Session 1520

The Use of MathCad in a Graduate Level Two-Phase Flow and Heat Transfer Course

Louay M. Chamra, Stephen T. McClain, and Soon-Seng Tang
Department of Mechanical Engineering
Mississippi State University

Abstract

In a graduate level two-phase flow and heat transfer course taught at Mississippi State University (MSU), students were encouraged to use MathCad for their projects and homework. Three example problems, the theory of the solutions, the MathCad solution, and student insights revealed about the problems are presented. The example problems cover two-phase pressure gradient calculations, subcooled boiling heat transfer, and condensation heat transfer. The students found that MathCad was very helpful in solving complex problems. In addition, MathCad helped enhance student understanding of boiling and condensation phenomena.

I. Introduction

The two-phase flow and heat transfer course at MSU is a graduate level course designed to provide students in Mechanical Engineering, Aerospace Engineering, and Chemical Engineering an understanding of the physical phenomena and specific models used in two-phase flow and heat transfer. This course concentrates on liquid-vapor two-phase hydrodynamics, boiling and condensation heat transfer, and pressure gradient and heat transfer models.

Boiling and condensation fundamentals are used in the design of air-conditioning components and power and petrochemical boilers and condensers, as well as many other devices. The implementation of two-phase fundamentals to real-life applications often leads to very complex equations. Solutions of such equations may require iteration and, in certain cases, a computer code to perform numerical integration. MathCad is a powerful tool that can be effectively used in two-phase flow to solve such complex problems.

Three example problems are presented in this paper. The example problems cover two-phase pressure gradient calculations, subcooled boiling heat transfer, and condensation heat transfer.

Proceedings of the 2001 American Society for Engineering Education Annual Conference &Exposition

II. Two-phase Annular Flow Frictional Pressure Gradient Problem

Problem Statement:

Determine the frictional pressure gradient of a steam/water system at 130 C and 2.7011 bar flowing through a 3-cm, vertical, well-insulated tube. The mass flow rate of water is 0.5 kg/s, while the mass flow rate of steam is 0.1 kg/s. For these conditions, assume that the flow is annular and that 40% of the water is entrained in the steam core.

III. Two-phase Annular Flow Frictional Pressure Gradient Solution

To solve the pressure gradient problem, the annular flow with entrainment model of Wallis (Collier and Thome, 1999) is used. To begin the solution, the flow pattern is checked using the flow map of Hewitt and Roberts (Collier and Thome, 1999). The quality is defined as the mass flow rate of vapor divided by the total mass flow rate of the system.

$$x = \frac{W_g}{W_f + W_g} \tag{1}$$

The mass fluxes of the liquid and vapor are defined as the mass flow rates divided by the area of the tube.

$$G_f = \frac{W_f}{A} \quad \text{and} \quad G_g = \frac{W_g}{A} \tag{2a,b}$$

The total mass flux is,

$$G = G_f + G_g \tag{3}$$

The superficial momentum fluxes are needed for use in the flow map of Hewitt and Roberts; these are defined in equations (4a) and (4b).

$$\Phi_f = \rho_f j_f^2 = \frac{G_f^2}{\rho_f} \quad \text{and} \quad \Phi_g = \rho_g j_g^2 = \frac{G_g^2}{\rho_g} \tag{4a,b}$$

The frictional pressure gradient is first estimated using the method of Lockhart and Martinelli (Collier and Thome, 1999). The Martinelli parameter is calculated using,

$$X = \left[\frac{\left(\frac{dP}{dz}\right)_{F,f}}{\left(\frac{dP}{dz}\right)_{F,g}} \right]^{\frac{1}{2}} \tag{5}$$

Inside the brackets of equation (5), the numerator is the pressure gradient associated with the liquid only flowing through the tube, and the denominator is the pressure gradient associated with vapor only flowing through the tube. These pressure gradients are defined in equations (6a) and (6b).

$$\left(\frac{dP}{dz}\right)_{F,f} = 2 f_f \frac{G_f^2}{\rho_f D} \quad \text{and} \quad \left(\frac{dP}{dz}\right)_{F,g} = 2 f_g \frac{G_g^2}{\rho_g D} \tag{6a,b}$$

Similar to the Martinelli parameter, the two-phase multiplier based on the vapor mass flux is described in equation (7).

$$\phi_g^2 = \frac{\left(\frac{dP}{dz}\right)_F}{\left(\frac{dP}{dz}\right)_{F,g}} \tag{7}$$

In terms of the Martinelli parameter, the vapor two-phase multiplier is expressed as

$$\phi_g = \left(1 + CX + X^2\right)^{0.5} \tag{8}$$

The constant C in equation (8) is dependent on the flow regime. For flow regimes of turbulent vapor-turbulent liquid, turbulent vapor-laminar liquid, laminar vapor-turbulent liquid, or laminar vapor-laminar liquid, the value of C is 20, 12, 10, or 5, respectively. The two-phase frictional pressure gradient can then be found using

$$\left(\frac{dP}{dz}\right)_F = \phi_g^2 \left(\frac{dP}{dz}\right)_{F,g} \tag{9}$$

Once the first estimate of the frictional pressure gradient has been made, an iterative procedure is used to find the actual frictional pressure gradient. To begin the iterative procedure, the pressure gradient based on liquid in the film is found using equation (10).

$$\left(\frac{dP}{dz}\right)_{fF} = 2 f_{fF} \frac{G_{fF}^2}{\rho_f D} \tag{10}$$

Then using the frictional pressure gradient, the two-phase multiplier based on the liquid in the film is computed from equation (11).

$$\phi_{fF} = \left[\frac{\left(\frac{dP}{dz}\right)_F}{\left(\frac{dP}{dz}\right)_{fF}}\right]^{\frac{1}{2}} \tag{11}$$

The void fraction, the fraction of the area through which vapor flows through the tube, is defined by the relationship of Lockhart and Martinelli,

$$\alpha = 1 - \frac{1}{\phi_{fF}} \tag{12}$$

In terms of the void fraction, the two-phase multiplier based on the vapor is found using equation (13).

$$\phi_g^2 = \left[\frac{1+75(1-\alpha)}{\alpha^{\frac{5}{2}}}\right]\left(\frac{W_g + e_n W_f}{W_g}\right)\left[1-2\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\rho_g}{\rho_f}\right)\left(\frac{W_{fF}}{W_g}\right)\right]^2 \tag{13}$$

With no relaxation, the new estimate for the frictional pressure gradient is then the product of the two-phase multiplier from equation (13) and the frictional pressure gradient based on the mass flow rate of vapor. If a relaxation is used, the frictional pressure gradient is computed using equation (14).

$$\left(\frac{dP}{dz}\right)_{F,new} = \left(\frac{dP}{dz}\right)_{F,old} + R\left[\phi_g^2\left(\frac{dP}{dz}\right)_{F,g} - \left(\frac{dP}{dz}\right)_{F,old}\right] \tag{14}$$

The iterative process proceeds by returning to equation (11) and calculating a new two-phase multiplier based on the liquid in the film. The process continues until the percentage difference between the new frictional pressure gradient and old frictional pressure gradient is less than a predetermined limit.

IV. Solution Scheme

1. Check that the flow is annular using equations (4a) and (4b).
2. Estimate the frictional pressure gradient using the method of Lockhart and Martinelli.
3. Calculate the frictional pressure gradient associated with the liquid in the annular region only using equation (10).
4. Calculate the two-phase multiplier based on the annular liquid with equation (11) using the annular liquid frictional pressure gradient from step (4) and the current estimate of the frictional pressure gradient.
5. Use The Lockhart and Martinelli relationship provided in equation (12) to find the void fraction.
6. Determine the vapor, two-phase multiplier using equation (13).
7. Calculate the new estimate of the pressure gradient with equation (14).
8. Repeat steps (4) through (7) until pressure gradient converges to a predetermined tolerance.

V. MathCAD Solution

The MathCAD provided in Appendix A, shows that the pressure gradient in the tube is 3301 Pa per meter of tube. The solution subroutine was modified and used to explore what value of relaxation worked best for the solution scheme. Figure A-1 shows the new pressure gradient after each iteration for relaxation values of 1 and 0.5. The process converges very slowly with a relaxation of 1, while it converges rapidly with a relaxation of 0.5.The solution also demonstrates the abilities of MathCad to handle units. The result is presented in correct units even after passing through the iteration program.

VI. Subcooled Boiling Heat Transfer Problem

A double-pipe counterflow heat exchanger is constructed with water flow in the tube side and oil flow in the annulus side.

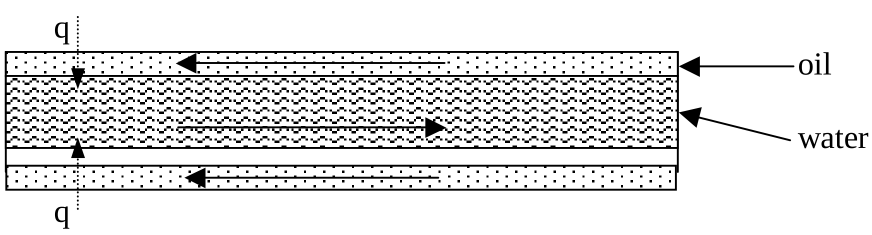

Figure 1. Double Pipe Heat Exchanger with Subcooled Boiling.

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The heat exchanger tubes are made of stainless steel with tube outside diameter of 25.0 mm and wall thickness of 1.0 mm. Heat is transferred from the oil to the water. The water flows at 0.42 kg/s into the heat exchanger at 1.433 bars and 90^oC. It exits the heat exchanger with a temperature 15 degree higher. The convective heat transfer coefficient for water in liquid phase only is found to be 6890 W/m^2-K. The oil has a convective heat transfer coefficient of 5000 W/m^2-K, and it flows into the heat exchanger at 160^oC and 4 bars with a flow rate of 0.67 kg/s. The oil exits the heat exchanger at 140^oC.

The objective of this problem is to determine if subcooled boiling, where the water temperature is measured at 100^oC, exist at a point along the heat exchanger length. The heat flux at this particular point is to be determined.

VII. Subcooled Boiling Heat Transfer Solution

To determine if subcooled boiling exist at a certain point along the heat exchanger length, the heat flux correlations developed by Bergles and Rohsenow (1963) and Rohsenow (1952) are implemented. An analytical solution at the incipient boiling condition, developed by Davis and Anderson (1966), is also adopted to generate the solution.

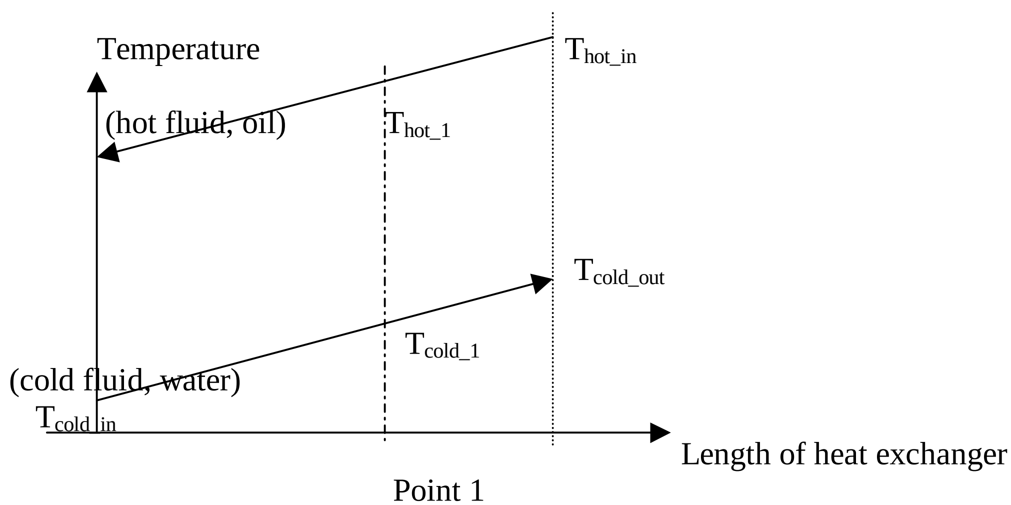

Figure 2. Temperature versus Heat Exchanger Length

Figure 2 shows the temperature change for the cold side (water) and hot side (oil) fluids in the counterflow double-pipe heat exchanger. At point 1, the water temperature is given as 100^oC.

The energy balance equation, which is shown in equation (15), is used to determine the oil temperature at point 1.

$$T_{hot\_1} = \frac{W_{cold} \cdot c_{p\_cold} \cdot \left(T_{cold\_out} - T_{cold\_1}\right) - W_{hot} \cdot c_{p\_hot} \cdot T_{hot\_in}}{-W_{hot} \cdot c_{p\_hot}} \qquad (15)$$

From the definition of convective heat transfer rate, q, the total convective heat transfer rate can be written as

$$q = h \cdot Area \cdot \Delta \ T \qquad (16)$$

To achieve an energy balance, the heat transfer rates for the hot side and the cold side fluids must be equal. For initial calculations, the water is assumed to undergo a subcooled boiling process. Instead of evaluating the heat transfer rate for the water using a single-phase (liquid-phase) convective heat transfer coefficient, two-phase heat transfer coefficient is used. The following equation states that the heat transfer rate of the hot side fluid is equal to the cold side fluid.

$$h_{hot} \cdot \left(T_{hot\_1} - T_w\right) \cdot Area_{hot} = h_{tp} \cdot \left(T_w - T_{cold\_1}\right) \cdot Area_{cold} \qquad (17)$$

With reference to the tube side (cold fluid side) heat transfer surface area, the heat flux can be determined as

$$q'' = h_{hot} \cdot \left(T_{hot\_1} - T_w\right) \cdot \frac{Area_{hot}}{Area_{cold}} \qquad (18)$$

To simply the calculation process, the ratio of the total heat transfer areas can be written as the ratio of the total heat transfer area per unit length of the heat exchanger.

$$q'' = h_{hot} \cdot \left(T_{hot\_1} - T_w\right) \cdot \frac{Area'_{hot}}{Area'_{cold}} \qquad (19)$$

where

$$Area'_{hot} = \pi \cdot OD_{tube} \qquad (m^2/m)$$

$$Area'_{cold} = \pi \cdot \left(OD_{tube} - 2th_{tube}\right) \qquad (m^2/m) \qquad (20a,b)$$

Based on the simplified empirical correlation for partial boiling developed by Bergles and Rohsenow (1963), the heat flux at the partial boiling region can be evaluated using equation (21).

$$q''^2 = q''^2_{FC} + \left(q''_{scb} - q''_{Bi}\right)^2 \tag{21}$$

The first term on the right hand side of equation (21) is the single-phase forced convection heat flux evaluated from the single-phase (liquid-phase) heat transfer coefficient.

$$q''_{FC} = h_{cold\_fo} \cdot \left(T_w - T_{cold\_1}\right) \tag{22}$$

The second and third terms of equation (21) represent the heat flux at subcooled boiling region and incipient boiling region, respectively.

Rohsenow (1952) proposed a correlation, which applied to various liquids, for subcooled boiling region. The correlation is shown as follows:

$$\frac{c_{p\_cold} \cdot \left(T_w - T_{cold\_sat}\right)}{i_{fg}} = C_{sf} \cdot \left[\frac{q''_{scb}}{\mu_f \cdot i_{fg}} \cdot \sqrt{\frac{\sigma}{g \cdot \left(\frac{1}{v_f} - \frac{1}{v_g}\right)}}\right]^{0.33} \cdot \left(\frac{c_{p\_cold} \cdot \mu_f}{k_f}\right)^{1.7} \tag{23}$$

The constant C_{sf} was determined as 0.020 by Bergles and Rohsenow (1963) using the experimental data for water flowing inside horizontal stainless steel pipe.

Solving equation (23) for the subcooled boiling heat flux results in

$$q''_{scb} = \left[\frac{c_{p\_cold} \cdot \left(T_w - T_{cold\_sat}\right)}{i_{fg}} \cdot \left(\frac{k_f}{c_{p\_cold} \cdot \mu_f}\right)^{1.7} \cdot \frac{1}{0.02}\right]^{\frac{1}{0.33}} \cdot \mu_f \cdot i_{fg} \cdot \sqrt{\frac{g \cdot \left(\frac{1}{v_f} - \frac{1}{v_g}\right)}{\sigma}} \tag{24}$$

Bergles and Rohsenow (1963) also developed a correlation for incipient boiling. The correlation is valid only for water at 1 to 138 bars. The correlation is in SI units with pressure, *P*, expressed in bar. The incipient boiling heat flux is determined by

$$q''_{Bi} = \left(\frac{\Delta T_{sat\_i}}{0.556}\right)^{\frac{1}{0.463 \cdot P_{cold}^{0.0234}}} \cdot 1082 \cdot P_{cold}^{1.156} \qquad (25)$$

ΔT_{sat_i} is the wall superheat at the incipient boiling condition and is the minimum wall superheat required to initiate nucleate boiling. Davis and Anderson (1966) carried out analytical solutions for the minimum wall superheat required to initiate nucleate boiling, and they found that

$$\Delta T_{sat\_i} = \left(\frac{8 \cdot \sigma \cdot q''_{Bi} \cdot T_{cold\_sat} \cdot v_{fg}}{i_{fg} \cdot k_f}\right)^{0.5} \qquad (26)$$

The incipient boiling heat flux, q''_{Bi}, is defined as the product of the single-phase heat transfer coefficient and the wall superheat at incipient condition. q''_{Bi} is written as

$$q''_{Bi} = h_{cold\_fo} \cdot \left(\Delta T_{sat\_i}\right) \qquad (27)$$

As a result, the wall superheat at incipient boiling condition can be rewritten as

$$\Delta T_{sat\_i} = \frac{8 \cdot \sigma \cdot T_{cold} \cdot v_{fg}}{i_{fg}} \cdot \frac{h_{cold\_fo}}{k_f} \qquad (28)$$

From all the equations above, the wall temperature, T_w, and the partial boiling heat flux, q'', remain unknown. An iteration process is required to determine the appropriate T_w that provides the equivalent q'' at the hot fluid side and the cold fluid side. In other words, the appropriate T_w is to be determined such that equations (18) and (21) are equal to each other.

VIII. Solution Scheme

1. Define the properties for fluids (hot and cold sides) and the dimensions of the tube.
2. For the given water temperature at point 1, determine the corresponding oil temperature at point 1 (T_{hot_1}) using equation (15).
3. Calculate the total heat transfer area per unit length of heat exchanger at hot and cold sides of the heat exchanger.
4. Determine the wall superheat at incipient boiling condition (ΔT_{sat_i}) using equation (28).
5. By iteration, determine the wall temperature (T_w) with the initial guessed value set as the oil inlet temperature. The iteration stops when the values generated from equations (18) and (21) are less than the pre-assigned tolerance value (*tol*). (As presented in the MathCAD worksheet).

6. With the known T_w from step (5), using equation (29) determine the wall superheat (ΔT_{sat}) at point 1.
7. With the result from step (6), determine if subcooled boiling exists at point 1.
8. Determine the local heat flux with equation (18) or (21).

IX. MathCAD Solution

The MathCad subroutine, required to solve the problem, is presented in Appendix B. The subroutine is used to carry out the iterative process to within a pre-assigned tolerance, *tol*. The initial guessed value for T_w is set as the oil inlet temperature. The iterative process starts by decreasing the wall temperature by 0.001 K increment. It stops when the difference between the values generated from equations (18) and (21) stays less than the tolerance.
With the known T_w, the local heat flux at location 1 is determined from equation (18). The wall superheat at location 1 then becomes

$$\Delta T_{sat} = T_{wall} - T_{cold\_sat} \tag{29}$$

For this particular example, the wall temperature is determined as 396.32 K (123.19°C) and the corresponding local heat flux is 164 kW/m^2. The minimum wall superheat required for incipient boiling is calculated from equation 26 as 0.951 K. The wall superheat at location 1 is13.175 K. Thus, the wall superheat at location 1 is higher than the minimum wall superheat required for incipient boiling. This result confirms that the subcooled boiling exists at location 1.

The differentiation of the various boiling regimes on the boiling process is crucial since the behavior of the fluid varies distinctively from the subcooled boiling regime to the saturated boiling regime. With the aid of MathCAD software, the boiling regimes can be simply and precisely. As a result, heat transfer performance of the fluid in a particular boiling process can be accurately determined.

X. Turbulent Film Condensation Heat Transfer Problem

Problem Statement:

Consider condensation of R-12 at 320 K on the outside of a vertical, 25.0 mm diameter tube, without vapor shear for ($T_s - T_w$) = 10 °C. Assume that the Reynolds number of the condensate leaving the bottom of the tube is 6,380. Use the turbulent film analysis, based on Dukler's (1960) analysis, to calculate the condensation coefficient at x = L.

XI. Turbulent Film Condensation Heat Transfer Solution

In order to determine the condensation heat transfer coefficient for turbulent film, Dukler (1960) presented a method of predicting the hydrodynamics and heat transfer in vertical film-wise condensation. The transport of heat in the film, neglecting downstream convection compared to cross-stream diffusion, is given by

$$q = -\left(K + \rho c_p \varepsilon_H\right)\frac{dT}{dy} \tag{30}$$

where ϵ_H is the eddy diffusivity of heat for turbulent flow. By substituting the thermal diffusivity, equation (30) becomes

$$q = -\rho c_p\left(\alpha_T + \varepsilon_H\right)\frac{dT}{dy} \Rightarrow q = -\rho c_p \nu\left(\frac{1}{\Pr} + \frac{\varepsilon_H}{\nu}\right)\frac{dT}{dy} \tag{31}$$

where α_T is the thermal diffusivity and Pr is the liquid Prandtl number. Equation (31) can be expressed in dimensionless form as follows:

$$\frac{q}{q_w} = \left(\frac{1}{\Pr} + \frac{\varepsilon_H}{\nu}\right)\frac{dT^+}{dy^+} \tag{32}$$

where

$$y^+ = y\frac{u^*}{\nu}\ldots where\ldots\ldots u^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\rho g} \tag{33}$$

$$T^+ = \frac{\rho c_p u^*}{q_w}\left(T_w - T\right) \tag{34}$$

If the turbulent Prandtl number, $\Pr_t$, is equal to one.

$$\Pr_t = \frac{\varepsilon_M}{\varepsilon_H} = 1 \Rightarrow \varepsilon_M = \varepsilon_H \tag{35}$$

where ϵ_M is the eddy diffusivity of momentum. Therefore, equation (32) becomes

$$\frac{q}{q_w} = \left(\frac{1}{\Pr} + \frac{\varepsilon_M}{\nu}\right)\frac{dT^+}{dy^+} \tag{36}$$

Assuming the shear stress and the heat transfer rate are constant, Equation (36) can be integrated from zero to δ^+, where δ^+ is the dimensionless liquid film thickness,

$$\delta^+ = \frac{\delta u^*}{\nu} \tag{37}$$

at y = δ, T= T_{sat} and at the surface, y= 0, T = T_w
Therefore,

$$\frac{T_{sat} - T_w}{\left(q_w / \rho c_p u^*\right)} = \int_0^{\delta^+} \frac{dy^+}{\frac{1}{\Pr} + \frac{\varepsilon_M}{\nu}} \tag{38}$$

Equation (38) can be integrated if the relationship between (ϵ_M/ν) and y^+ is known. Dukler (1960) used two distribution based on the local value of y^+.

For $y^+ \leq 26$, Dukler used the Deissler approximation for (ϵ_M/ν),

$$\frac{\varepsilon_M}{\nu} = \frac{u^+ y^+}{100}\left[1 - \exp\left(-\frac{u^+ y^+}{100}\right)\right] \tag{39}$$

For $y^+ > 26$, Dukler used the following relation:

$$\frac{\varepsilon_M}{\nu} = \frac{y^+}{2.5} - 1 \tag{40}$$

By using the Dukler analysis, the condensation heat transfer coefficient can be found by

$$h = \frac{q}{T_{sat} - T_w} \Rightarrow \frac{\rho c_p u^*}{h} = \int_0^{26} \frac{dy^+}{\frac{1}{\Pr} + \frac{u^+ y^+}{100}\left[1 - \exp\left(-\frac{u^+ y^+}{100}\right)\right]} + \int_{26}^{\delta^+} \frac{dy^+}{\frac{1}{\Pr} + \left(\frac{y^+}{2.5} - 1\right)} \tag{41}$$

Using equation (41), together with the universal velocity profile for turbulent flow,

$$0 \leq y^+ \leq 5 \Rightarrow u^+ = y^+ \Rightarrow \frac{\varepsilon_M}{\nu} = 0 \quad \text{Viscous sublayer} \tag{42a}$$

$$5 \leq y^+ \leq 30 \Rightarrow u^+ = -3.05 + 5\ln(y^+) \quad \text{Buffer sublayer} \tag{42b}$$

$$y^+ > 30 \Rightarrow u^+ = 5.5 + 2.5\ln(y^+) \quad \text{Turbulent region} \tag{42c}$$

where

$$u^+ = \frac{\overline{u}}{u^*} \tag{42d}$$

The integrals in equation (41) must be numerically integrated to solve for the condensation heat transfer coefficient, h.

$$\frac{\rho c_p u^*}{h} = I = \underbrace{\int_0^5 \frac{dy^+}{1/\mathrm{Pr}}}_{Viscous,sublayer} + \underbrace{\int_5^{26} \frac{dy^+}{\frac{1}{\mathrm{Pr}} + \frac{u^+ y^+}{100}\left[1 - \exp\left(-\frac{u^+ y^+}{100}\right)\right]}}_{Buffer,sublayer} + \underbrace{\int_{26}^{\delta^+} \frac{dy^+}{\frac{1}{\mathrm{Pr}} + \left(\frac{y^+}{2.5} - 1\right)}}_{Turbulent,region} \qquad (43)$$

For turbulent flow, the dimensionless film thickness can be found by using the Ganchev et al. (1976) empirical correlation,

$$\delta^+ = 0.051 \mathrm{Re}^{0.87} \qquad (44)$$

XII. Solution Scheme

1. Given the Reynolds number, calculate δ^+ from equation (44).

2. Numerically integrate Equation (43) to obtain I.

3. Calculate δ from

$$\delta^+ = \frac{\delta\sqrt{g\delta}}{\nu}$$

4. Calculate u^* from equation (37)

5. Calculate the condensation heat transfer coefficient, $h = \frac{\rho c_p u^*}{I}$

XIII. MathCad Solution

The MathCad subroutine, required for the numerical integration, is given in Appendix C. The subroutine is written for the numerical integration process required for step (2). The solution shows the ease of using MathCad to numerically integrate a complex expression.

XIV. Conclusion

In many graduate classes, theory is usually dominant over engineering application. Application of graduate level theory may be lengthy and/or difficult without computer codes. The students in the class with prior computer language experience preferred using MathCad for their projects. Computer codes such as FORTRAN or C^+ require users to enter algorithms with a distinct alphanumeric structure that only vaguely looks like the equations upon which the algorithm is based. MathCad provides the same basic programming structures available in compiler based programming languages while maintaining the look of the mathematics. The ability to use Greek symbols and common mathematical operators makes programs look like the equations provided by theory. The common mathematical appearances of MathCad worksheets help a student program and understand the solution algorithm more easily. Consequently, the students felt that the use of MathCad in the application projects helped them understand the theory more easily than with compiler based programming languages.

The students also embraced MathCad's abilities to make quick and accurate calculations and to handle units. The unit handler allows quick checks of the formulas entered in the MathCad worksheets. An unusual unit output or an error message concerning incompatible units quickly identifies that some equation has an error. MathCad can be used as a calculator that correctly handles units and it can be used to perform complex mathematical algorithms in the same worksheet.

The three examples provided and the comments received from the students demonstrates that the use of MathCad had a very positive effect on the student understanding of boiling and condensation phenomena. The iteration processes needed in certain solutions required a relatively short setup times, and the results were generated with minimal efforts in writing MathCad subroutines. The illustrative nature of the MathCad solutions provided the students with the ability to quickly grasp the solution process and to maintain correct units. The MathCad solutions were also used to quickly evaluate the best relaxation for the iterative solution for the pressure gradient example. The software was also effective in evaluating complex integrations, which would otherwise require writing and debugging a lengthy computer code.

XV. Nomenclatures

Symbol	MathCad	Description
A	A	cross sectional area (m^2)
Area	Area	heat transfer area heat exchanger (m^2)
Area'	Area'	heat transfer area per unit length of heat exchanger (m^2/m)
c_p	c_p, C_p	specific heat (joule/kg-K)
D	D	diameter (m)
e	e	fraction of liquid entrained (dimensionless)
$(dP/dz)_F$	$dPdz_f$	pressure gradient due to friction (Pa/m)

$(dP/dz)_{fF}$	$dPdz_{fF}$	frictional pressure gradient assuming liquid alone (Pa/m)
f	f	friction factor (dimensionless)
f_{fF}	f_{fF}	friction factor based on liquid alone flow (dimensionless)
G	G	mass velocity (kg/m^2-s)
G_{fF}	G_{fF}	mass velocity of liquid phase alone (kg/m^2-s)
g	g	gravitation acceleration (m/s^2)
h	h	heat transfer coefficient ($watt/m^2$-K)
i_{fg}	i_{fg}	latent heat of vaporization (joule/kg)
j	j	volumetric flux (superficial velocity) (m/s)
k	k	thermal conductivity (watt/m-K)
L	L	length of heat exchanger (m)
OD	OD	outside diameter of tube (m)
P	P	pressure (bar)
Pr	Pr	Prandtl number (dimensionless)
Pr_t	Pr_t	turbulent Prandtl number (dimensionless)
q	q	total local heat transfer rate (watt)
q”	q”	total local heat flux ($watt/m^2$)
Re	Re	Reynolds number (dimensionless)
T	T	temperature (K)
T^+	Tplus	dimensionless temperature
tol	tol	tolerance
th	th	wall thickness of tube (m)
ΔT	ΔT	wall superheat (K)
u^*	ustar	friction velocity (m/s)
u^+	uplus	dimensionless velocity
v	v	specific volume (m^3/kg)
W	W	mass flow rate (kg/s)
x	x	mass vapor quality (dimensionless)
X	X	Martinelli parameter (dimensionless)
y	y	distance measured from boundary (m)
y^+	yplus	dimensionless distance from the wall
Greek		
α	α	void fraction (dimensionless)
δ	δ	liquid film thickness (m)
δ^+	δplus	dimensionless liquid film thickness
ϵ_H	ϵ_H	eddy diffusivity of heat (m^2/s)
ϵ_M	ϵ_M	eddy diffusivity of momentum (m^2/s)
μ	μ	kinematic viscosity ($N-s/m^2$)
ν	ν	kinematic viscosity (m^2/s)
Φ	Φ	superficial momentum flux (kg/m^2-s)
ϕ	ϕ	two-phase multiplier (dimensionless)

ϕ_{fF}	ϕ_{fF}	two-phase multiplier based on liquid alone flow
ρ	ρ	density (kg/m^3)
σ	σ	surface tension (N/m)

Subscripts

1	location 1 as specified
a	dummy subscript
b	dummy subscript
Bi / i	incipient boiling
cold	cold fluid (water)
f	liquid phase
FC	forced convection
fo	liquid only
g	vapor phase
hot	hot fluid (oil)
HX	heat exchanger
in	inlet of heat exchanger
out	outlet of heat exchanger
sat	saturation
scb	subcooled boiling
tp	two-phase
tube	tube side
w, wall	wall

XVI. Bibliography

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LOUAY M CHAMRA

Dr. Chamra received his B.S. in Mechanical Engineering at the University of Texas at Austin, his MS at University of Portland, and his Ph.D. at the Pennsylvania State University. He has worked as a Research Associate at Penn State University. Currently, he is an assistant Professor of Mechanical Engineering at Mississippi State University where he teaches graduate and undergraduate courses in the thermal sciences and conducts related research. Since joining the faculty, Dr. Chamra has developed two new courses.

STEPHEN T. MCCLAIN

Stephen T. McClain is a Lecturer and Undergraduate Laboratory Manager at Mississippi State University. He received his B.S. in Mechanical Engineering from The University of Memphis in 1995, and he received his M.S. in Mechanical Engineering from Mississippi State University in 1997. He is a part-time doctoral student and hopes to finish his degree in May of 2002. He has taught classes in instrumentation, experimental uncertainty analysis, and thermodynamics.

SOON SENG TANG

Soon Seng Tang is currently a master candidate in mechanical engineering department, Mississippi State University. He received his B.S in mechanical engineering from Mississippi State University on 1997 with Summa Cum Laude. Upon his graduation, he was granted the Outstanding International Student Award. Before he returned to the graduate school, he worked as a project engineer for printing and packaging industries in Malaysia. Currently, he is a graduate research assistant in the university and his research project is in the area of heat transfer modeling for micro-fin tubes.

APPENDIX A Two-Phase Annular Flow Frictional Pressure Gradient Solution

This worksheet calculates the frictional pressure gradient associated with annular two-phase flow in a circular tube using the annular flow with entrainment model of Wallis. The example problem provided concerns a water-steam system at 130 deg C and 2.7011 bar flowing in a vertical 3 cm bore tube. The mass flow rate of water is 0.5 kg/s, while the mass flow rate of steam is 0.1 kg/s.

To begin the problem, the mixture properties are entered . The subscript "f" denotes the liquid properties, while the subscript "g" denotes vapor or gas properties.

$\rho_f := 1000 \cdot \frac{kg}{m^3}$ $\quad \rho_g := 1.64 \cdot \frac{kg}{m^3}$

$\mu_f := 1 \cdot 10^{-3} \cdot \frac{N \cdot s}{m^2}$

$\mu_g := 1.8 \cdot 10^{-5} \cdot \frac{N \cdot s}{m^2}$

$\sigma := 7.2 \cdot 10^{-2} \cdot \frac{N}{m}$

More flow parameters are entered .

$W_f := 0.5 \cdot \frac{kg}{s}$ $\quad W_g := 0.1 \cdot \frac{kg}{s}$

$W := W_f + W_g$ $\quad W = 0.6 \frac{kg}{s}$

$D := 0.03 \cdot m$ $\quad A := \frac{\pi \cdot D^2}{4}$

$A = 7.069 \times 10^{-4} m^2$

$G_f := \frac{W_f}{A}$ $\quad G_f = 707.355 \frac{kg}{m^2 s}$

$G_g := \frac{W_g}{A}$ $\quad G_g = 141.471 \frac{kg}{m^2 s}$

$G := G_f + G_g$

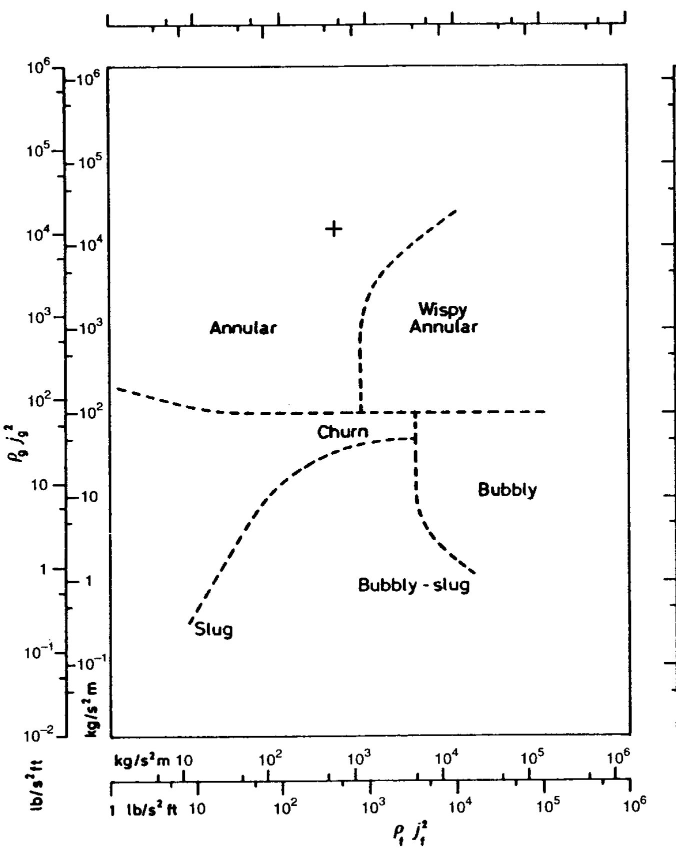
$G = 848.826 \frac{kg}{m^2 s}$

$x := \frac{W_g}{W}$ $\quad x = 0.167$

To verify that the flow is annular, the flow pattern will be determined using the flow pattern chart of Hewitt and Roberts. To use the pattern chart of Hewitt and Roberts, the superficial momentum fluxes of the liquid and vapor must be determined.

$\Phi_f := \frac{G_f^{\,2}}{\rho_f}$ $\quad \Phi_f = 500.352 \frac{kg}{s^2 \cdot m}$

$\Phi_g := \frac{G_g^{\,2}}{\rho_g}$ $\quad \Phi_g = 1.22 \times 10^4 \frac{kg}{s^2 \cdot m}$



The flow map of Hewitt and Roberts shows that the flow is indeed annular.

To use the annular flow model of Wallis, which is iterative, the frictional pressure gradient must first be estimated using some other method. The separated flow method of Lockhart and Martinelli is used to obtain the first estimate of the frictional pressure gradient.

$$Re_f := \frac{G_f \cdot D}{\mu_f}$$

$$Re_f = 2.122\times 10^4$$

$$Re_g := \frac{G_g \cdot D}{\mu_g}$$

$$Re_g = 2.358\times 10^5$$

$$f_f := 0.079\, Re_f^{-0.25}$$

$$f_f = 6.545\times 10^{-3}$$

$$f_g := 0.079\, Re_g^{-0.25}$$

$$f_g = 3.585\times 10^{-3}$$

The frictional pressure gradients associated with liquid and the vapor are calculated .

$$dPdz_f := 2\cdot f_f \cdot \frac{G_f^2}{\rho_f \cdot D} \qquad dPdz_f = 218.334\frac{Pa}{m} \qquad dPdz_g := 2\cdot f_g \cdot \frac{G_g^2}{\rho_g \cdot D}$$

$$dPdz_g = 2.917\times 10^3 \frac{Pa}{m}$$

The Martinelli parameter is calculated .

$$X_{tt} := \left(\frac{dPdz_f}{dPdz_g}\right)^{0.5} \qquad X_{tt} = 0.274$$

The two-phase multiplier based on the vapor frictional pressure gradient is now calculated.

$$\phi_g := \left(1 + 20\cdot X_{tt} + X_{tt}^2\right)^{0.5}$$

$$\phi_g = 2.559$$

The initial estimate of the frictional pressure gradient is calculated .

$$dPdz_I := \phi_g^2 \cdot dPdz_g$$

$$dPdz_I = 1.91\times 10^4 \frac{Pa}{m}$$

The frictional pressure gradient function for a given entrainment percentage is provided .

$$
\begin{aligned}
dPdz(e, dPdz, Relax) := \;& W_{fF} \leftarrow (1-e)\cdot W_f \\
& G_{fF} \leftarrow \frac{W_{fF}}{A} \\
& Re_{fF} \leftarrow Re_f \cdot (1-e) \\
& f_{fF} \leftarrow 0.079\, Re_{fF}^{-0.25} \\
& dPdz_{fF} \leftarrow 2\cdot f_{fF} \frac{G_{fF}^2}{\rho_f \cdot D} \\
& \%diff \leftarrow 1 \\
& \text{while } \%diff > 0.000001 \\
& \quad dPdz_{old} \leftarrow dPdz \\
& \quad \phi_{fF} \leftarrow \left(\frac{dPdz}{dPdz_{fF}}\right)^{\frac{1}{2}} \\
& \quad \alpha \leftarrow 1 - \frac{1}{\phi_{fF}} \\
& \quad \phi g2 \leftarrow \left[\frac{1 + 75\cdot(1-\alpha)}{\alpha^{\frac{5}{2}}}\right] \cdot \left(\frac{W_g + e\cdot W_f}{W_g}\right) \cdot \left[1 - 2\cdot\left(\frac{\alpha}{1-\alpha}\right)\cdot\left(\frac{\rho_g}{\rho_f}\right)\cdot\left(\frac{W_{fF}}{W_g}\right)\right]^2 \\
& \quad dPdz \leftarrow \phi g2 \cdot dPdz_g \\
& \quad dPdz \leftarrow dPdz_{old} + Relax\cdot(dPdz - dPdz_{old}) \\
& \quad \%diff \leftarrow \left|1 - \frac{dPdz_{old}}{dPdz}\right| \\
& dP \leftarrow dPdz \\
& dP
\end{aligned}
$$

The assumed entrainment percentage is entered .

$e_n := 0.4$

The frictional pressure gradient is then determined using the function presented above.

$dPdz_f := dPdz\left(e_n, dPdz_I, 0.5\right)$

$dPdz_f = 3.301\times 10^4 \frac{Pa}{m}$

Thus, for 0.5 kg/s of water and 0.1 kg/s of steam at 130 deg C and 2.7011 bar flowing in a vertical 3 cm bore tube, the frictional pressure gradient is 3301 Pa per meter of tube.

$$
\begin{aligned}
dPdz2(e, dPdz, Relax, N) := \; & i \leftarrow 0 \\
& W_{fF} \leftarrow (1 - e)\cdot W_f \\
& G_{fF} \leftarrow \frac{W_{fF}}{A} \\
& Re_{fF} \leftarrow Re_f\cdot(1 - e) \\
& f_{fF} \leftarrow 0.079\, Re_{fF}^{-0.25} \\
& dPdz_{fF} \leftarrow 2\cdot f_{fF}\cdot\frac{G_{fF}^{2}}{\rho_f\cdot D} \\
& \text{while } i < N \\
& \quad dPdz_{old} \leftarrow dPdz \\
& \quad \phi_{fF} \leftarrow \left(\frac{dPdz}{dPdz_{fF}}\right)^{\frac{1}{2}} \\
& \quad \alpha \leftarrow 1 - \frac{1}{\phi_{fF}} \\
& \quad \phi g2 \leftarrow \left[\left[\frac{1 + 75\cdot(1-\alpha)}{\alpha^{\frac{5}{2}}}\right]\cdot\left(\frac{W_g + e\cdot W_f}{W_g}\right)\cdot\left[1 - 2\cdot\left(\frac{\alpha}{1-\alpha}\right)\cdot\left(\frac{\rho_g}{\rho_f}\right)\cdot\left(\frac{W_{fF}}{W_g}\right)\right]\right]^2 \\
& \quad dPdz \leftarrow \phi g2\cdot dPdz_g \\
& \quad dPdz \leftarrow dPdz_{old} + Relax\left(dPdz - dPdz_{old}\right) \\
& \quad i \leftarrow i + 1 \\
& dPdz
\end{aligned}
$$

$i := 0..\, 25$

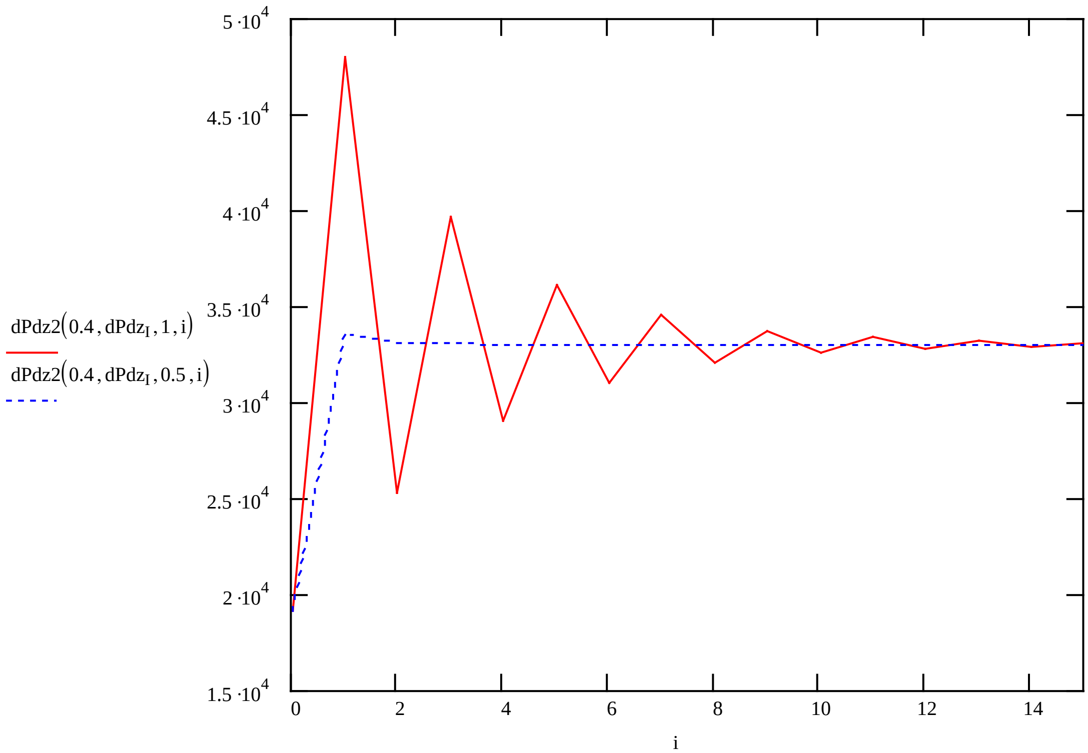


Figure A-1 Pressure gradient convergence

APPENDIX B Subcooled Boiling Heat Transfer Solution

Properties of Fluids and Heat Exchanger Material

Water Properties

$T_{cold\_in} := 90 + 273.15$ $T_{cold\_out} := 105 + 273.15$ $T_{cold\_sat} := 110 + 273.15$ $T_{cold\_1} := 100 + 273.15$

$P_{cold} := 1.433$ $W_{cold} := 0.42$ $c_{p\_cold} := 4.2 \cdot 10^3$ $\mu_f := 254.8 \cdot 10^{-6}$ $\sigma := 56.83 \cdot 10^{-3}$

$k_f := 0.684$ $v_f := 1.0515 \cdot 10^{-3}$ $v_g := 1.2101$ $v_{fg} := v_g - v_f$ $i_{fg} := (2691 - 461.3) \cdot 10^3$

$h_{cold\_fo} := 6890$ $g := 9.807$

Oil Properties

$T_{hot\_in} := 160 + 273.15$ $P_{hot} := 4$ $W_{hot} := 0.67$ $c_{p\_hot} := 2 \cdot 10^3$ $h_{hot} := 5000$

Tube Properties

$OD_{tube} := 25 \cdot 10^{-3}$ $th_{tube} := 1 \cdot 10^{-3}$

Based on energy balance equation,

$$T_{hot\_1} := \frac{W_{cold} \cdot c_{p\_cold} \cdot \left(T_{cold\_out} - T_{cold\_1}\right) - W_{hot} \cdot c_{p\_hot} \cdot T_{hot\_in}}{-W_{hot} \cdot c_{p\_hot}} \qquad T_{hot\_1} = 426.568 \ (\ 153.42^{o}C \ )$$

Define the total heat transfer area per unit length of heat exchanger,

$Area'_{hot} := \pi \cdot OD_{tube}$ $Area'_{cold} := \pi \cdot \left(OD_{tube} - 2 \cdot th_{tube}\right)$

According to Davis and Anderson (1966),

$$\Delta T_{sat\_i} := \frac{8 \cdot \sigma \cdot T_{cold\_sat} \cdot v_{fg}}{i_{fg}} \cdot \frac{h_{cold\_fo}}{k_f} \qquad \Delta T_{sat\_i} = 0.951$$

MathCad subroutine to calculate the corresponding wall temperature, T_w.

$$T_w(tol) := \left| \begin{array}{l} T_w \leftarrow T_{hot\_in} \\ q''_a \leftarrow 10000 \\ q''_b \leftarrow 30000 \\ \text{while } \left|q''_a - q''_b\right| > tol \\ \quad \left| \begin{array}{l} q''_a \leftarrow h_{hot} \cdot (T_{hot\_1} - T_w) \cdot \dfrac{Area'_{hot}}{Area'_{cold}} \\ q''_b \leftarrow \sqrt{\left[h_{cold\_fo} \cdot (T_w - T_{cold\_1})\right]^2 + \left[\left[\dfrac{c_{p\_cold} \cdot (T_w - T_{cold\_sat})}{i_{fg}} \cdot \left(\dfrac{k_f}{c_{p\_cold} \cdot \mu_f} \right)^{1.7} \cdot \dfrac{1}{0.02} \right]^{\frac{1}{0.33}} \cdot \mu_f \cdot i_{fg} \cdot \sqrt{\dfrac{g \cdot \left(\dfrac{1}{v_f} - \dfrac{1}{v_g} \right)}{\sigma}} + - \left[\left(\dfrac{\Delta T_{sat\_i}}{0.556} \right)^{\frac{1}{0.463 \cdot P_{cold}^{0.0234}}} \cdot 1082 \cdot P_{cold}^{1.156} \right] \right]^2} \\ T_w \leftarrow T_w - 0.001 \\ \text{error("Tolerance (tol) is too small, larger value required") if } T_w \le T_{cold\_1} \end{array} \right. \\ T_w \end{array} \right.$$

With the subroutine, the calculated wall temperature is,

$T_{wall} := T_w(1)$ $\quad$ $T_{wall} = 396.325$ $\quad$ (123.19 ^oC)

At the incipient condition, $\Delta T_{sat\_i} = 0.951$

At location 1, the wall superheat is, $\Delta T_{sat} := T_{wall} - T_{cold\_sat}$ $\quad$ $\Delta T_{sat} = 13.175$

Since ΔT_{sat} at location 1 is greater that ΔT_{sat_i} , subcooled boiling does exist.

The local heat flux at location 1 is

$$q''_a := h_{hot} \cdot (T_{hot\_1} - T_{wall}) \cdot \frac{Area'_{hot}}{Area'_{cold}} \qquad q''_a = 164364$$

APPENDIX C

This worksheet calculates the heat transfer coefficient for condensation outside a vertical tube without vapor shear. The solution is based on Dukler (1960) turbulent film analysis.

To begin the solution, R-12 properties are entered. The subscript "f" denotes the liquid properties.

$OD := 25\ mm$ $\quad$ $T_{sat} := 320\ K$ $\quad$ $Re_L := 6380$ $\quad$ $\mu_f := 185.7 \cdot 10^{-6} \frac{N \cdot s}{m^2}$ $\quad$ $\rho_f := 1226 \frac{kg}{m^3}$

$\nu_f := 1.515 \cdot 10^{-7} \frac{m^2}{s}$ $\quad$ $Cp_f := 1040 \frac{J}{kg \cdot K}$ $\quad$ $k_f := 0.0613 \frac{W}{m \cdot K}$ $\quad$ $Pr_f := 3.15$

For turbulent flow,

$\delta plus := 0.051 \cdot Re_L^{0.87}$ $\quad$ $\delta plus = 104.175$

uplus can be found from the the universal velocity profiles, $uplus(yplus) := -3.05 + 5 \cdot \ln(yplus)$

To calculate the value of I, we need to numerically integrate,

$$I := \int_0^5 Pr_f \, dyplus \; ...$$

$$+ \int_5^{26} \frac{1}{\frac{1}{Pr_f} + \frac{uplus(yplus)}{100} \cdot yplus \cdot \left(1 - \exp\left(-uplus(yplus) \cdot \frac{yplus}{100}\right)\right)} \, dyplus \; ...$$

$$+ \int_{26}^{\delta plus} \frac{1}{\frac{1}{Pr_f} + \left(\frac{yplus}{2.5} - 1\right)} \, dyplus$$

$I = 37.863$

Calculate the liquid film thickness, δ

$$\delta plus = \frac{\delta}{\nu_f} \cdot \sqrt{g \cdot \delta} \qquad \delta := \left(\frac{\delta plus^2}{g} \cdot \nu_f^{\,2}\right)^{\frac{1}{3}} \qquad \delta = 2.94 \times 10^{-4}\, m$$

Calculate the friction velocity, ustar

$$ustar := \frac{\delta plus \cdot \nu_f}{\delta} \qquad ustar = 0.054 \frac{m}{s}$$

Calculate the condensation heat transfer coefficient, h

$$h := \rho_f \cdot Cp_f \cdot \frac{ustar}{I} \qquad h = 1.808 \times 10^3 \frac{W}{m^2 \cdot K}$$