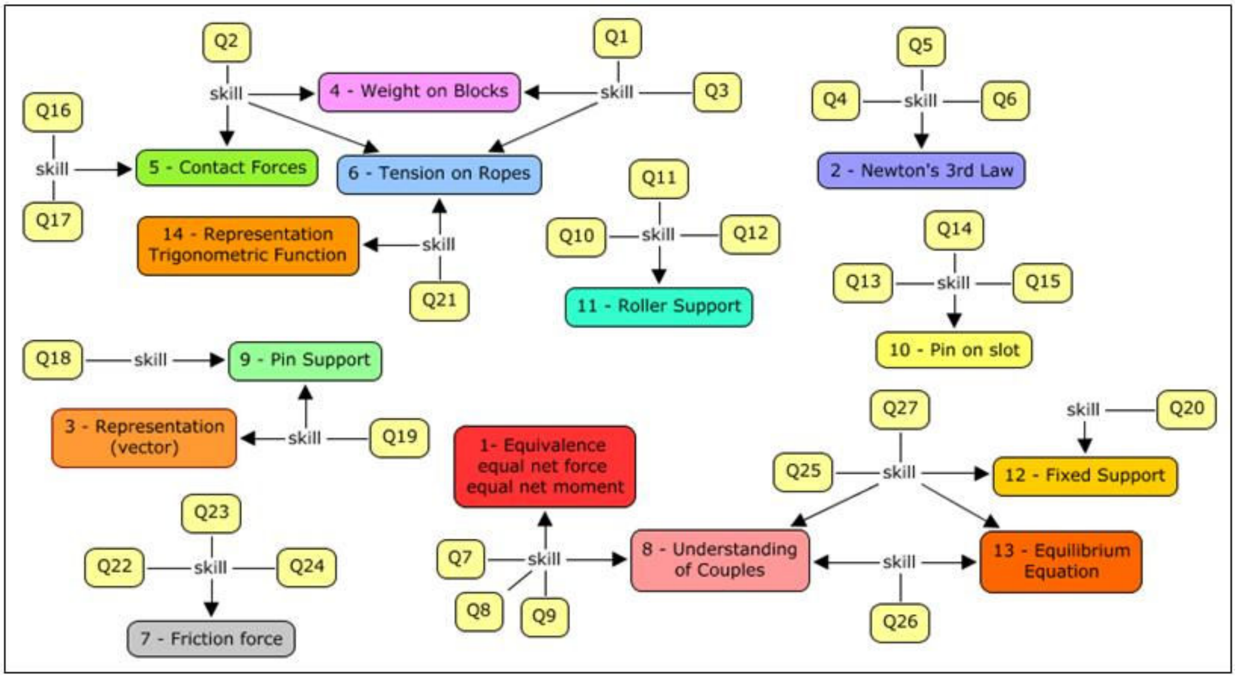

Figure 3 – Cognitive attributes identified for each item

3. Organize the cognitive attributes (facets) within each cluster – facet clusters,

Once the SCA were identified for each of the items, a relationship between the item concepts (27), the concept clusters (4), and conceptual errors (11) was established. Specifically, each SCA relates to an item(s) from CATS (Appendix 6), which also relates to a statics concept cluster(s) and common error(s) (Appendix 7). Mapping all of these tables resulted in Appendix 8. This information allowed the researchers to rank the SCA based on students' responses to CATS items (refer to the next section).

4. Rank facet clusters with respect to how problematic they are.

Minstrell's framework[11] establishes that problematic ideas in a cluster of facets can be identified according to a combination of students' difficulties, teachers' perceptions, and classroom observations. Therefore, SCA identified for the Q-matrix needed to be ranked according to how problematic they were. This was achieved by analyzing participants' responses and identifying the items the majority of the students answered incorrectly, resulting on items 4, 16, 17, 22, 23, 24, and 26 (refer to Appendix 9). The percentage of students, from the sample studied, that got the correct answer for these items were: 25%, 28%, 26%, 29%, 29%, 33%, and 16% respectively. These items are expected to be the most problematic and therefore, the corresponding cognitive attributes should be problematic as well. The cognitive attributes that correspond to these items are: 2 – *Newton's 3rd* Law (item 4), 5 – contact forces (items 16 and 17), 9 – pin support (items 16 and 17), 7 – friction force (items 22, 23, and 24), 8 – couples (item 26) and 13 – equilibrium (item 23).

Once the problematic cognitive attributes were identified they needed to be ranked. This was done by averaging the percentage of correct response of the corresponding items and ranking the attributes accordingly (refer to Table 1). From the results, it is expected attributes 8 (couples) and 13 (equilibrium) to be the most problematic, followed by attribute 2 (*Newton's 3rd* Law), then attributes 5 (contact force) and 9 (pin support), and lastly attribute 7 (friction force). The remaining attributes are expected to be less problematic.

Table 1. Problematic cognitive attributes

Item	% correct response	Cognitive Attribute	Average % correct response	Rank
4	25%	2 – Newton's 3rd Law	25%	2
16	28%	5 – contact forces 9 – pin support	27%	3
17	26%			
22	29%	**7 – friction force**	**30%**	4
23	29%			
24	33%			
26	16%	8 – couples 13 – equilibrium	16%	1

Conclusions

The objective of this study was to identify the SCA necessary for an examinee to answer correctly an item. The completion of this phase required two questions to be answered: (1) what are the cognitive attributes required for each of the items in CATS and (2) how can these attributes be determined? Each question will be addressed individually on the following sections.

What are the cognitive attributes required for each of the items in CATS

The objective of the first question was to generate the initial Q-matrix that would allow the researcher to apply the Fusion Model to CATS to determine: (1) if the model can be applied to CI's and (2) CATS cognitive capability[5]. The SCA identified were: equivalence, Newton's Third Law, representation of vectors, weight on blocks, contact forces, tension in ropes, friction force, couples, pin support, pin on slot, roller support, fixed support, equilibrium, representation of forces. A description of each is presented in Appendix 6. (Refer to Appendix 10 for a graphical representation of the Q-matrix.)

How can the process of determining cognitive attributes be generalized to work for other concept inventories?

The objective of this question was to establish a procedure that could be repeated with other CI's (refer to Figure 4). A four step procedure was established based on Minstrell's work (refer to Figure 1) and the research done by the developer of CATS, Dr. Steif. The first step consisted of identifying the domain cluster concepts. In the case of statics, Dr. Steif identified four main clusters (refer to Appendix 2). He used textbooks, course syllabus, textbook authors, and experienced faculty in the domain (statics). Other developers of CI's have used other techniques such as Delphi processes, focus groups among others[5]. The second step – identification of cognitive attributes – consisted on identifying for each item in the instrument the concepts required for students to choose the correct answer. Also we considered what common errors students would have to choose one of the distractors. A cognitive attribute(s) resulted from the analysis. The results were validated with content experts. Next step, step 3, consisted of organizing the cognitive attributes with the cluster concepts. In our case we mapped each of the SCA identified in step 2 with the concept clusters (4) identified in step 1, statics common errors (11), and the item concepts (9). The final step consisted of ranking the cognitive attributes according to difficulty level based on students' responses to the inventory items. The most difficult attributes were ranked according to difficulty resulting on six cognitive attributes (refer to Table 1).

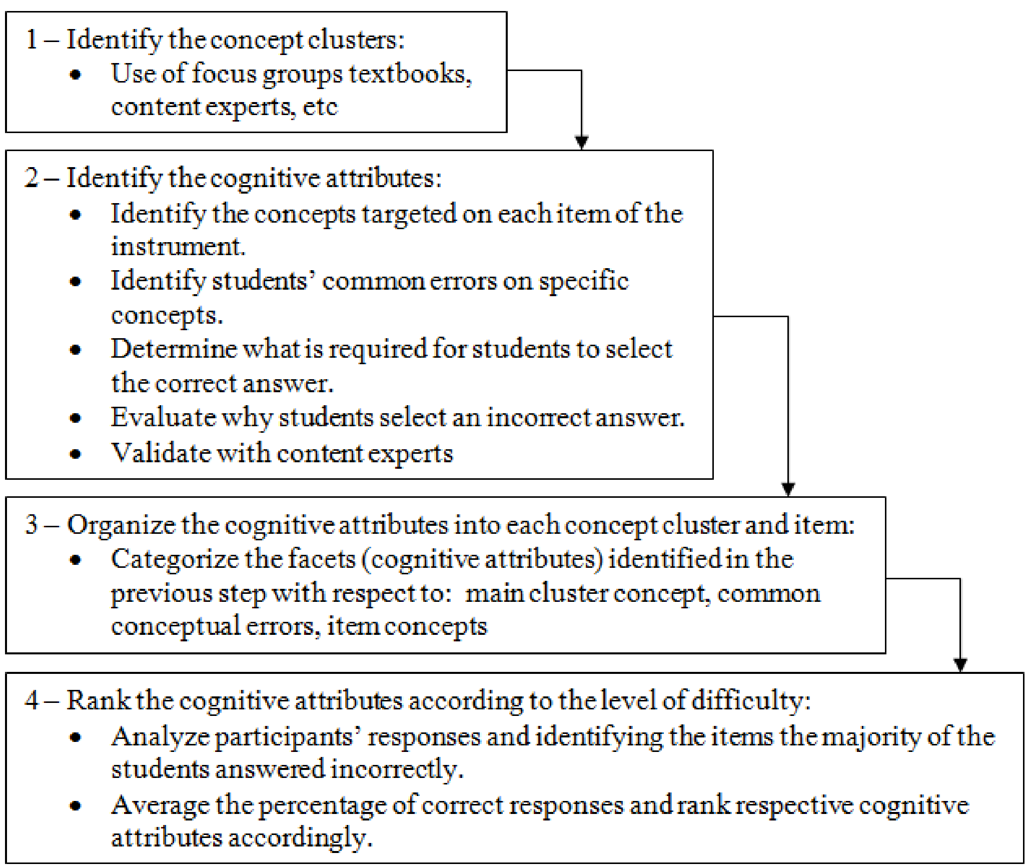

Figure 4 – Procedure applied to identify SCA

The cognitive attributes identified for the Q-matrix were consistent with Minstrell's work on facets of understanding, or students' ideas about a concept and how they make meaning of it. Similar to the facets in Minstrell's work, the cognitive attributes were identified for each item according to the problem stated and the different distractors. Identification of the SCA provide additional diagnostic information to instructors because instructors will know not only which concepts the students find more difficult, but also what specific attributes contribute to making the concept difficult. With this added information, instruction can be targeted to those specific attributes or concepts. Various faculty members with expertise teaching Statics (including the developer of the instrument) were consulted to bolster reliability of results and maintain objectivity during the process.

Implications

The contribution made by this study can be identified within two main areas: the field of engineering education research, and the application of the Fusion Model. Each one will be discussed in the following sections.

Field of Engineering Education Research

One would expect that other CI's that have been designed similarly to CATS would also be appropriate for diagnostic assessment. Listed below are two recommendations about CI development that should improve the success of applying the Fusion Model to CI's.

1. Categorizing questions according to the concepts and common errors is helpful in identifying the cognitive attributes for each of the items that conforms the Q-matrix.

2. Grouping items according to general concepts and having more than one item per concept (and ideally the same number of items per group), helps in creating a Q-matrix that is more balanced (in other words, has the same number of cognitive attributes per items).

Fusion Model

For the study population, five cognitive attributes were identified that are expected to be more difficult among the studied population (refer to Table 1). The attributes include: 1 – equivalence, 2 - *Newton's 3*rd Law, 5 - contact forces, 7 - friction force, and 21 - couples and equilibrium (consisting of attributes 8 and 13). It should be noted that cluster concept # 4 and error #8 occur most frequently and therefore they should be considered in detail. Cluster concept 4 indicates that the equilibrium conditions always pertain to the external force acting directly on a chosen body, and a body is in equilibrium if the summation of forces on it is zero and the summation of moments on it is zero. Similarly, common error 8 relates to not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces. Other studies have shown the difficulty of this error among senior mechanical and civil engineering students[12]. Therefore, the current study validates such findings.

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Appendixes

Appendix 1 – Concepts addressed by each CATS item[3] (p. 206)

Concept	Name	Description	Questions
A	Drawing forces on separate bodies	Identifying forces acting on a subset of a system of bodies.	1, 2, 3
B	Newton's 3rd Law	Forces between two contacting bodies must be equal and opposite.	4, 5, 6
C	Static Equivalence	Static equivalence between forces, couples, and combinations.	7, 8, 9
D	Roller joint	Direction of force between the roller and the rolled surface.	10, 11, 12
E	Pin-in-slot joint	Direction of force between pin and slot of a member.	13, 14, 15
F	Loads at surfaces with negligible friction	Direction of force between frictionless bodies in point contact.	16, 17, 18
G	Representing loads at connections	Representing unknown loads at various connections.	19, 20, 21
H	Limits on friction force	Sorting out implications of equilibrium and Coulomb's Law of friction force.	22, 23, 24
I	Equilibrium	Consideration of both force and moment balance in equilibrium.	25, 26, 27

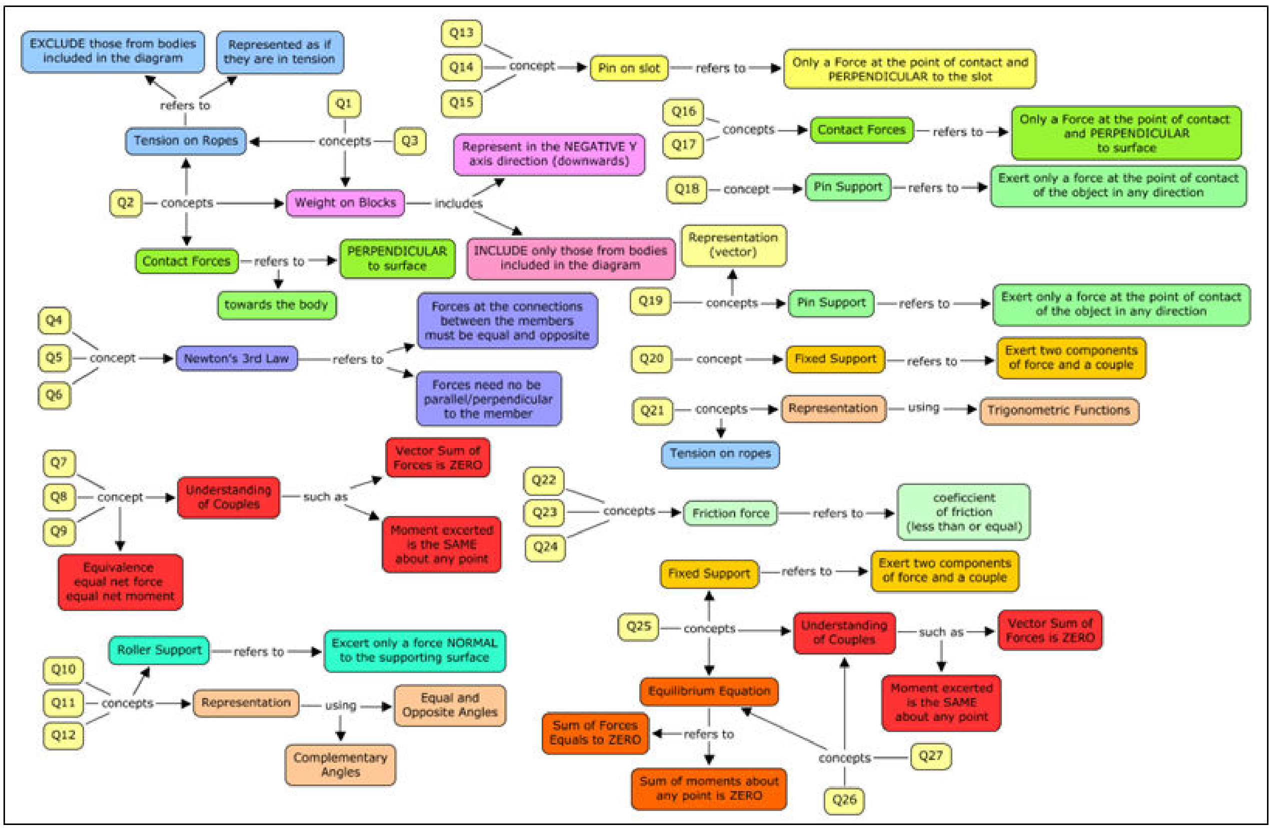
Appendix 2 – Clusters of Concepts for Statics[3] (p. 363)

Cluster	Description
1	Forces are always equal and opposite pairs acting between bodies, which are usually in contact.
2	Distinction must be drawn between a force, a moment due to a force about a point, and a couple. Two combinations of forces and couples are statically equivalent to one another if they have the same net force and moment.
3	The possibilities of forces between bodies that are connected to, or contact, one another can be reduced by virtue of the bodies themselves, the geometry of the connection and/or assumptions on friction.
4	Equilibrium conditions always pertain to the external force acting directly on a chosen body, and a body is in equilibrium if the summation of forces on it is zero and the summation of moments on it is zero.

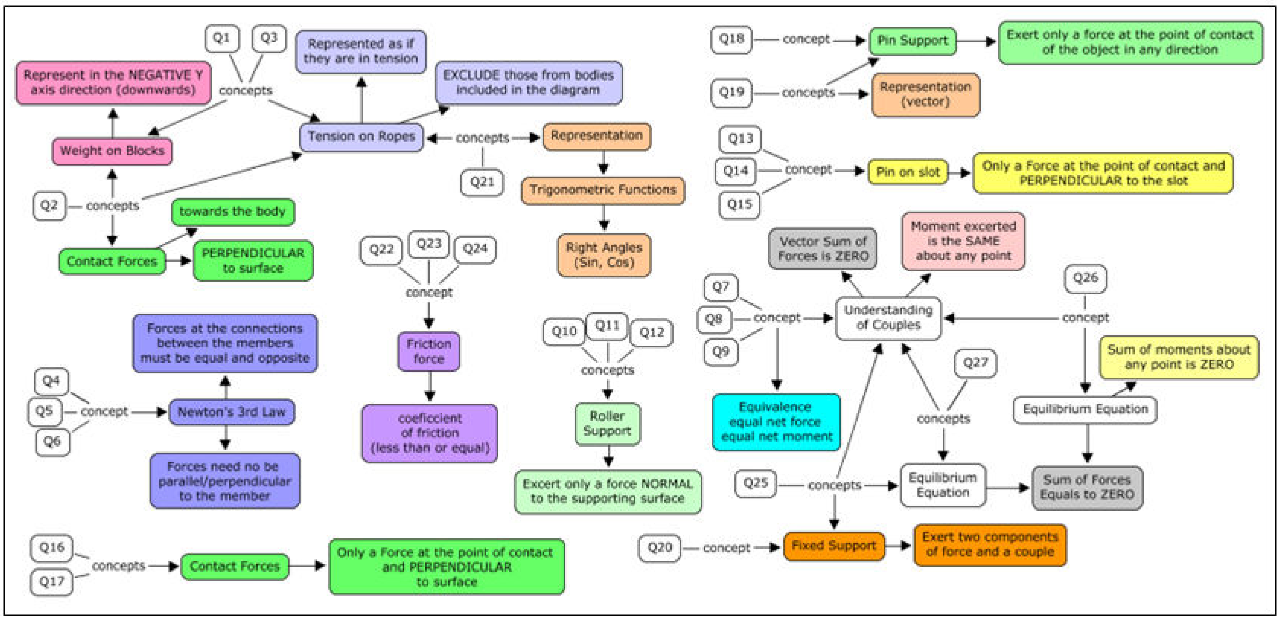
Appendix 3 – Conceptual Errors in Statics[3] (p. 364)

Error	Description
1	Failure to be clear as to which body is being considered for equilibrium.
2	Failure to take advantage of the options of treating a collection of parts as a single body, dismembering a system into individual parts, or dividing a part into two.
3	Leaving a force off the free body diagram (FBD) when it should be acting.
4	Drawing a force as acting on the body in the FBD, even though that force is exerted by a part which is also included in the FBD.
5	Drawing a force as acting on the body of the FBD, even though that force does not act directly on the body.
6	Failing to account for the mutual (equal and opposite) nature of forces between connected bodies that are separated for analysis.
7	Ignoring a couple that could act between two bodies or falsely presuming its presence.
8	Not allowing for the full range of possible forces between connected bodies, or not sufficiently restricting the possible forces.
9	Presuming a friction force is at the slipping limit (μN), even though equilibrium is maintained with a friction force of lesser magnitude.
10	Failure to impose balance of forces in all directions and moments about all axes.
11	Having a couple contribute to a force summation or improperly accounting for a couple in a moment summation.

Appendix 4 – Graphical representation of CATS' initial set of cognitive attributes



Appendix 5 – Resultant representation of CATS' cognitive attributes (after conversations with Dr. Steif)



Appendix 6 – Description of initial cognitive attributes

Cognitive Attribute	Name	Description	Items
1	Equivalence	Static equivalence between forces, couples, and combinations	7-9
2	Newton's 3rd Law	Forces between two contacting bodies must be equal and opposite	4-6
3	Representation of Vectors	Representation of forces according to the geometry of the representation (equal and opposite angles).	19
4	Weight on Blocks	Identifying forces acting on the corresponding blocks.	1-3
5	Contact Forces	Direction of force between frictionless bodies in point of contact.	2, 16-18
6	Tension in Ropes	Identifying forces on the corresponding ropes.	1-3, 21
7	Friction Force	Implication of equilibrium and Columbus Law of friction force (the force must be less than or equal to the coefficient of friction)	22-24
8	Couples	Sum of forces must be equal to zero. The moment exerted is the same about any point.	7- 9, 25-27
9	Pin Support	Representation of pin support (two forces in the x-y direction).	16-19
10	Pin on Slot	Representation of pin-on-slot support (same as roller support).	13- 15
11	Roller Support	Representation of roller support (one force perpendicular to the contact surface).	10- 12
12	Fixed Support	Representation of fixed support (two forces in the x-y direction and a moment).	20, 25
13	Equilibrium	Consideration of both force ($\sum F = 0$) and moment ($\sum M = 0$) balance in equilibrium.	25- 27
14	Representation of Forces	Representation of forces as vectors (Pythagoras theorem)	21

Appendix 7 – Relationship between CATS items and statics' cluster concepts and conceptual errors.

Item	Concept	Name	Cluster Concept	Common Error
1 – 3	A	Drawing forces on separate bodies	1, 3	3, 4, 5, 6, 8
4 – 6	B	Newton's 3rd Law	1	6
7 – 9	C	Static equivalence	2, 4	7, 10, 11
10 – 12	D	Roller joint	3	2, 3, 8
13 – 15	E	Pin-in-slot joint	3	2, 3, 8
16 – 18	F	Load at surfaces with negligible friction	3	9
19 – 21	G	Representing loads at connections	1, 3	2, 6, 8
22 – 24	H	Limits on friction force	3	9
25 – 27	I	Equilibrium	4	10, 11

Appendix 8 – Relationship between the cognitive attributes, CATS, and Statics conceptual frameworks

Cognitive Attribute (facet)	Name	CATS		Statics Frameworks	
		Items	Concept	Cluster Concept	Conceptual Error
1	Equivalence	7-9	C	2, 4	2
2	Newton's 3rd Law	4-6	B	1, 4	2, 6, 8
3	Representation of Vectors	19	G	1, 3, 4	3
4	Weight on Blocks	1-3	A	1, 3, 4	1, 2, 3, 4, 5
5	Contact Forces	2, 16-18	A, F	1, 3, 4	2, 3, 8
6	Tension in Ropes	1-3, 21	A, G	1, 3, 4	1, 2, 3, 4, 5
7	Friction Force	22-24	H	1, 3, 4	9
8	Couples	7- 9, 25-27	C, I	2, 4	7, 10, 11
9	Pin Support	16-19	F, G	1, 3, 4	1, 2, 3, 8
10	Pin on Slot	13- 15	E	1, 3, 4	2, 3, 8
11	Roller Support	10- 12	D	1, 3, 4	2, 3, 8
12	Fixed Support	20, 25	G, I	1, 3, 4	1, 2, 3, 8
13	Equilibrium	25- 27	I	1, 3, 4	10
14	Representation of Forces	21	G	1, 3, 4	3

Appendix 9 – Percentage distribution of students' responses to CATS (bolded items indicate critical questions)

Item	Correct Answer	Mode	% of responses					Cognitive Attributes
			A	B	C	D	E	
1	D	D	12	11	23	51	3	4, 6
2	E	E	23	12	8	15	42	4, 5, 6
3	C	C	8	15	59	11	7	4, 6
4	**D**	B	19	26	13	**25**	16	**2**
5	D	D	29	8	11	44	8	2
6	D	D	25	11	13	33	18	2
7	E	E	10	30	8	21	32	1, 8
8	B	B	30	35	11	9	15	1, 8
9	D	D	11	22	13	37	17	1, 8
10	C	C	9	5	68	11	7	11
11	C	C	4	5	78	5	8	11
12	C	C	9	5	58	23	5	11
13	B	B	5	70	4	15	6	10
14	B	B	6	70	12	5	7	10
15	E	E	4	14	2	7	74	10
16	**D**	C	13	16	40	**28**	2	**5, 9**
17	**D**	C	16	11	45	**26**	2	**5, 9**
18	C	C	18	14	58	7	3	5, 9
19	A	A	63	4	5	22	5	3, 9
20	A	A	71	4	9	9	6	1
21	A	A	52	18	6	16	8	6, 14
22	**B**	E	4	**29**	16	14	37	**7**
23	**D**	E	2	9	13	**29**	47	**7**
24	**B**	D	7	**33**	14	41	5	**7**
25	D	D	7	8	8	62	15	8 12, 13
26	**B**	D	28	**16**	26	29	1	**8, 13**
27	A	A	49	3	16	22	11	8, 13

Appendix 10 – Initial Q-matrix

Items	Cognitive Attributes													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	1	0	1	0	0	0	0	0	0	0	0
2	0	0	0	1	1	1	0	0	0	0	0	0	0	0
3	0	0	0	1	0	1	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	1	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	1	0	0	0	0	0	0
8	1	0	0	0	0	0	0	1	0	0	0	0	0	0
9	1	0	0	0	0	0	0	1	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0
12	0	0	0	0	0	0	0	0	0	0	1	0	0	0
13	0	0	0	0	0	0	0	0	0	1	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0
16	0	0	0	0	1	0	0	0	1	0	0	0	0	0
17	0	0	0	0	1	0	0	0	1	0	0	0	0	0
18	0	0	0	0	1	0	0	0	1	0	0	0	0	0
19	0	0	1	0	0	0	0	0	1	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	1	0	0
21	0	0	0	0	0	1	0	0	0	0	0	0	0	1
22	0	0	0	0	0	0	1	0	0	0	0	0	0	0
23	0	0	0	0	0	0	1	0	0	0	0	0	0	0
24	0	0	0	0	0	0	1	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	1	0	0	0	1	1	0
26	0	0	0	0	0	0	0	1	0	0	0	0	1	0
27	0	0	0	0	0	0	0	1	0	0	0	0	1	0