The Exploded View: A Simple and Intuitive Approach to Teaching the Free-body Diagram

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Abstract

One of the most fundamental concepts in all of mechanics is the free-body diagram (FBD) and teaching students how to draw the FBD correctly can be a fairly challenging prospect. In order to facilitate and unify the approach to drawing virtually any free-body diagram, the exploded-view method is presented in this paper. In short, the exploded-view approach is a four step process in which all the external forces and moments acting on the system are drawn, the particles and bodies in the system are separated from one another and any support, the knowns and unknowns are identified and the correct free-body diagram is selected for analysis. Three examples involving equilibrium of particles (2D and 3D) and equilibrium of rigid bodies (2D) are presented in the paper along with a comparison between the exploded-view approach and the methods employed by the authors of two different Statics textbooks. The dependability of this approach compared to traditional methods has been assessed based on its implementation in a section of a Statics class and the results are compared to that of a control group for a common Statics final exam and a Statics assessment in the Strength of Materials class in the following semester. Based on the results, there is a discernible improvement in the scores of students who were taught to analyze the FBD's using the exploded-view approach, even though a more comprehensive assessment is needed to study the veracity of this method in the long run.

Introduction

The free-body diagram (FBD) is perhaps the most fundamental concept in all of mechanics and mechanical engineering. Drawing an FBD which shows the correct external forces and moments acting on a body isolated from the rest of system is a key step in solving virtually any solid mechanics problem. It is for this reason that one of the first major courses for any mechanical or civil engineering student is Statics (or its equivalent), which is almost entirely based around the concept of the FBD. The FBD can be a rather challenging concept to teach, especially to freshmen and sophomore undergraduate students who may not have yet developed a physics and engineering perspective in their approach to solving problems. With this in mind, the *exploded-view* approach, which is a simple and intuitive way to teach the concept of the FBD for virtually any mechanical system, is explored in this paper.

Motivation

The exploded-view approach to free-body diagrams was utilized by the author primarily due to a Spring semester Statics teaching assignment, where many of the students are typically repeating the course and require more attention. It was initially observed that while the traditional approach to teaching FBDs, in which particles or bodies are separated from their supports and attachments, was effective for some students who had already developed the intuition, there were still a significant number of students who struggled to apply the process to different types of problems and would resort to memorizing and patterning similar problems. As such, in order to simplify and streamline the process for students who were struggling, a unified and relatively simple approach to drawing the correct free-body diagrams was incorporated based on the manner in which FBDs for trusses, frames and machines are treated in most Statics textbooks.

Exploded-View Approach to Free-Body Diagrams

The exploded-view approach to free-body diagrams is not a completely new concept and as mentioned previously, it is inspired by the manner in which the FBDs for trusses, frames and machines are typically treated. This approach involves using Newton's 3^{rd} law of motion (for every action, there is an equal and opposite reaction) to express the forces and moments internal to a system as external forces and moments acting on components of that system. The purpose of presenting this approach in this context is to develop a uniform series of steps that allows for the FBDs of two- or three-dimensional particles, single rigid bodies and structures to be correctly diagrammed and analyzed from a mechanics perspective. The steps are outlined below and will be applied to three example static cases.

Step 1: External forces/moments Define a coordinate system and draw every external force and moment, including the weight (if applicable), that is acting on the entire mechanical system. Also, find the equivalent concentrated load for any external distributed forces present in the system.

Step 2: Separate everything! Separate the particles and bodies present in the system from one another and the ground or any other support and draw all of the forces and moments acting on each of the individual entities present in the system, including the bodies, ground and supports. There are several rules of thumb that should be considered during this step:

- *Ropes, cables and strings:* Draw the tension force due to any cable as pointing away from each end, as shown in Figure 1. Do this for every rope or cable (even those that are internal to a particular body). If a cable goes over a pulley, the magnitude of the tension on either side of the pulley will be the same unless friction in the pulley is not neglected.
- *Springs:* If the displacement of the spring with respect to equilibrium is known, identify whether the spring is being stretched or compressed and draw the spring force in opposite directions at each end, as shown in Figure 2.

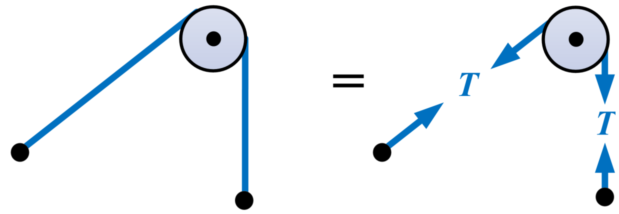

Figure 1 Tension force FBD for cable

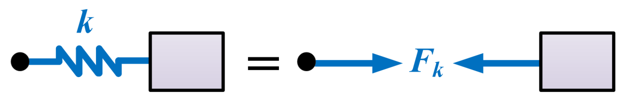

Figure 2 Spring force FBD

- *Two-force members:* Replace a two-force member (a rigid member that is attached to other bodies only at its two ends with no other external force acting on it) by outward-pointing forces of the same magnitude at each end and oriented along the imaginary line that connects the two ends, as shown in Figure 3.

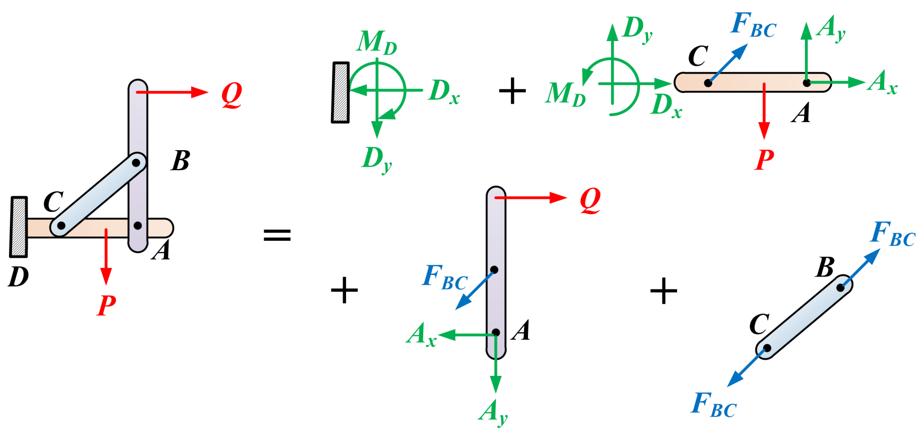

Figure 3 Two-force member FBD

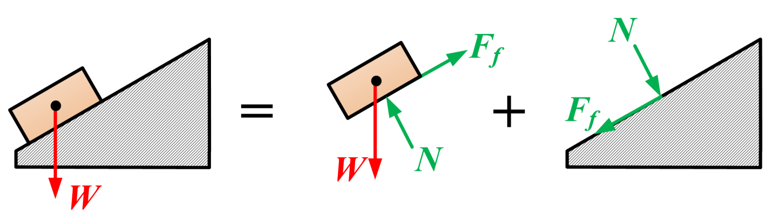

Figure 4 Contact forces FBD

- *Contact forces:* Contact forces consist of the normal force that pushes against both surfaces of contact and the friction force that acts against two surfaces moving against one another, as shown in Figure 4. The friction force acts opposite to the direction of impending motion.

- *Internal forces of multi-force members:* Cut the multi-force member that is of interest in two and draw the axial, shear and bending moments in opposite directions on each end that results from the cut, as shown in Figure 5.

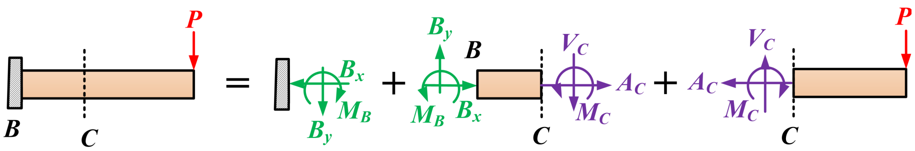

Figure 5 FBD of internal forces in a member

- *Supports:* For any two- or three-dimensional problem involving rigid bodies, the correct treatment of the supports is vital since the supports keep the bodies or structure in place. Reaction forces or moments are generated if the motion of a rigid body is constrained along or about a certain direction. When separating the support from a body, reaction forces and moments that are equal in magnitude and opposite in direction should be drawn on the body and the support, as shown in Figure 6.

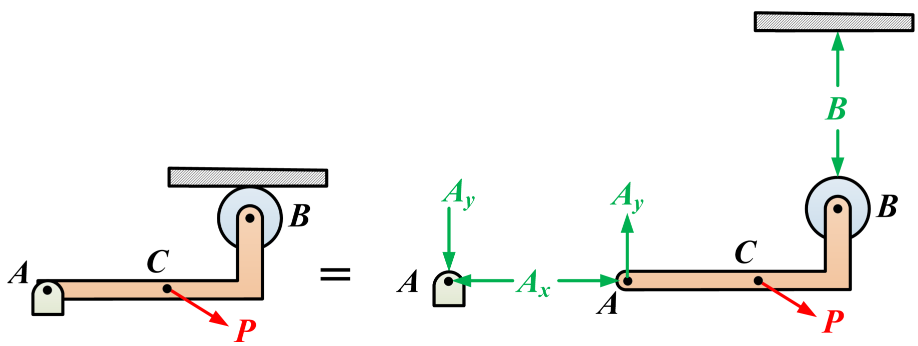

Figure 6 FBD of supports

- It is important to note that in many instances the supports themselves can be further analyzed. For example, the pin support can be separated from a pinned pulley and the reaction forces required to keep the pinned pulley in place can be obtained from equilibrium equations. However, if a problem does not explicitly ask for this reaction force, the pinned pulley should be treated as one support and not be separated further.

It is important to reiterate that a student is not expected to memorize these rules of thumb and the best way to learn them is by solving practice problems that focus on each case.

Step 3: Identify the knowns and the unknowns After the free-body diagram is completed, the magnitude and direction of all the forces/moments in the problem should either be known or be determined from Newton's/Euler's 2nd law of motion. All of the knowns and the unknowns should be clearly identified by the student.

Step 4: Choose the correct free-body diagram If the previous steps are followed correctly, there will be two or more FBDs present. To check to see if the free-body diagrams have been drawn correctly, one can "add" all of the FBDs together and the result should be identical to the figure in Step 1. In order to apply the equilibrium equations, one of the FBDs should be selected to be analyzed. There are two simple rules of thumb that should be taken into account when choosing the FBD:

- Neglect the ground and the supports.
- Start with the FBD with the least number of unknowns. If two FBDs have the same number of unknowns, it is usually more convenient to start with the one with fewer external forces or moments.

Examples

Example 1: Equilibrium of particles (2D) Knowing that the traffic signal at *B* in Figure 7 weighs 200 N, determine the weight of the signal at *C*.

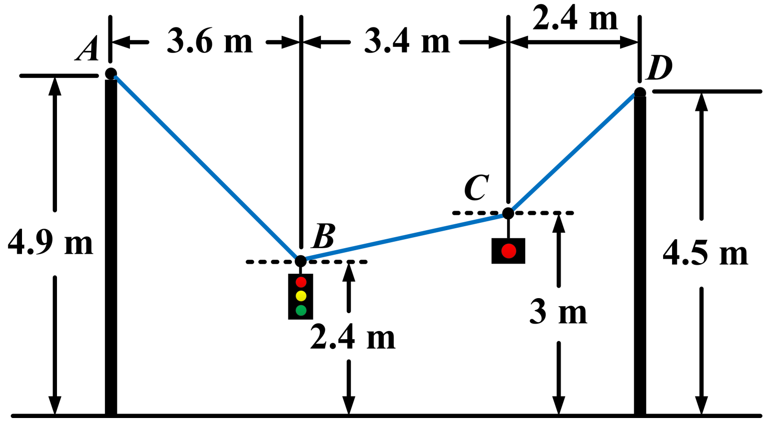

Figure 7 Equilibrium of 2D particles example

Example 1 Solution: The exploded-view approach can be utilized to solve this example. *Step 1*, which depicts all the external forces (two weights in this case) acting on the entire system, is on left and *Step 2*, in which all the forces in the exploded-view system are shown, is on the right of the equal sign in Figure 8. As a check, the sum of the individual forces in *Step 2* should yield the diagram in *Step 1*, which is true in this case. In *Step 3*, all of the known and unknown values for the magnitude and direction of the forces in the problem should be determined. For the tension forces, the angles shown in Figure 8 can be determined. From the given geometry,

$$\alpha = \tan^{-1}\left(\frac{4.9\text{ m} - 2.4\text{ m}}{3.6\text{ m}}\right) = 34.78^\circ \tag{1}$$

$$\beta = \tan^{-1}\left(\frac{3\text{ m} - 2.4\text{ m}}{3.4\text{ m}}\right) = 10^\circ \tag{2}$$

$$\gamma = \tan^{-1}\left(\frac{4.5\text{ m} - 3\text{ m}}{2.4\text{ m}}\right) = 32^\circ \tag{3}$$

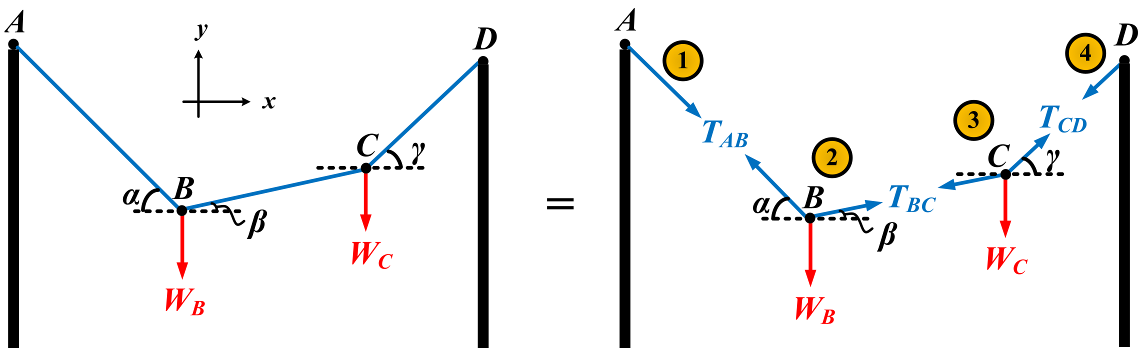

Figure 8 FBD of 2D equilibrium of particle example

The weight of the traffic signal at *B* is given in the problem statement as $W_B = 500$ N. Therefore, T_{AB}, T_{BC}, T_{CD} and W_C are the unknowns in this problem. To determine these forces, the correct FBD must be selected (*Step 4*). In Figure 8, segments 1 and 4 should be neglected since they are connected to supports. Between FBDs 2 and 3, the one that has the least number of unknowns should be selected. FBD 2 has two unknowns and FBD 3 has three unknowns. Therefore, FBD 2 is analyzed first.
Applying force equilibrium in the *x*-direction yields:

$$-T_{AB}\cos\alpha + T_{BC}\cos\beta = 0$$

$$T_{BC} = T_{AB}\frac{\cos\alpha}{\cos\beta} \tag{4}$$

Applying force equilibrium in the *y*-direction and substituting (4) results in:

$$T_{AB}\sin\alpha + T_{BC}\sin\beta - W_B = 0$$

$$T_{AB} = \frac{W_B}{\sin\alpha + \tan\beta\cos\alpha} = \frac{200\ \mathrm{N}}{\sin 34.78^\circ + \tan 10^\circ \cos 34.78^\circ} = 277.76\ \mathrm{N} \tag{5}$$

Substituting (5) into (4) yields:

$$T_{BC} = \frac{W_B}{\tan\alpha\cos\beta + \sin\beta} = \frac{200\ \mathrm{N}}{\tan 34.78^\circ \cos 10^\circ + \sin 10^\circ} = 231.76\ \mathrm{N} \tag{6}$$

Applying force equilibrium in the *x*-direction for FBD 3 yields:

$$-T_{BC}\cos\beta + T_{CD}\cos\gamma = 0$$

$$T_{CD} = T_{BC}\frac{\cos\beta}{\cos\gamma} = (231.76\ \mathrm{N})\frac{\cos 10^\circ}{\cos 32^\circ} = 269.04\ \mathrm{N} \tag{7}$$

Applying force equilibrium in the *y*-direction and substituting (7) results in:

$$T_{CD}\sin\gamma - T_{BC}\sin\beta - W_C = 0$$

$$W_C = T_{CD}\sin\gamma - T_{BC}\sin\beta = (269.04\ \mathrm{N})\sin 32^\circ - (231.76\ \mathrm{N})\sin 10^\circ = 102.33\ \mathrm{N} \tag{8}$$

Therefore, the weight of the traffic signal at *C* is $\boxed{102.33\ \mathrm{N}}$.

Example 1: Comparison of solution methods This problem is similar to 2.48 in Vector Mechanics for Engineers by Beer, et al.[1] The solution approach posed by the authors is to draw the FBD of point *B*, solve for the tension *BC*, draw the FBD of point *C* and solve for the weight W_C. While this is perfectly valid and very similar to what the steps in the exploded-view approach lead to, there is no explanation on why point *B* was the first FBD to be analyzed or how the internal forces look like for the whole system. The exploded-view, on the other hand, provides a logical, physics-based step-by-step methodology that can be used by students struggling to understand how to approach this problem.

Example 2: Equilibrium of rigid bodies (2D) Determine the minimum mass *m* required to cause loss of contact between the wall and the uniform rod of mass M at point *A*.

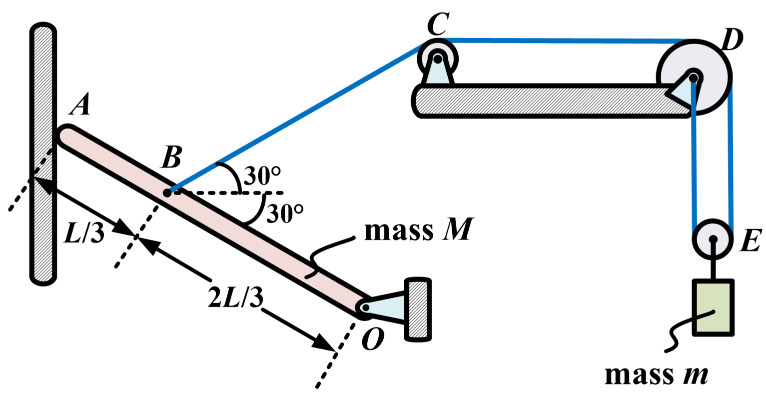

Figure 9 Equilibrium of 2D rigid body example

Example 2 Solution: The exploded-view approach can also be utilized to solve this example. Figure 10 depicts *Step 1* with all the external forces (two weights in this case) acting on the entire system and *Step 2* with all the forces in the exploded-view system. As a check, the sum of the individual forces in *Step 2* should yield the diagram in *Step 1*, which is true in this case.

In *Step 3*, all of the known and unknown values for the magnitude and direction of the forces in the problem should be determined. The direction of every force is known. The magnitude of $N_A = 0$ when the rod loses contact with the wall at point *A*. The magnitudes of the forces O_x, O_y, T and mg are unknown. To determine these forces, the correct FBD must be selected (*Step 4*). In Figure 10, segments 1, 3 and 4 should be neglected since they are connected to the ground/supports. Between FBDs 2 and 5, the one that has the least number of unknowns should be selected. FBD 2 has three unknowns and FBD 5 has two unknowns. Therefore, FBD 5 is analyzed first.

Applying force equilibrium in the *y*-direction yields:

$$2T - mg = 0$$

$$T = \frac{mg}{2} \tag{9}$$

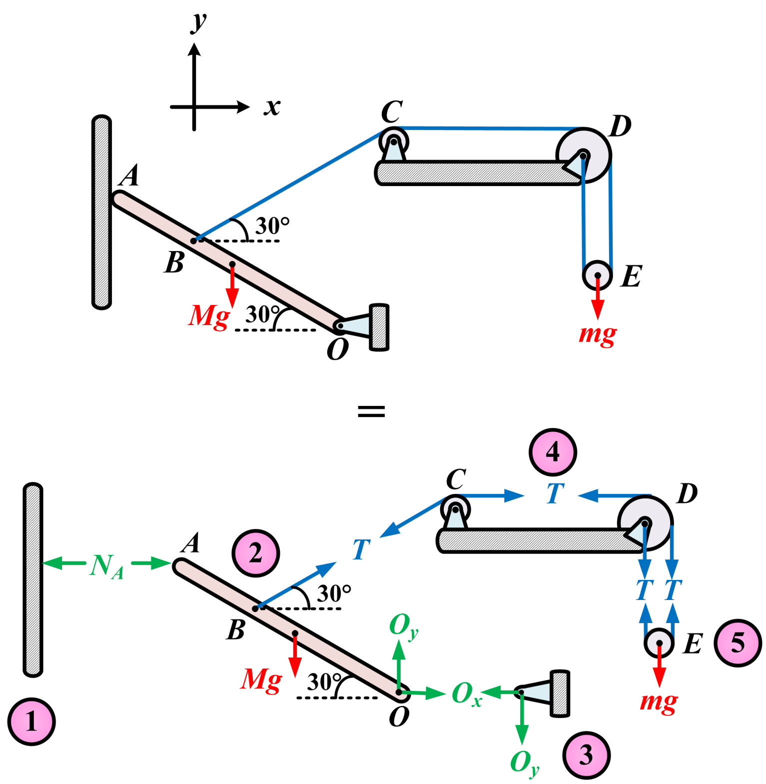

Figure 10 FBD of 2D equilibrium of rigid body example

For FBD 2, applying moment equilibrium about point O results in:

$$-N_A\left(L\sin 30°\right) - T\cos 30°\left(\frac{2L}{3}\sin 30°\right) - T\sin 30°\left(\frac{2L}{3}\cos 30°\right) + Mg\left(\frac{L}{2}\cos 30°\right) = 0$$

$$-\frac{T}{3} + \frac{Mg}{4} = 0 \tag{10}$$

Substituting (9) into (10) yields:

$$m = 1.5M \tag{11}$$

Therefore, the mass m should be at least $\boxed{1.5M}$ to cause the rod to lose contact with the wall at point A.

Example 2: Comparison of solution methods This is similar to Problem 3/19 in Engineering Mechanics by Meriam and Kraige[2]. The solution approach posed by the authors is to draw the FBD of point E, solve for the tension in the cable, draw the FBD of the rod and take a moment of the rod about point O to establish a relationship between the normal force at A and the tension in the cable. Once again, this is a correct approach, but especially for problems that involve analyzing multiple free-body diagrams, the exploded-view approach is more advantageous in showing the student the entire picture. It is for this reason that in most Statics textbooks the exploded-view approach is used to analyze trusses, frames and machines.

Example 3: Equilibrium of particles (3D) If each cable can support 600 N, determine the maximum force P before one cable breaks.

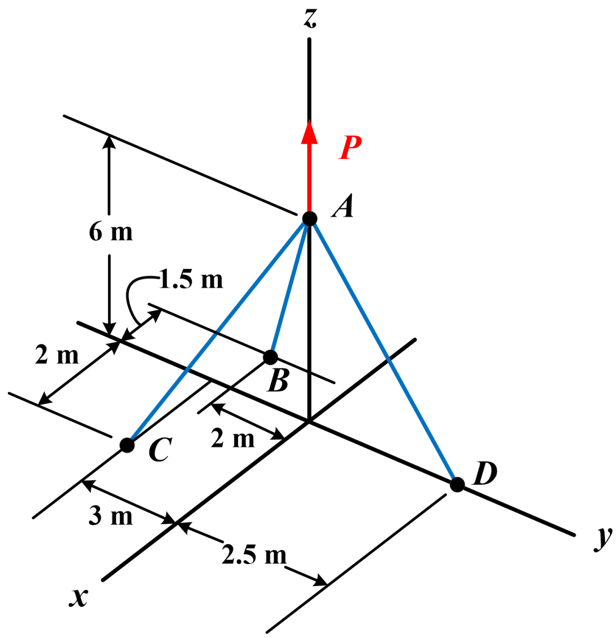

Figure 11 3D equilibrium of particle example

Example 3 Solution: *Step 1* and *Step 2* of the exploded-view approach are depicted in Figure 12. Once again, as a check, the sum of the individual forces in *Step 2* do yield the diagram in *Step 1*. In *Step 3*, all of the known and unknown values for the magnitude and direction of

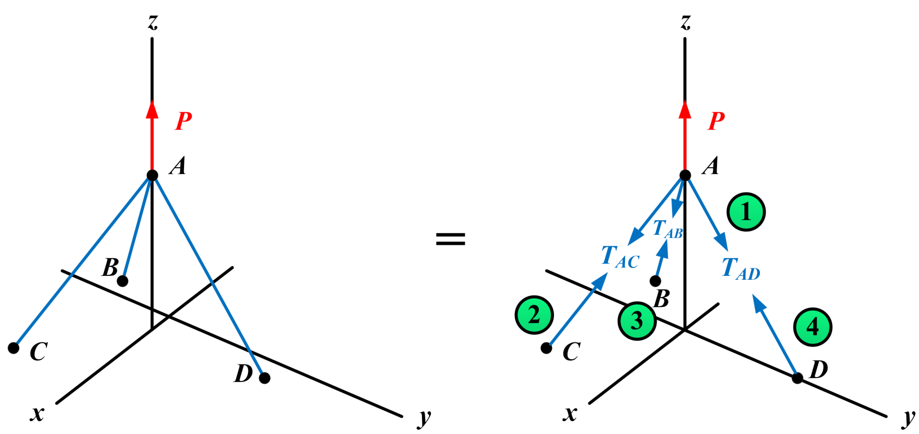

Figure 12 FBD of 3D equilibrium of particle example

the forces in the problem should be determined. The directions of the three forces shown in

Figure 12 can be determined. The coordinates of the points involved are:

$$A = (0, 0, 6)\ \text{m} \tag{12}$$
$$B = (-1.5, -2, 0)\ \text{m} \tag{13}$$
$$C = (2, -3, 0)\ \text{m} \tag{14}$$
$$D = (0, 2.5, 0)\ \text{m} \tag{15}$$

The relevant direction vectors are:

$$\vec{r}_{AB} = B - A = -1.5\hat{\imath} - 2\hat{\jmath} - 6\hat{k}\ \text{m} \tag{16}$$
$$\vec{r}_{AC} = C - A = 2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}\ \text{m} \tag{17}$$
$$\vec{r}_{AD} = D - A = 2.5\hat{\jmath} - 6\hat{k}\ \text{m} \tag{18}$$

From the direction vectors, the relevant unit vectors, which specify the direction of the tension forces, are:

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = -\frac{1.5}{6.5}\hat{\imath} - \frac{2}{6.5}\hat{\jmath} - \frac{6}{6.5}\hat{k} \tag{19}$$
$$\vec{u}_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|} = \frac{2}{7}\hat{\imath} - \frac{3}{7}\hat{\jmath} - \frac{6}{7}\hat{k} \tag{20}$$
$$\vec{u}_{AD} = \frac{\vec{r}_{AD}}{|\vec{r}_{AD}|} = \frac{2.5}{6.5}\hat{\jmath} - \frac{6}{6.5}\hat{k} \tag{21}$$

Therefore, the relevant force vectors in the problem are:

$$\vec{T}_{AB} = T_{AB}\vec{u}_{AB} = -\frac{1.5T_{AB}}{6.5}\hat{\imath} - \frac{2T_{AB}}{6.5}\hat{\jmath} - \frac{6T_{AB}}{6.5}\hat{k} \tag{22}$$
$$\vec{T}_{AC} = T_{AC}\vec{u}_{AC} = \frac{2T_{AC}}{7}\hat{\imath} - \frac{3T_{AC}}{7}\hat{\jmath} - \frac{6T_{AC}}{7}\hat{k} \tag{23}$$
$$\vec{T}_{AD} = T_{AD}\vec{u}_{AD} = \frac{2.5T_{AD}}{6.5}\hat{\jmath} - \frac{6T_{AD}}{6.5}\hat{k} \tag{24}$$
$$\vec{P} = P\hat{k} \tag{25}$$

The unknowns are T_{AB}, T_{AC}, T_{AD} and P. To determine these forces, the correct FBD must be selected (*Step 4*). In Figure 12, segments **2**, **3** and **4** should be neglected since they are connected to supports. Therefore, FBD **1** is analyzed first.
Applying force equilibrium in the *x*-direction yields:

$$-\frac{1.5}{6.5}T_{AB} + \frac{2}{7}T_{AC} = 0$$

$$T_{AB} = \frac{13}{10.5}T_{AC} \approx 1.24T_{AC} \tag{26}$$

Applying force equilibrium in the *y*-direction and substituting (26) results in:

$$-\frac{2}{6.5}T_{AB} - \frac{3}{7}T_{AC} + \frac{2.5}{6.5}T_{AD} = 0$$

$$T_{AD} = \left(\frac{26}{26.25} + \frac{19.5}{17.5}\right) T_{AC} \approx 2.105 T_{AC} \tag{27}$$

Based on (26) and (27), the tensions in the cables are $T_{AD} > T_{AB} > T_{AC}$. Since the tension in cable AD is largest, it will be the first cable to fail. Therefore, to find the maximum value for *P*, the tension in this cable should be set to its maximum value, namely $T_{AD} = 600$ N. From this value and using (26) and (27), one finds that:

$$T_{AD} = 600 \text{ N} \tag{28}$$

$$T_{AC} = \left(\frac{26}{26.25} + \frac{19.5}{17.5}\right)^{-1} T_{AD} = 285.07 \text{ N} \tag{29}$$

$$T_{AB} = \frac{13}{10.5} T_{AC} = 352.94 \text{ N} \tag{30}$$

Applying force equilibrium in the *z*-direction and substituting (28)-(30) results in:

$$-\frac{6}{6.5} T_{AB} - \frac{6}{7} T_{AC} - \frac{6}{6.5} T_{AD} + P = 0$$

$$P = \frac{6}{6.5} T_{AB} + \frac{6}{7} T_{AC} + \frac{6}{6.5} T_{AD} = 1.124 \text{ kN} \tag{31}$$

Therefore, the maximum force P is $\boxed{1.124 \text{ kN}}$.

Example 3: Comparison of solution methods This is similar to Problem 3-49 in Engineering Mechanics (Statics) by Hibbeler[3]. This case has been included here to demonstrate that the exploded-view approach may not have a distinct advantage over the standard textbook approach to FBDs if the problem ultimately involves only one FBD that is considered, even though some students might still prefer to see the whole picture with the forces that are exerted at ends B, C and D. The solution approach, which involves drawing the FBD of point *A*, finding the unit vectors of the tension forces, obtaining the tension vectors and finally applying equilibrium equations to point *A* to solve for *P*, is fairly similar in both sets of approaches.

Assessment

The exploded-view approach to teaching the free-body diagram in the Statics class was first implemented in the Spring 2016 semester at Penn State Behrend. During the Spring 2016 semester, two sections of Statics were offered: *Section A*, which was taught by the author and employed the exploded-view approach to the FBD, and *Section B*, which was taught by a colleague and did not use the exploded-view approach to the FBD. Statistics relating to GPA of the students (calculated before Spring 2016) and the final exam score are provided in Table 1. As the values in the table indicate, the average GPA between the two sections is fairly similar. If the average GPA is used as an indicator of the "strength" of a class (and there can be legitimate discussions on how effective an indicator the GPA actually is), then the two sections are assumed to have been fairly even at the beginning of the Spring 2016 semester.

Table 1 Statistics of *Section A* and *Section B* of Statics in Spring 2016

		Section A	*Section B*
Number of students		39	42
GPA (out of 4)	Average	2.94	2.88
	Standard deviation	0.61	0.53
Final exam score	Average	75.08%	71.23%
	Standard deviation	18.45%	15.68%

For the two sections, a common final exam was held at the same time. Four of the five problems on the exam required drawing free-body diagrams (the other problem involved finding the centroid of a particular shape). Of the aforementioned four problems, *Problem 1* involved the 2D equilibrium of a rigid body, *Problem 2* dealt with finding the forces in truss members, *Problem 3* was a 3D equilibrium of a rigid body and *Problem 4* tested the students on the 2D equilibrium of particles with friction. The results for the two sections are presented in Table 2.

Table 2 Final exam problem comparison between *Section A* and *Section B*

		Section A	*Section B*
Problem 1	Average	83.59%	76.19%
	Standard deviation	21.85%	27.52%
Problem 2	Average	80.17%	75.48%
	Standard deviation	20.89%	25.58%
Problem 3	Average	63.08%	55.71%
	Standard deviation	31.97%	48.05%
Problem 4	Average	64.62%	67.5%
	Standard deviation	33.31%	43.69%

As the results in Table 2 show, the performance of the students who were taught the FBD using the exploded-view approach is significantly better in three of the four problems than the students in *Section B*. The standard deviation in the grades of both sets of students is fairly high, indicating that the scatter in scores is quite significant for all four problems. It is important to reiterate that the values displayed in Tables 1 and 2 relate to only a single instance where the exploded-view approach was implemented. It is expected that more assessment data will be gathered in future semesters so that a better overview of the advantages of this approach will be demonstrated.

An effort was also made to assess student retention of the exploded-view approach to free-body diagrams. On the first day of the Strength of Materials class in the Fall 2016 semester, a Statics assessment of student learning was conducted. Of the 14 questions on the assessment, 9 directly tested drawing free-body diagrams. Only the students who were registered in either *Section A* or

Section B of Statics in Spring 2016 and were enrolled in Strength of Materials in the Fall 2016 semester were evaluated further. Information regarding the GPA's of these students entering the Fall 2016 semester, the results of the free-body diagram portion of the assessment along with their final letter grade converted to numeric form is tabulated in Table 3.

Table 3 Strength of Materials evaluation of Statics *Section A* and *Section B* students

		Section A	*Section B*
Number of students		11	16
GPA (out of 4)	Average	2.80	2.91
	Standard deviation	0.52	0.66
FBD assessment score	Average	76.3%	78.13%
	Standard deviation	12.93%	10.03%
Final class grade (out of 4)	Average	2.51	2.14
	Standard deviation	0.73	1.16

Only one-third of the original Statics class enrolled in Strength of Materials in Fall 2016. This is primarily due to students taking the class in the summer, transferring to the University Park campus, not passing Statics in Spring 2016 or some majors not requiring Strength of Materials. As the values in Table 3 show, the students who registered in Strength of Materials in Fall 2016 from *Section A* had lower GPA's than their *Section B* counterparts. The assessment scores for *Section A* students were slightly lower, but this can be attributed to one outlier grade in *Section A*. It is also interesting to note that the final class grade in Strength of Materials was significantly higher on average for *Section A* than *Section B*, even though the student GPA's were lower on average for *Section A*. It should be noted that two sections of Strength of Materials were offered in Fall 2016 at Penn State Behrend and about half of the students from *Section A* and *Section B* were enrolled in each Strength's section.

As previously noted, the exploded-view approach to teaching free-body diagrams is relatively new at Penn State Behrend and to be able to adequately judge the effectiveness of this method, further assessment of individual Statics classes and higher level classes that involve utilizing free-body diagrams is required. As such, the progress of this group of Statics students will be tracked as they move towards graduation and the performance of current and prospective Statics students will be similarly considered from the Spring 2017 semester onwards.

References

[1] F. P. Beer, E. R. Johnston, and E. R. Eisenberg, *Vector Mechanics for Engineers: Statics*. New York: McGraw-Hill, 8th ed., 2007.

[2] J. L. Meriam and L. G. Kraige, *Engineering Mechanics*. New York: J. Wiley, 7th ed., 2012.

[3] R. C. Hibbeler, *Engineering Mechanics: Statics*. Upper Saddle River, N.J: Pearson, 13th ed., 2013.