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Tolerance Stack-up Analysis in Manufacturing-Based Capstone Projects

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Abstract

Most manufactured products typically consist of multiple components assembled in specified configurations. Such components have specifications for important dimensional characteristics to ensure adequate performance once assembled. The specifications are typically given as tolerances within which components must be produced. Consequently, parts produced outside the specifications are rejected and not used in the assembled product. As dimensional variation is to be expected in produced components, it may accumulate, or stack up, to cause unwanted variation in the assembled product. When this happens, additional costs in scrap and rework would result and may extend to warranty charges and customer dissatisfaction should unacceptable products find their way to the customer.

In this paper, issues resulting from tolerance stack-up in capstone production will be addressed. The questions this paper will attempt to answer are: (1) Are students aware of the impact of tolerance stack-up in product development, and (2) What can be done to minimize the potential effects of tolerance stack-up before product launch.

Introduction

A tolerance is defined as the range of acceptable variance from the nominal value of a dimensional characteristic while still allowing the part to fulfill its functional requirements. This concept is relatively straightforward for engineers, technicians, and students to understand, the phenomenon of tolerance stacking is not as easily understood. Tolerances might be expressed bilaterally (in both positive and negative directions from the nominal dimension) or unilaterally (in only one direction from the nominal dimension). In general parts with greater dimensional tolerances allow companies to manufacture parts less expensively, because the dimensional requirements can be fulfilled using less precise tooling and equipment.

During the engineering design phase of the new product development process, design engineers develop the three-dimensional computer aided designs (CAD) for a product. This geometry is designed at nominal dimensions, that is, the system of parts is digitally designed based on theoretically perfect parts. However, the implementation of perfect parts is not reasonable because of myriad reasons and therefore the design engineers must develop a tolerance. This tolerance is generally determined by two things: 1.) Part functionality and 2.) process capability. To be successful, design engineers must respect the process capability of their company or supplier, or they risk specifying parts that are expensive due to tolerances which are too constraining [1] [2] [3].

Just as parts have standard tolerances, assemblies comprised of those parts have a tolerance as well. The total tolerance for the assembly is directly driven by the tolerances of the parts which comprise the assembly. For example, if the assembly were comprised of parts which are all at the upper end of their total tolerance, the assembly might not function as intended. Therefore, design engineers must be mindful of the total tolerance with respect to the assembly. This phenomenon is known as tolerance stacking. However, the implementation of a proactive plan to avoid production issues caused by assemblies which are out of tolerance due to tolerance stacking is not as straight forward to understand because the effects of tolerance staking are a result of interactions between parts. Once production begins, this problem is further compounded because of interactions across parts with respect to process capability [2].

Perhaps the easiest way for designers to combat the possibility of tolerance stack interactions is by using the worst-case method of tolerance stack analysis. When completing this type of analysis, the designer considers the extreme conditions of every part involved in the assembly. For example, if each part in an assembly is at the upper end of the tolerance; would the assembly still function with respect to the designed requirements? Alternatively, the designer could consider the situation where all parts are at the lower end of their tolerance and ask the same question. While this is perhaps the easiest way to perform tolerance stack analysis, it is not the most efficient from a perspective of cost savings. The worst-case scenario for tolerance stack analysis also assumes that if an assembly functions at both ends of the spectrum (largest and smallest) the middle will take care of itself [1]. In reality, the performance of an assembly of parts may be significantly degraded as it approaches either the upper or lower end of the tolerance spectrum. Design engineers must be careful communicating the concept of total tolerance of an assembly to avoid the perception that product performance is equal across the tolerance spectrum [4]

As an example, refer to Figure 1. All of the example parts in this illustration have a standard tolerance of ± .015. This means that the largest vertical dimension of PART A could be 2.015, while the smallest acceptable vertical dimension of PART A could be 1.985. When this bilateral tolerance of ± .015 is applied across the system (assembly) of parts, one can see the stacking effect of the part tolerances against the total tolerance. In this case, the worst case (largest) tolerance of the assembly is 10.000+.015+.015+.015+.015+.015=10.075, while the worst case (smallest) tolerance of the assembly is 10.000-.015-.015-.015-.015-.015=9.925, Therefore, the total tolerance range of this assembly is 9.925 -10.075 or 10.000 ± .075. This is achieved with each part involved in the assembly being within the part tolerance.

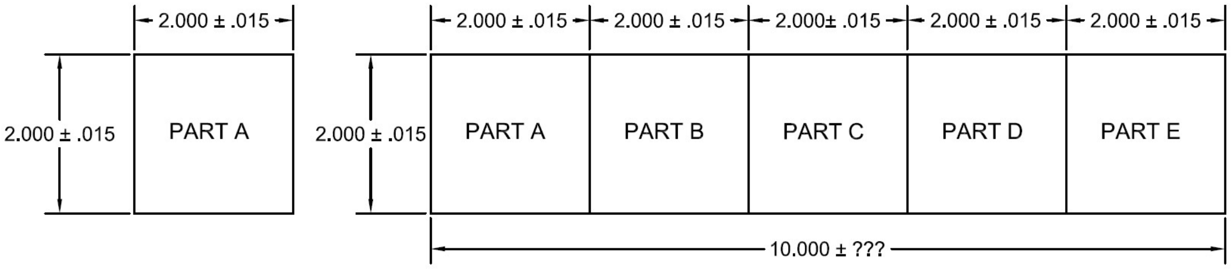

Figure 1: Tolerance Stack Illustration

As companies seek to produce parts of high quality as well as lower costs, the tolerances of parts become a driver of both cost and quality. Additionally, different configurations of parts may drive tighter tolerances because of requirements within different applications using the same part, further compounding production issue. As companies design products with advanced features and shorter design cycles, the role of tolerance stacking becomes a more urgent matter for design engineers to address early in the design lifecycle [5].

Researchers have explored tolerance stack-up analysis using different methods or a combination of methods. In addition to the conservative method of worst case (WC) method, the root sum squares (RSS) which started in the 1950s continued to be explored for a variety of stack-up conditions and modifications. M. Spotts suggested adding a safety factor by averaging the RSS and WC methods [5]. Additional RSS models were suggested by others to account for mean shift during production [6] [7]. Since the RSS assumes that component data follows normal distribution, more research was conducted for data that follow other distributions such as, but not limited to, uniform distribution, triangular distribution [8] [9] [10].

This study seeks to explore student understanding of the role of tolerances as a component of their educational experience at a Midwestern university within an engineering technology program. Students in the capstone course are often engaged in designing products that are assembled of several components. In some cases, components are chained together and could create some issues for fit and function for the assembled product should the potential of accumulation of variation be ignored. As a result of this paper, a process for conducting tolerance stack-up analysis will be developed and tested with students to determine its effectiveness.

Methodology Development

Only two popular alternatives are introduced in this study; the WC and RSS. Generally, engineers and designers would rather have tight tolerances to ensure fit, function and better quality of the assembled product. On the other hand, the aim of manufacturing is to introduce products so tight tolerances may cause issues when variation exists in the process.

As shown in Figure 1, the five components (A, B, C, D, and E) are assembled together create the final product. If the worst-case scenario is used, the assembled product would be assumed to have as large of the tolerance as the sum of the individual tolerances or ±0.075 inches. If the designer would like to keep the tolerance as ±0.050, then tighter tolerances for components would be sought. If this is applied to all components equally over the five assembled components, then the tolerance of each component will have to be set to ± 0.010 inches which would increase the cost of components, perhaps unnecessarily.

Using the RSS method, the variation of the assembled product is calculated using the squared values of the individual components. Let $\pm T_i$ denotes the individual tolerance, the assembled tolerance, T_A, can be given by Equation 1.

$$T_A = \sqrt{\sum_i {T_i}^2} \qquad (1)$$

To illustrate, consider the example in Figure 1 where the individual tolerances are ±0.015 inches for each of the five components. The assembly tolerance can be calculated to be ±0.034 inches using Equation 1 by taking the square root of the sum of squared tolerances. Compare this to the assembly tolerance obtained by WC method of ±0.075. Assuming the designer's tolerance of ±0.050, it is within the desired limits.

While WC method is non-statistical in nature, the RSS method is based on the normal distribution with the variation expressed in units of standard deviations (σ). The normal distribution spread is based on adding / subtracting three standard deviations on each side of the mean (μ) or (±3σ) which would include 99.73% of the data (Figure 2).

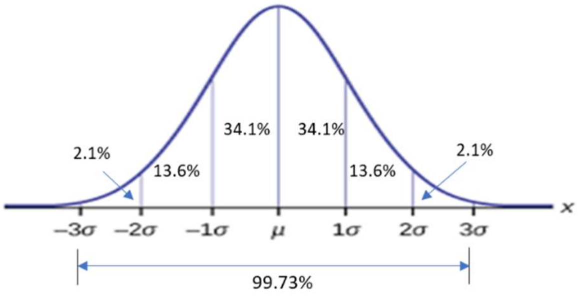

Figure 2: Normal Distribution

In tolerance stack analysis, a reliable estimate of the standard deviation of each of the chained components will be needed. This can be accomplished using one of the following methods:

1. Production data: This is the most reliable method as the standard deviation may be calculated from available data which may be in the form of inspection records or statistical process control (charts).
2. Similar products using the same process: When a new product is being introduced with no production data history, it may be necessary to see how similar products performed over time.
3. Engineering tolerances: This method is commonly used to determine the assembly tolerance. Two-sided specifications are equated to the process spread to estimate the variation. For a symmetrical tolerance, one side of the tolerance is equated to three standard deviations. This also depends on the capability desired for the assembled product.

It should be noted here that in many cases, some components of the assembly may already be utilized as components in other assemblies. Therefore, production data for those components may already be available. Therefore, the use of a combination of the above methods is also possible.

Whenever available, estimating variation from available data is preferred given that the process is stable. Absence of process stability (having special causes of variation) may cause problems in reliably predicting process performance over time, at least in the foreseeable future [11]. When data from similar products engineering tolerances are utilized, validation of data should occur using a limited production run. Further validation and update of tolerances should occur when full production data is available as well.

Once the component tolerances are reliably estimated, the analysis can be conducted to essentially answer two questions: (1) does the current assembly tolerance meet requirements? and (2) which components should be targeted to tighten tolerances if needed? To answer the first question, the analysis of rejection fraction (or parts per million) may be performed using the standard normal distribution. This should be done with some allowance for process mean shift during production of chained components. Utilizing the Six Sigma methodology in estimating the fraction nonconforming with 1.5 standard deviation allowable shift in the long run, the 4σ capability would result in a rejection rate of 6.7% or 66,807 defects per million opportunities (DPMO). This can be calculated using the standard normal distribution with Z=2.5 standard deviations (4.0 -1.5). When the process is operating at the Six Sigma capability, it is actually at the 4.5σ with the mean shift is accounted for and the rejection rate is 0.00034% or 3.4 DPMO. Table 1 displays other capability levels.

The second question as to which components should be targeted to reduce variation is not obviously decided based on the variability contribution of each component to the overall assembly, but also other variables including costs. Methods for such allocations have been introduced in the literature as demonstrated by Chase et al [2]

Table 1: Six Sigma Capability Scale

Sigma Level	% Rejects	DPMO
3σ	6.7%	66,807
4σ	0.62%	6,210
5σ	0.023%	233
6σ	0.00034%	3.4

When variation is excessive in the components, it is reflected in the resulting assembly and a fraction of which could be rejected. To improve performance of the process and reduce the overall assembly variation, it is imperative to identify which components need to be tackled first. This can be achieved by calculating the contribution each component makes to the overall variation using the variances. For example, to calculate the contribution of any of component, Equation 2 can be used:

$$\%\ Component\ Contribution = \frac{\sigma^2_{Component}}{\sigma^2_{Assembly}}\ x\ 100\% \qquad (2)$$

Once contribution for each component is known, order of improvement can be made based on size of contribution as well as cost of improvement. This cost is not easy to determine given that costs of dissatisfied customer may not be known. The plan-do-study-act (PDSA) continuous improvement cycle can be used to as means for carrying out such improvements.

Stack-up Analysis Process

In the preceding section, details about conducting tolerance stack-up analysis was presented. The aim here is to develop a usable methodology for students when they perform stack-up analysis. It begins with identifying components and their chained characteristics on the overall (assembly) characteristic. The individual (component) means of the characteristics are added to determine the overall mean. Similarly, using the RSS method, the variances are added to determine the overall variance then standard deviation.

The overall (assembly) variation can then be compared against the tolerance set by the customer or the designer to determine process capability. This will depend on the sigma scale (quality level) of interest. If the process is not capable or the variation does not quite meet established capability requirements, then process or product design changes may be necessary. If capability requirements are met, the process ends. Figure 3 presents a flow diagram that students will use to ensure that possible tolerance stack-up issues are resolved early in the product design phase. As the model shows, if no production data is available for the components, validation using initial or limited production run can be performed to ensure that the estimated standard deviations were similar and process capability requirements are met.

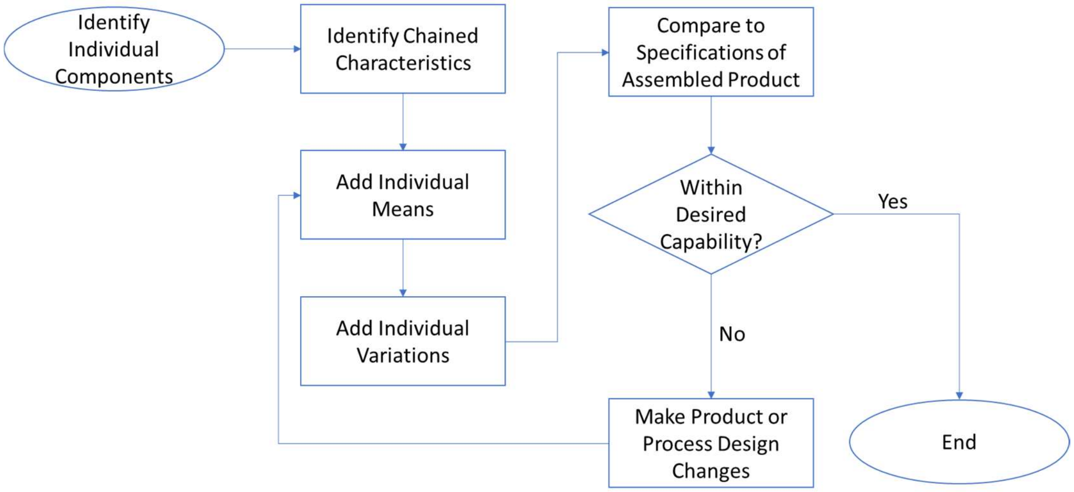

Figure 3: Proposed Stack-up Analysis Process

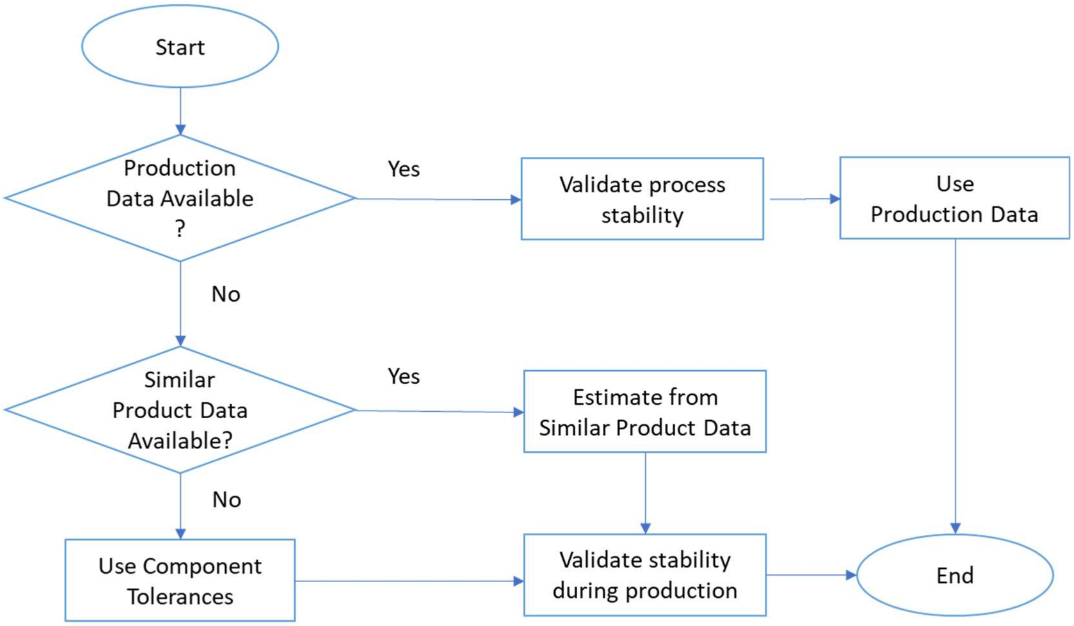

Figure 4: Determining Means & Standard Deviations

Model Testing

The aim of these tests is to validate the adequacy of the topic coverage and the provided model (flow diagrams) in guiding the students through tolerance stack-up analysis. The students were divided into their capstone teams and asked to conduct tolerance analysis. Using the models in Figures 4 and 5 above, teams were asked to:

1. Identify tolerance stack-up potential on their current projects. This will be carried out for each team on the capstone projects
2. Conduct tolerance stack-up analysis for the following cases:
   a. Case 1: Assembled components have both tolerances and production data
   An assembly is comprised of 3 components. The parts have been produced before and both production data and tolerances are available as shown in the Table 2. Perform tolerance stack-up analysis and make recommendations.

Table 2: Case 1 Tolerance Analysis Data

Component	Current Statistics	Tolerance
A	$\mu = 2.003$ $\sigma = 0.006$	2.000 ± 0.015
B	$\mu =2.000$ $\sigma =0.010$	2.000 ± 0.015
C	$\mu =1.998$ $\sigma = 0.008$	2.000 ± 0.015

b. Case 2: A combination of new components with tolerance only, similar components with data, and in-production components with data
If the assembly, comprised of 3 components, has only component A available and in production while components B and C are new. For part C, a similar product that is slightly larger in size is producing a standard deviation of 0.009 inches (Table 3). Perform tolerance stack-up analysis and make recommendations.

Table 3: Case 2 Tolerance Analysis Data

Component	Current Statistics	Tolerance
A	μ = 2.003 σ = 0.006	2.000 ± 0.015
B	Not available	2.000 ± 0.015
C	Similar Product Available σ = 0.009	2.000 ± 0.015

Each of the five capstone project teams presented their analysis based on the two questions asked. For the first question regarding potential tolerance stacking for the product being designed, teams had their discussions and identified potential tolerance stacking then briefly presented to the rest of the class. As for the second question with the two cases, students followed the model presented in the flowcharts of Figures 3 and 4:

- All teams conducted the analysis correctly:
  - For Case 1, they correctly used the current statistics provided from production data and not the tolerance.
  - For Case 2, they correctly used production data estimates for component A, tolerance estimates for component B, and similar product estimates for component C.
- All but one team correctly identified % contribution of variation by each component for both cases using Equation 2. One team used standard deviations instead of variances to make the calculations which was incorrect.
- As a result, all teams made recommendations as to which component to target first to minimize the variation in assembly.

Concluding Remarks

Accounting for tolerance stacking in manufacturing is a critical skill for graduating engineers to have. As a deliverable item of this study, the senior capstone course will be modified to include a section dedicated for tolerance stack-up analysis. This section will include coverage of the topic, including the process through the developed flow diagrams in this study. Their understanding of the concepts will be verified by working through a variety of cases. They will also identify any potential tolerance stack-up in their capstone product.

Future work may include introducing Monte Carlo simulation techniques and compare against the proposed process. Additional work may include using the Taguchi loss function in determining which components to target based on total cost to society.

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