AC 2009-65: THE USE OF SPREADSHEETS IN TEACHING THE POWER-FLOW PROBLEM

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The Use of Spreadsheets in Teaching the Power-Flow Problem

1. Introduction

The solution to the power-flow problem is of fundamental importance in power system analysis and design. In transient stability studies and fault analysis, solutions to a power-flow problem constitute a necessary initial step in such analyses.

The objective of the power-flow problem is to calculate the voltage magnitudes and phase angles at each bus or node in a given power system. Calculations are performed under the assumption of balanced three-phase steady-state conditions. In addition to voltages and angles, real and reactive power flows in equipment such as transformers and transmission lines can be also obtained from these calculations.

The topics in this paper follow the treatment found in standard reference material on power system analysis and design.[2–4] In particular, two numerical methods, namely, the Gauss-Seidel and the Newton-Raphson methods are used to determine the power flows in a small-scale power system. The examples are simple enough so that readers can replicate hand calculations and reproduce the spreadsheet implementations. The application of spreadsheets for solving power flows and other related problems has been reported in the literature.[1, 5–7] The emphasis of this paper is on the educational value of spreadsheets in the analysis of power systems.

The paper is organized as follows: Section 2 provides a spreadsheet implementation for solving the power-flow problem using the Gauss-Seidel method; Section 3 presents an implementation of the Newton-Raphson method for solving the power-flow problem; Section 4 discusses the authors' experience in the classroom and their pedagogical insights; and finally Section 5 presents some concluding remarks. Since the mathematical underpinnings of the power flow problem are well known, the reader is referred to standard books[2–4] for details.

2. Power-flow solution by the Gauss-Seidel method

In this section we use the Gauss-Seidel method to determine the power flows in the three-bus network of Figure 1. Bus and transmission line data are summarized in Tables 1 and 2, respectively.

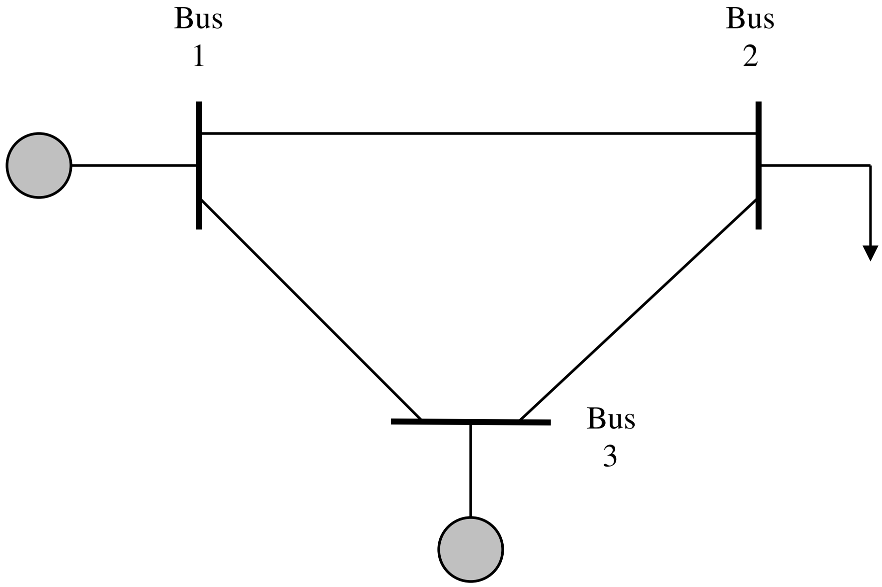

Figure 1: Three-bus power system.

Table 1: Bus input data.

Type	Bus i	Bus voltage $\lvert V_i \rvert$ (p.u.)	δ_i (deg.)	Complex power (p.u.) P_{gi}	Q_{gi}	P_{di}	Q_{di}
Slack	1	1.0	0°	—	—	0.0	0.0
Load	2	—	—	0.0	0.0	1.8	0.6
Constant voltage	3	1.0	—	1.0	—	0.0	0.0

Table 2: Transmission line parameters.

Line bus i to bus k	Impedance Z_{ik} (p.u.)	Shunt admittance $B_{ik}/2$ (p.u.)
1—2	$0.01 + j0.1$	$j0.02$
2—3	$0.02 + j0.2$	$j0.04$
1—3	$0.01 + j0.2$	$j0.03$

We first construct the bus admittance matrix $\mathbf{Y}_{\text{bus}}$ for the power system in Figure 1. The bus admittance matrix offers a convenient representation of the topology of a power network. In the example at hand, we calculate the primitive admittance of each branch in per unit:

$$y_{12} = \frac{1}{0.01 + j0.1} = 0.9901 - j9.9010,$$
$$y_{23} = \frac{1}{0.02 + j0.2} = 0.4950 - j4.9505,$$
$$y_{13} = \frac{1}{0.01 + j0.2} = 0.2494 - j4.9875.$$

With the primitive admittances just calculated, we determine the elements of $\mathbf{Y}_{\text{bus}}$:

$$Y_{11} = y_{12} + y_{13} + \frac{B_{12}}{2} + \frac{B_{13}}{2} = 1.2395 - j14.8385,$$
$$Y_{22} = y_{12} + y_{23} + \frac{B_{12}}{2} + \frac{B_{23}}{2} = 1.4851 - j14.7915,$$
$$Y_{33} = y_{23} + y_{13} + \frac{B_{23}}{2} + \frac{B_{13}}{2} = 0.7444 - j9.8680,$$
$$Y_{12} = Y_{21} = -y_{12} = -0.9901 + j9.9010,$$
$$Y_{23} = Y_{32} = -y_{23} = -0.4950 + j4.9505,$$
$$Y_{13} = Y_{31} = -y_{13} = -0.2494 + j4.9875.$$

Therefore, the bus admittance matrix for the network of Figure 1 is given by

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 1.2395 - j14.8385 & -0.9901 + j9.9010 & -0.2494 + j4.9875 \\ -0.9901 + j9.9010 & 1.4851 - j14.7915 & -0.4950 + j4.9505 \\ -0.2494 + j4.9875 & -0.4950 + j4.9505 & 0.7444 - j9.8680 \end{bmatrix}. \tag{1}$$

For an N-bus system, the Gauss-Seidel method calculates the voltage at any bus i at iteration m, $V_i^{(m)}$, according to

$$V_i^{(m)} = \frac{1}{Y_{ii}} \left[\frac{P_{i,\text{sch}} - jQ_{i,\text{sch}}}{V_i^{(m-1)*}} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(m)} - \sum_{k=i+1}^{N} Y_{ik} V_k^{(m-1)} \right], \quad i = 2, 3, \ldots, N \tag{2}$$

where Y_{ik} is the (i,k) entry in the $\mathbf{Y}_{\text{bus}}$ matrix, $P_{i,\text{sch}}$ is the *net scheduled real power*, and $Q_{i,\text{sch}}$ is the *net scheduled reactive power* being injected into the network at bus i. The net scheduled real power is defined as the difference between the scheduled power P_{gi} being generated at bus i and the scheduled power demand P_{di} of the load at that bus; the net scheduled reactive power is defined in a similar manner. That is,

$$P_{i,\text{sch}} = P_{gi} - P_{di}, \tag{3}$$
$$Q_{i,\text{sch}} = Q_{gi} - Q_{di}. \tag{4}$$

In Equation (2) the notation $(\cdot)^*$ denotes complex conjugation and bus 1 is designated as the *slack bus*. After each iteration of Equation (2) the *power flows* are calculated:

$$P_i - jQ_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k = \sum_{k=1}^{N} | Y_{ik} V_i V_k | \, e^{j(\theta_{ik} + \delta_k - \delta_i)} . \tag{5}$$

where θ_{ik} is the argument (angle) of the (i,k) entry in the $\mathbf{Y}_{\text{bus}}$ matrix, δ_i is the angle of bus voltage V_i, and the notation $|\cdot|$ denotes modulus (magnitude). The iterations stop when the power mismatches $\Delta P_i = P_{i,\text{sch}} - P_i$ and $\Delta Q_i = Q_{i,\text{sch}} - Q_i$ at each bus are zero, or within a prescribed precision index.

In the following we present the calculations for the first iteration of the Gauss-Seidel method applied to the system in Figure 1. With the slack bus designated as number 1, we start computations at bus 2. If $V_2^{(0)}$ and $V_3^{(0)}$ are initial estimates for the voltages at buses 2 and 3, respectively, we have

$$V_2^{(1)} = \frac{1}{Y_{22}} \left[\frac{P_{2,\text{sch}} - jQ_{2,\text{sch}}}{V_2^{(0)*}} - (Y_{21} V_1 + Y_{23} V_3^{(0)}) \right].$$

The corrected voltage $V_2^{(1)}$ is then used to calculate the value of $V_3^{(1)}$

$$V_3^{(1)} = \frac{1}{Y_{33}} \left[\frac{P_{3,\text{sch}} - jQ_{3,\text{sch}}}{V_3^{(0)*}} - (Y_{31} V_1 + Y_{32} V_2^{(1)}) \right].$$

The procedure is repeated until the amount of correction in voltage at every bus is less than some predetermined precision index.

The bus admittance matrix $\mathbf{Y}_{\text{bus}}$ for this system is given by Equation (1). The input data and unknowns at each bus can be inferred from Table 1 and are summarized in Table 3.

Table 3: Input data and unknowns for the power system of Figure 1.

Type	Bus i	Input data	Unknowns
Slack	1	$\|V_1\| = 1.0$ p.u., $\delta_1 = 0°$	$P_{\text{sch},1} = P_{g1} - P_{d1} = P_{g1}$, $Q_{\text{sch},1} = Q_{g1} - Q_{d1} = Q_{g1}$
Load	2	$P_{\text{sch},2} = P_{g2} - P_{d2} = -1.8$ p.u., $Q_{\text{sch},2} = Q_{g2} - Q_{d2} = -0.6$ p.u.	$\|V_2\|$, δ_2
Constant voltage	3	$\|V_3\| = 1.0$ p.u., $P_{\text{sch},3} = P_{g3} - P_{d3} = 1.0$ p.u.	δ_3, $Q_{\text{sch},3} = Q_{g3} - Q_{d3} = Q_{g3}$

We assume initial guesses for the voltages $V_2^{(0)} = 1.0e^{j0^\circ} = 1.0$ and $V_3^{(0)} = 1.0e^{j0^\circ} = 1.0$ at buses 2 and 3, respectively. Using Equation (2) we determine an improved value for V_2:

$$V_2^{(1)} = \frac{1}{1.4851 - j14.7915}\left[\frac{-1.8 + j0.6}{1.0} - (-0.9901 + j9.9010)(1.0) - (-0.4950 + j4.9505)(1.0)\right]$$
$$= 0.9589e^{-j6.999^\circ}.$$

With $V_2^{(1)}$ at hand, we then proceed to find $Q_3^{(1)}$ and $V_3^{(1)}$. Since bus 3 is a voltage-controlled bus, we can use Equation (5) to compute the reactive power at that bus. Observe that the real power is the real part of the right-hand side of Equation (5), while the reactive power is given by the negative of the imaginary part of the right-hand side of that equation. Here we show how to use Equation (5) to compute the reactive power at bus 3:

$$Q_3^{(1)} = -\mathrm{Im}\left\{ 1.0e^{-j0^\circ} \left[(-0.2494 + j4.9875)(1.0) + (-0.4950 + j4.9505)(0.9589e^{-j6.999^\circ}) + (0.7444 - j9.8680)(1.0) \right]\right\}$$
$$= 0.1110\,\text{p.u.}$$

With the preceding value of reactive power, we use Equation (2) once again to calculate an improved value of V_3:

$$V_3^{(1)} = \frac{1}{0.7444 - j9.8680}\left[\frac{1.0 - j0.1110}{1.0} - (-0.2494 + j4.9875)(1.0) - (-0.4950 + j4.9505)(0.9589e^{-j6.999^\circ})\right]$$
$$= 1.0038e^{j2.288^\circ}.$$

As indicated in Table 3, $|V_3| = 1$ at bus 3 and, therefore, we set $V_3^{(1)} = 1.0e^{j2.288^\circ}$ for the next iteration. This concludes the first iteration of the Gauss-Seidel method. The calculations are repeated in a similar manner with updated values until convergence can be discerned. We next show how to implement the above calculations using a spreadsheet.

We begin by inputting the entries that form the bus admittance matrix given in Equation (1) . Also, we input the known data shown in Table 3. This step is shown in the screen capture of Figure 2. The actual Microsoft Excel commands used to generate the input data is presented in Figure 3.

	A	B	C	D	E	F
1	**Bus admittance matrix Ybus**					
2	**Bus**	**1**	**2**	**3**		
3	**1**	1.2395-14.8385j	-0.9901+9.901j	-0.2494+4.9875j		
4	**2**	-0.9901+9.901j	1.4851-14.7915j	-0.495+4.9505j		
5	**3**	-0.2494+4.9875j	-0.495+4.9505j	0.7444-9.868j		
6						
7	**Input data**					
8	**Type**	**Bus**	**V_i (p.u.)**	**d_i (deg)**	**P_i (p.u.)**	**Q_i (p.u.)**
9	Swing	**1**	1	0		
10	Load	**2**			-1.8	-0.6
11	Const V	**3**	1		1	

Figure 2: Bus admittance matrix and input data for the power system of Figure 1.

	A	B	C	D	E	F
1	**Bus adı**					
2	**Bus**	**1**	**2**	**3**		
3	**1**	=COMPLEX(1.2395,-14.8385,"j")	=COMPLEX(-0.9901,9.901,"j")	=COMPLEX(-0.2494,4.9875,"j")		
4	**2**	=COMPLEX(-0.9901,9.901,"j")	=COMPLEX(1.4851,-14.7915,"j")	=COMPLEX(-0.495,4.9505,"j")		
5	**3**	=COMPLEX(-0.2494,4.9875,"j")	=COMPLEX(-0.495,4.9505,"j")	=COMPLEX(0.7444,-9.868,"j")		
6						
7	**Input d**					
8	**Type**	**Bus**	**V_i (p.u.)**	**d_i (deg)**	**P_i (p.u.)**	**Q_i (p.u.)**
9	Swing	**1**	=COMPLEX(1,0,"j")	=180*IMARGUMENT(C9)/PI()		
10	Load	**2**			-1.8	-0.6
11	Const V	**3**	1		1	

Figure 3: Microsoft Excel commands used to generate the input data in Figure 2.

The calculations of the Gauss-Seidel method are shown in Figure 4 as the cell range H1:T11 in the spreadsheet. The entries in row 3 extending from cell H3 to T3 correspond to initial estimates for the numerical algorithm. The basic formulas are found in row 4 and extend from H4 to T4. The formulas in row 4 are copied to the rows below as many times as needed until convergence is reached. Some columns were omitted in Figure 4 for clarity; these columns simply contain intermediate calculations. The calculations reveal that convergence is reached after 5 iterations for a precision index of 10^{-4} for the bus voltages and 10^{-3} for the angles. A final calculations section is also shown in Figure 4; the section covers the cell range V1:AC3, but columns V and W were left out as these columns contain intermediate calculations. The solution to the power-flow problem at hand is summarized in Table 4.

To conclude this section, we provide in Table 5 all the formulas used in the implementation of the Gauss-Seidel method. Formulas corresponding to hidden columns are also included for completeness.

	H	L	M	P	S	T
1	**Power-flow solutions by Gauss-Seidel method**					
2	**Iteration m**	**| V_2 |**	**d_2**	**Q_3**	**| V_3 |**	**d_3**
3	0	1.0000	0.000		1.0000	0.000
4	1	0.9589	-6.999	0.1110	1.0000	2.288
5	2	0.9403	-6.162	0.2017	1.0000	2.761
6	3	0.9406	-6.191	0.2022	1.0000	2.760
7	4	0.9405	-6.188	0.2024	1.0000	2.762
8	5	0.9405	-6.188	0.2024	1.0000	2.762

	X	Y	Z	AA	AB	AC
1						
2	**P_2**	**Q_2**	**P_3**	**Q_3**	**P_1**	**Q_1**
3	-1.8000	-0.6000	1.0000	0.2024	0.8000	0.3976

Figure 4: Gauss-Seidel iterations showing relevant quantities in the power system of Figure 1.

Table 4: Steady-state power flows and bus voltages for the system in Figure 1. Except for the angles, all other quantities are expressed in per unit (p.u.).

Bus i	Input data (given)	Unknowns (calculated)
1	$\lvert V_1\rvert = 1.0$, $\delta_1 = 0^\circ$	$P_{sch,1} = 0.8$, $Q_{sch,1} = 0.3976$
2	$P_{sch,2} = -1.8$, $Q_{sch,2} = -0.6$	$\lvert V_2\rvert = 0.9405$, $\delta_2 = -6.188^\circ$
3	$\lvert V_3\rvert = 1.0$, $P_{sch,3} = 1.0$	$\delta_3 = 2.762^\circ$, $Q_{sch,3} = 0.2024$

Table 5: Microsoft Excel formulas for implementing the Gauss-Seidel method.

Cell	Formula	Comments
H3:H11	List of numbers from 0 to 8.	Iteration number.
H2:T2	Labels for various quantities.	Gauss-Seidel section.
I3, J3	Blank cells.	No calculations.
K3	=COMPLEX(1,0,"j")	Initial guess for V_2.
L3	=IMABS(K3)	Magnitude of initial V_2.
M3	=180*IMARGUMENT(K3)/PI()	Angle of initial V_2 in deg.
N3:P3	Blank cells.	No calculations.
Q3	=COMPLEX(1,0,"j")	Initial value for calculated V_3.
R3	=COMPLEX(1,0,"j")	Initial value for corrected V_3; magnitude must be constant.
S3	=IMABS(R3)	Magnitude of initial corrected V_3.
T3	=180*IMARGUMENT(Q3)/PI()	Angle of initial calculated V_3 in deg.

Table 5 (*continued*): Microsoft Excel formulas for implementing the Gauss-Seidel method.

Cell	Formula	Comments
I4	=IMDIV(COMPLEX(E10,-F10,"j"),IMCONJUGATE(K3))	The term $\frac{P_{2,\text{sch}} - jQ_{2,\text{sch}}}{V_2^{(m-1)*}}$ of Equation (2).
J4	=IMSUM(IMPRODUCT(B4,C9),IMPRODUCT(D4,R3))	The term $Y_{21}V_1 + Y_{23}V_3^{(m-1)}$ of Equation (2).
K4	=IMDIV(IMSUB(I4,J4),C4)	Completes the calculation of $V_2^{(m)}$ in Equation (2) using intermediate results in cells I4 and J4.
L4	=IMABS(K4)	Magnitude of updated V_2.
M4	=180*IMARGUMENT(K4)/PI()	Angle of updated V_2 in deg.
N4	=IMSUM(IMPRODUCT(B5,C9), IMSUM(IMPRODUCT(C5,K4), IMPRODUCT(D5,R3)))	The term $\sum_{k=1}^{N} Y_{3k}V_k$ of Equation (5).
O4	=IMDIV(COMPLEX(E11,-P4,"j"), IMCONJUGATE(R3))	The term $\frac{P_{3,\text{sch}} - jQ_{3,\text{sch}}}{V_3^{(m-1)*}}$ of Equation (2).
P4	=-IMAGINARY(IMPRODUCT(IMCONJUGATE(R3),N4))	Q_3 computed as the negative of the imaginary part of $V_3^* \sum_{k=1}^{N} Y_{3k}V_k$ as indicated in Equation (5).
Q4	=IMDIV(IMSUM(IMSUB(O4,N4), IMPRODUCT(D5,R3)),D5)	Updated value $V_3^{(m)}$ calculated from Equation (2). Observe that in the cell formula $Y_{33}V_3$ was added back since this term is not present explicitly in Equation (2). The reason is that cell N4 uses $Y_{33}V_3$ when computing Q_3 according to Equation (5), but this term has to be excluded when computing V_3.
R4	=COMPLEX(S4*COS(IMARGUMENT(Q4)),S4*SIN(IMARGUMENT(Q4)),"j")	Corrects $V_3^{(m)}$ to match the voltage magnitude at bus 3. The angle (argument) is the same as in cell Q4.

Table 5 (*continued*): Microsoft Excel formulas for implementing the Gauss-Seidel method.

Cell	Formula	Comments
S4	=S3	Copies the constant voltage magnitude at bus 3.
T4	=180*IMARGUMENT(Q4)/PI()	Angle of updated V_3 in deg.
I5:T11	Copies of cell range I4:T4.	Replicates cell formulas in rows below until convergence is achieved.
V2:AC2	Labels for various quantities.	Final calculations section.
V3	=IMSUM(IMPRODUCT(B4,C9),IMSUM(IMPRODUCT(C4,K11), IMPRODUCT(D4,R11)))	The term $\sum_{k=1}^{N} Y_{2k} V_k$ of Equation (5).
W3	=IMSUM(IMPRODUCT(B5,C9), IMSUM(IMPRODUCT (C5,K11), IMPRODUCT(D5,R11)))	The term $\sum_{k=1}^{N} Y_{3k} V_k$ of Equation (5).
X3	=IMREAL(IMPRODUCT(IMCONJUGATE(K11),V3))	P_2 computed as the real part of $V_2^* \sum_{k=1}^{N} Y_{2k} V_k$ according to Equation (5).
Y3	=-IMAGINARY(IMPRODUCT(IMCONJUGATE(K11),V3))	Q_2 computed as the negative of the imaginary part of $V_2^* \sum_{k=1}^{N} Y_{2k} V_k$ according to Equation (5).
Z3	=IMREAL(IMPRODUCT(IMCONJUGATE(R11),W3))	P_3 computed as the real part of $V_3^* \sum_{k=1}^{N} Y_{3k} V_k$ according to Equation (5).
AA3	=-IMAGINARY(IMPRODUCT(IMCONJUGATE(R11),W3))	Q_3 computed as the negative of the imaginary part of $V_3^* \sum_{k=1}^{N} Y_{3k} V_k$ according to Equation (5).
AB3	=-X3-Z3	Real power P_1 at slack bus (balance of real power).
AC3	=-Y3-AA3	Reactive power Q_1 at slack bus (balance of reactive power).

3. Power-flow solution by the Newton-Raphson method

To set up the Newton-Raphson numerical method, we employ the power-flow expression given by Equations (5). For the three-bus system in Figure 1, the Newton-Raphson method leads to

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} \\ \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} & \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} \\ \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} & \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} \\ \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} & \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} \end{bmatrix} \begin{bmatrix} \Delta |V_2| \\ \Delta |V_3| \\ \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix}. \tag{6}$$

The 4 x 4 matrix in Equation (6) is the *Jacobian* and it will be denoted by **J**. Since bus 3 is a constant voltage bus and not a load bus, we need to modify the preceding formulation slightly. For this power system, the variables of interest are $|V_2|$, δ_2, and δ_3. Taking into account that $|V_1| = 1.0$ p.u., $\delta_1 = 0°$, $|V_3| = 1.0$ p.u., and the $\mathbf{Y}_{\text{bus}}$ matrix of Equation (1), we write out the equations that need to be solved in accordance with Equation (2):

$$\begin{aligned} -1.8 = {} & (9.9504)(|V_2|)(1.0)\cos(95.711° + 0° - \delta_2) + (14.8659)(|V_2|)(|V_2|)\cos(-84.266° + \delta_2 - \delta_2) \\ & + (4.9752)(|V_2|)(1.0)\cos(95.711° + \delta_3 - \delta_2), \end{aligned}$$

$$\begin{aligned} -0.6 = {} & -(9.9504)(|V_2|)(1.0)\sin(95.711° + 0° - \delta_2) - (14.8659)(|V_2|)(|V_2|)\sin(-84.266° + \delta_2 - \delta_2) \\ & - (4.9752)(|V_2|)(1.0)\sin(95.711° + \delta_3 - \delta_2), \end{aligned}$$

$$\begin{aligned} 1 = {} & (4.9938)(1.0)(1.0)\cos(92.862° + 0° - \delta_3) + (4.9752)(1.0)(|V_2|)\cos(95.711° + \delta_2 - \delta_3) \\ & + (9.8961)(1.0)(1.0)\cos(-85.686° + \delta_3 - \delta_3). \end{aligned}$$

Simplifying the right-hand sides of the preceding expressions yields

$$P_2 = 9.9504|V_2|\cos(95.711° - \delta_2) + 1.4851|V_2|^2 + 4.9752|V_2|\cos(95.711° + \delta_3 - \delta_2),$$

$$Q_2 = -9.9504|V_2|\sin(95.711° - \delta_2) + 14.7915|V_2|^2 - 4.9752|V_2|\sin(95.711° + \delta_3 - \delta_2),$$

$$P_3 = 4.9938\cos(92.862° - \delta_3) + 4.9752|V_2|\cos(95.711° + \delta_2 - \delta_3) + 0.7444.$$

Thus, the elements of the Jacobian matrix are given by

$$\frac{\partial P_2}{\partial |V_2|} = 9.9504\cos(95.711^\circ - \delta_2) \ + \ 2.9703|V_2| \ + \ 4.9752\cos(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial P_2}{\partial \delta_2} = 9.9504\,|V_2|\sin(95.711^\circ - \delta_2) + \ 4.9752|V_2|\sin(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial P_2}{\partial \delta_3} = -4.9752|V_2|\sin(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial Q_2}{\partial |V_2|} = -9.9504\sin(95.711^\circ - \delta_2) \ + \ 29.5830|V_2| \ - \ 4.9752\sin(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial Q_2}{\partial \delta_2} = 9.9504\,|V_2|\cos(95.711^\circ - \delta_2) + \ 4.9752|V_2|\cos(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial Q_2}{\partial \delta_3} = -4.9752|V_2|\cos(95.711^\circ + \ \delta_3 - \delta_2)\,,$$

$$\frac{\partial P_3}{\partial |V_2|} = 4.9752\cos(95.711^\circ + \delta_2 - \delta_3)\,,$$

$$\frac{\partial P_3}{\partial \delta_2} = -4.9752\,|V_2|\sin(95.711^o + \delta_2 - \delta_3)\,,$$

$$\frac{\partial P_3}{\partial \delta_3} = 4.9938\sin(92.862^\circ - \delta_3) + 4.9752\,|V_2|\sin(95.711^\circ + \delta_2 - \delta_3)\,.$$

The above partial derivatives form the Jacobian matrix

$$\mathbf{J} = \begin{bmatrix} \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} \\ \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} \\ \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} \end{bmatrix}.$$

For the initial estimates $|V_2|$ = 1.0 p.u., $\delta_2 = 0^\circ$, and $\delta_3 = 0^\circ$, the initial Jacobian matrix becomes

$$\mathbf{J}^{(0)} = \begin{bmatrix} 1.4851 & 14.8515 & -4.9505 \\ -0.4950 & -4.9505 & 9.9380 \\ 14.7315 & -1.4851 & 0.4950 \end{bmatrix}.$$

The initial power mismatches are

$$\Delta P_2 = P_2 + 1.8 = 0 + 1.8 = 1.8\,,$$

$$\Delta Q_2 = Q_2 + 0.6 = -0.06 + 0.6 = 0.54\,,$$

$$\Delta P_3 = P_3 - 1.0 = 0 - 1.0 = -1.0\,.$$

Consequently, the corrected values after the first iteration are

$$\begin{bmatrix} |V_2|^{(1)} \\ \delta_2^{(1)} \\ \delta_3^{(1)} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1.4851 & 14.8515 & -4.9505 \\ -0.4950 & -4.9505 & 9.9380 \\ 14.7315 & -1.4851 & 0.4950 \end{bmatrix}^{-1} \begin{bmatrix} 1.8 \\ -1.0 \\ 0.54 \end{bmatrix} = \begin{bmatrix} 0.9516 \\ -0.1003 \\ 0.0483 \end{bmatrix}.$$

Although we assumed that $\delta_2 = 0^{\circ}$, and $\delta_3 = 0^{\circ}$ for simplicity, these angles must be converted to radians before performing the necessary matrix operations. The procedure is repeated until the variables of interest satisfy a prescribed precision index. Once this is achieved, we use the values of $|V_2|$, δ_2, and δ_3 given by the algorithm to compute Q_3 from Equation (5). This completes the solution process.

In the following we present a spreadsheet implementation of the Newton-Raphson method. Again, we take the power system of Figure 1. The input data section is identical to that of the Gauss-Seidel method discussed in Section 2 (see Figures 2 and 3).

The power-flow solution is shown in Figure 5. The calculations reveal that acceptable solutions are attained after 3 iterations for a precision index of 10^{-4}. As expected, the solutions agree with those obtained by the Gauss-Seidel method (see Table 4). In general, the Newton-Raphson method converges to the solution faster than the Gauss-Seidel method. In some instances, ill-conditioned problems lead to divergence by either method.

As can be seen in Figure 5, the Newton-Raphson method covers the cell range H1:T15. A complete list of Microsoft Excel formulas is given in Table 6.

	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	**Power-flow solutions by Newton Raphson method**												
2	**Iter.**	**Var.**	**Value**	**Value**		**Jacobian**			**Inverse of Jacobian**		**Power mismatch**		**Corr.**
3		**|V_2|**	1.0000	1.0000	1.4851	14.8515	-4.9505	0.0067	0.0000	0.0672	**dP_2**	1.8000	0.0484
4	0	**d_2**	0.000	0.000	-0.4950	-4.9505	9.9380	0.0801	0.0402	-0.0067	**dP_3**	-1.0000	0.1003
5		**d_3**	0.000	0.000	14.7315	-1.4851	0.4950	0.0402	0.1207	0.0000	**dQ_2**	0.5400	-0.0483
6		**|V_2|**	0.9516	0.9516	-0.3720	13.8697	-4.5894	0.0158	-0.0014	0.0741	**dP_2**	0.1012	0.0109
7	1	**d_2**	-0.1003	-5.745	0.2431	-4.7288	9.7225	0.0863	0.0405	0.0016	**dP_3**	-0.0328	0.0076
8		**d_3**	0.0483	2.765	13.5767	-3.0437	1.1630	0.0416	0.1226	-0.0011	**dQ_2**	0.1251	0.0001
9		**|V_2|**	0.9407	0.9407	-0.5149	13.6874	-4.5279	0.0168	-0.0014	0.0760	**dP_2**	0.0015	0.0002
10	2	**d_2**	-0.1079	-6.181	0.2806	-4.6727	9.6664	0.0875	0.0407	0.0025	**dP_3**	-0.0004	0.0001
11		**d_3**	0.0482	2.762	13.2786	-3.1127	1.1839	0.0418	0.1232	-0.0010	**dQ_2**	0.0019	0.0000
12		**|V_2|**	0.9405	0.9405	-0.5170	13.6846	-4.5270	0.0168	-0.0014	0.0760	**dP_2**	0.0000	0.0000
13	3	**d_2**	-0.1080	-6.188	0.2811	-4.6719	9.6656	0.0875	0.0407	0.0025	**dP_3**	0.0000	0.0000
14		**d_3**	0.0482	2.762	13.2740	-3.1137	1.1842	0.0418	0.1232	-0.0010	**dQ_2**	0.0000	0.0000
15		**NOTE:**	**d_i (rad)**										

Figure 5: Spreadsheet implementation of the Newton-Raphson method for determining the power flows in the network of Figure 1.

Table 6: Microsoft Excel formulas for the Newton-Raphson method.

Cell	Formula	Comments
H3:H14	List of numbers from 0 to 3.	Iteration number. Some cells are blank.
H2:T2	Labels for various quantities.	Newton-Raphson section.
I15:K15	Labels to highlight angles in radians and degrees.	No calculations.
I3:I14	Labels for variables of interest.	$\lvert V_2\rvert$, δ_2, δ_3.
J3:J14	Numerical values corresponding to variables of interest.	δ_2 and δ_3 in radians.
K3:K14	Numerical values corresponding to variables of interest.	δ_2 and δ_3 in degrees.
L3:N14	Jacobian matrices.	All matrices are 3 x 3.
O3:Q14	Inverse of Jacobian matrices.	All matrices are 3 x 3.
R3:R14	Labels for power mismatches.	ΔP_2, ΔP_3, ΔQ_2.
S3:S14	Numerical values corresponding to power mismatches.	ΔP_2, ΔP_3, ΔQ_2.
T3:T14	Correction terms.	$\Delta \lvert V_2 \rvert$, $\Delta\delta_2$, $\Delta\delta_3$.
J3:J5	Initial estimates for variables of interest.	$\lvert V_2 \rvert^{(0)}$, $\delta_2^{(0)}$, $\delta_3^{(0)}$.

Table 6 (*continued*): Microsoft Excel formulas for the Newton-Raphson method.

Cell	Formula	Comments
K3	=J3	Copies the value of $\lvert V_2 \rvert^{(0)}$.
K4	=J4*180/PI()	Converts $\delta_2^{(0)}$ to degrees.
K5	=J5*180/PI()	Converts $\delta_3^{(0)}$ to degrees.
L3	=IMABS(B4)*IMABS(C9)*COS (IMARGUMENT(B4) +IMARGUMENT(C9)-J4)+ 2*IMABS(C4)*J3 *COS(IMARGUMENT(C4))+ IMABS(D4)*C11*COS(IMARG UMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial P_2}{\partial \lvert V_2 \rvert}$.
M3	=IMABS(B4)*J3*IMABS(C9)* SIN(IMARGUMENT(B4)+ IMARGUMENT(C9)-J4)+ IMABS(D4)*J3*C11* SIN(IMARGUMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial P_2}{\partial \delta_2}$.
N3	=-IMABS(D4)*J3*C11*SIN(IMARGUMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial P_2}{\partial \delta_3}$.
L4	=IMABS(C5)*C11*COS(IMAR GUMENT(C5)+J4-J5)	Jacobian element $\dfrac{\partial P_3}{\partial \lvert V_2 \rvert}$.
M4	=-IMABS(C5)*C11*J3*SIN(IMARGUMENT(C5)+J4-J5)	Jacobian element $\dfrac{\partial P_3}{\partial \delta_2}$.
N4	=IMABS(B5)*C11*IMABS(C9)*SIN(IMARGUMENT(B5)+ IMARGUMENT(C9)- J5)+IMABS(C5)*C11*J3* SIN(IMARGUMENT(C5)+J4-J5)	Jacobian element $\dfrac{\partial P_3}{\partial \delta_3}$.
L5	=-IMABS(B4)*IMABS(C9)* SIN(IMARGUMENT(B4)+ IMARGUMENT(C9)-J4)- 2*IMABS(C4)*J3*SIN(IMARGUM ENT(C4))-IMABS(D4)*C11* SIN(IMARGUMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial Q_2}{\partial \lvert V_2 \rvert}$.
M5	=IMABS(B4)*J3*IMABS(C9)* COS(IMARGUMENT(B4)+IMAR GUMENT(C9)- J4)+IMABS(D4)*J3*C11*COS (IMARGUMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial Q_2}{\partial \delta_2}$.
N5	=-IMABS(D4)*J3*C11* COS(IMARGUMENT(D4)+J5-J4)	Jacobian element $\dfrac{\partial Q_2}{\partial \delta_3}$.
O3:Q5	=MINVERSE(L3:N5)	Inverse of Jacobian matrix in cell range L3:N5.
S3	=IMABS(B4)*J3*IMABS(C9)* COS(IMARGUMENT(B4)+ IMARGUMENT(C9)-J4)+ IMABS(C4)* J3*J3*COS(IMARGUMENT(C4))+ IMABS(D4)*J3*C11*COS(IMA RGUMENT(D4)+J5-J4)-E10	Power mismatch ΔP_2.

Table 6 (*continued*): Microsoft Excel formulas for the Newton-Raphson method.

S4	=IMABS(B5)*C11*IMABS(C9)*COS(IMARGUMENT(B5)+IMARGUMENT(C9)-J5)+IMABS(C5)*C11*J3*COS(IMARGUMENT(C5)+J4-J5)+IMABS(D5)*C11*C11*COS(IMARGUMENT(D5))-E11	Power mismatch ΔP_3.
S5	=-IMABS(B4)*J3*IMABS(C9)*SIN(IMARGUMENT(B4)+IMARGUMENT(C9)-J4)-IMABS(C4)*J3*J3*SIN(IMARGUMENT(C4))-IMABS(D4)*J3*C11*SIN(IMARGUMENT(D4)+J5-J4)-F10	Power mismatch ΔQ_2.
T3:T5	=MMULT(O3:Q5,S3:S5)	Computes correction terms $\Delta \lvert V_2 \rvert^{(0)}$, $\Delta\delta_2^{(0)}$, $\Delta\delta_3^{(0)}$. Multiplies the inverse of the Jacobian in O3:Q5 by the vector of power mismatch in S3:S5.
J6	=J3-T3	New estimate $\lvert V_2 \rvert^{(1)}$.
J7	=J4-T4	New estimate $\delta_2^{(1)}$.
J8	=J5-T5	New estimate $\delta_3^{(1)}$.
K6:T8	Copies of the formulas in cell range K3:T5.	Completes calculations for iteration 1.
J9:T14	Copies of the formulas in cell range J6:T8.	Replicate formulas as needed until solutions are within some precision index.

4. Classroom experience and pedagogical insights

The Gauss-Seidel and the Newton-Raphson methods are among the most popular numerical techniques encountered in power system courses. The authors introduce these techniques to their undergraduate students when discussing the power-flow problem. To reinforce conceptual understanding of these methods, the students are asked to analyze the power flows in a small three-bus system, such as the one given in Figure 1. To facilitate computer implementation, the authors make the spreadsheets described in this paper available to their students. This approach has pedagogical advantages since students are relieved from the burden of learning new software. By simply modifying the necessary spreadsheet cells, the students can easily determine solutions to their assigned problem.

From the authors' experience throughout the years, students express positive attitudes toward the spreadsheet implementation of their power-flow project. Ease of implementation; widespread availability of spreadsheets; convenient tracking and displaying of numerical results; transparency of results that are often obscured by specialized power system analysis programs, are among the most cited comments. These responses were obtained from informal conversations with students who enrolled in the power courses.

The authors recommend the use of the spreadsheet approach to solve only small power systems, perhaps up to three buses. Larger systems such as four- or five-bus systems may still be accommodated by adding the necessary rows and columns. However, the size of the spreadsheet may become unwieldy, thereby rendering the spreadsheet approach ineffective.

Large systems are more suitably analyzed using specialized software such as PSS/E, PowerWorld, or EMTP. These programs are highly sophisticated and requires several hours of training. These programs provide the user with a graphical interface so that virtually any power system may be simulated. Problems that may be solved using these programs include power flows, fault analysis, economic dispatch, among others. The sophistication of such programs may at times obscure the inner workings of the numerical methods that produce the simulation results. In this regard, the spreadsheet approach offers a more transparent platform for learning fundamentals at a formative stage, albeit for small systems.

The spreadsheet approach is effective in other respects such as gaining insight into the numerical techniques, making sense of the convergence or divergence of computer-generated solutions, developing intuition about well- or ill-conditioned systems, and handling “what if” questions with relative ease. If the emphasis of a power system course is on fundamentals, spreadsheets offer an attractive approach to learning a difficult topic such as the power flow problem.

5. Conclusions

In this paper we have presented spreadsheet implementations of two widely used methods for solving power-flow problems. The Gauss-Seidel and the Newton-Raphson numerical methods are introduced to students in power system analysis courses. Although the mathematical underpinnings are found in courses such as numerical analysis, power systems provide a suitable real-world application upon which constructivist activities can be designed by instructors. Spreadsheets provide students with an easily accessible tool with which mathematical models of real systems can be built and analyzed. Furthermore, spreadsheets lend themselves to answering “what if” questions when quantities such as real power, reactive power, or bus voltage change to new quiescent operating conditions.

Interested readers who wish to obtain an electronic copy of the spreadsheets presented in this paper are welcome to contact the first author at mlau@suagm.edu or see Ref. 5 for another example.

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