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Torsion Tests to Study Plastic Deformation in Ductile Materials

Prof. Somnath Chattopadhyay, University at Buffalo, SUNY

Dr. Somnath Chattopadhyay teaches mechanics, manufacturing and design at the University at Buffalo. He has authored a text on Pressure Vessel s and till recently was an Associate Editor of the ASME Journal of Pressure Vessel Technology. His research interests are in the areas of fatigue and fracture of metals, carbon nanotubes, multi-scale material modeling and engineering education. He had a very successful industrial career with Westinghouse Electric where he directed and performed structural design of power generation equipment and is a registered professional engineer. He received his Ph.D. in mechanics from Princeton University.

Mr. Jerry Qu, University at Buffalo

Jerry Qu is currently a senior undergraduate student at the University at Buffalo's School of Engineering and Applied Sciences. He is pursing a Bachelors of Science in Industrial and Systems Engineering.

Mr. Adres Nuri, University at Buffalo

Adres Nuri is currently a senior undergraduate student at the State University of New York at Buffalo. He will be graduating in May 2017 with a Bachelor of Science in Mechanical Engineering.

Mr. Syed Ali Hasan, University at Buffalo

I am currently a senior undergraduate student at the University at Buffalo's School of Engineering and Applied Sciences. I am pursing a Bachelors of Science in Computer Science and a Bachelors of Science in Mechanical Engineering.

Mr. Nilanjan Kundu, University at Buffalo

Nilanjan Kundu is currently a senior pursuing a Bachelor of Science in Mechanical Engineering at the University at Buffalo's School of Engineering and Applied Sciences.

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ABSTRACT

This project is an experimental study on plastic deformation in ductile materials such as aluminum and steel. The objective is to stimulate interest in engineering undergraduate students the importance of plasticity in structural design and metal forming. It is proposed as a supplemental lab activity for the junior level Mechanics of Materials course. Torsion tests were performed on circular cylindrical bars to obtain torque-twist curves (the torsional shear stress vs torsional shear strain plots). The 0.2% offset yield shear strength, k, were estimated from these curves. The bars were twisted well into their plastic regions, and as the elastic/plastic torsion continued, the torques seemed to approach their limiting values. Experimental estimates for the limiting torques were in reasonable agreement with the values predicted by the so-called sand heap analogy. This states that the stress surface for a fully plastic cross-section is obtained by piling dry sand on a horizontal base whose shape is identical to the cross-section of the bar. For a circular cross-section, the sand heap is a cone, and the limiting torque is equal to ($k\ a\ /G$) times the volume of the cone, where a is the radius. The specimens were twisted to a predetermined maximum value. The directions of twist were subsequently reversed. After unloading and reloading from the initial loading phase, the materials seemed to yield in the reverse direction with lesser yield strength values. This shows that the torsional plastic deformation for metallic materials is one direction affects plastic deformation in the reversed direction and demonstrates the Bauschinger Effect in torsion.

INTRODUCTION

In this work, prismatic bars of circular cross sections have been loaded in torsion to study their responses in elastic and elastic-plastic domains. The purpose was to experimentally demonstrate the plastic deformation that takes place when the torque exceeds the amount that causes the bar to yield. This is an extension of the torsion test where the torques are such that yielding does not take place, and the torque twist characteristic is linear. When a bar is twisted beyond the elastic range (yielding), there is a non-linear torque-twist behavior. Simple analytical solutions for the deformation and stresses in a bar subject to axial torsion exist only for circular cross-sections. One of the simplest ways to study elastic as well as elastic-plastic behavior analytically is to assume an elastic-perfectly plastic representation of the stress-strain curve. For the case of torsion, this would be the shear stress vs. shear strain curve as shown in Figure 1. The shear stress varies linearly with shear strain in the elastic region, and reaches a constant value and stays constant in the plastic region. Sometimes this representation is termed as one of "zero hardening." However, most ductile materials exhibit strain hardening where in the plastic region, the shear stress monotonically increases with shear strain.

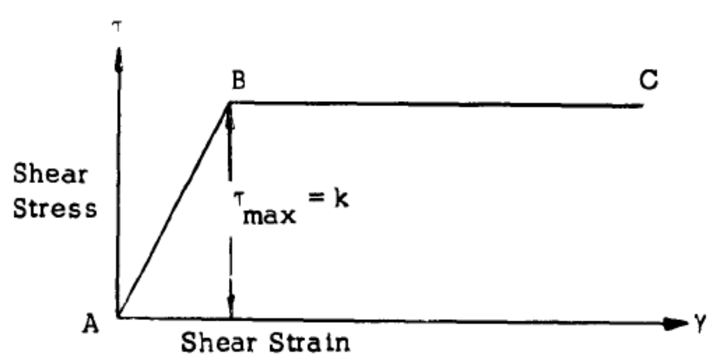

Figure 2. Idealized Shearing Stress-Strain Curve

Figure 1

There are two analogies to study elastic and elastic-plastic torsion, and are described below.

The Membrane Analogy for Elastic Torsion

An analogy between the elastic torsion of a bar and the small deflection of a laterally loaded membrane has been pointed out by (Prandtl, 1903). The membrane is stretched by a uniform tension F per unit length of its boundary, and is attached to a die whose edge plane is of the same shape as the cross section of the twisted bar. A uniform lateral pressure is then applied to the membrane to produce a deflection w at a generic point. The boundary condition is $w = 0$ along the edge of the die. It turns out that the contours of constant deflection correspond to the lines of shearing of the twisted bar. Also, the applied torque is proportional to the volume bounded by the deflected membrane and the xy plane. Since the membrane everywhere concaves to the applied pressure, the greatest value of the shear stress must occur somewhere on the boundary.

Consider a cylindrical or prismatic bar of constant cross section which is twisted and held in equilibrium by twisting moments applied at its ends. The bar is considered to be composed of an isotropic material possessing the idealized stress-strain relationship for an elastic, perfectly plastic material shown in Figure 1. Increasing torque causes the material to pass from the elastic region (line AB, Figure 1) into the perfectly plastic range (line BC, Figure 1). After a point in the cross section reaches the yield stress in shear (point B), this maximum shearing stress remains a constant value k as increasing torque causes an increase in the plastic region of the bar. Before examining the plastic behavior of the prismatic cross section, we will consider the stress characteristics in the elastic range.

The Sand-Heap Analogy for Plastic Torsion

An extension of the membrane analogy to elastic/plastic torsion has also been suggested by (Nadai, 1950). It is necessary to erect a roof of constant slope while having its base similar to the boundary of the cross section.

If the bar of certain cross-section is twisted beyond the yield point, certain parts of the bar will be deformed plastically. Similar to the case of the elastic torsion, the shearing stresses are directed tangentially to the contour lines of the plastic stress surface. The plastic stress function may be considered as a "roof" under which the membrane, geometrically the same as the cross section, expands. When the membrane touches the "roof," the condition of plasticity is satisfied and plastic yielding begins at that point in the cross section. As the torque is increased, as represented by increasing air pressure on the membrane, the membrane expands and touches more of the "roof." At the limit, the membrane fills the entire volume under the roof. The cross section is considered to have attained a fully plastic state. Following the membrane analogy, the torque required to achieve the fully plastic state is proportional to the volume under the "roof." The mathematical and physical interpretation of the plastic response for the case of complete yielding of the entire prismatic bar can be demonstrated by sand heaps covering a plate similar in cross section to the twisted bar. This analogy was first presented by (Nadai, 1923) at a meeting of the German Society of Applied Mathematics and Mechanics in Marburg, Germany in 1923.

> "A plate whose shape is geometrically similar to the cross section of the twisted bar serves as a horizontal tray to hold a heap of dry uniform sand. The heap is to be as big as it is possible to pile by pouring a gentle stream of sand on top of the model, the excessive sand rolling freely down the slopes of the heap and falling off the elevated tray." (Nadai, 1950)

In a solid bar made of non-hardening material, the fully plastic stress distribution represents a limiting state which is approached in an asymptotic manner as the angle of twist increases. The fully plastic value of the torque has a physical significance, since it is very closely attained while the deformation is still of the elastic order of magnitude. The stress surface for a fully plastic cross section can be obtained experimentally by piling dry sand on a horizontal base whose shape is geometrically similar to that of the cross section of the bar. The limiting torque is proportional to the volume of sand forming the hill. Once the limiting torque has been reached, the bar is free to twist in an unrestricted manner.

TORSION EXPERIMENTS

MTS axial and torsion testing machine (Figure 2) was used. The torsion tests were performed on 6061-T6 extruded aluminum and low alloy steel A-36 bars of circular cross section (1.0 inch in diameter). The loading was increased and continued through the inelastic region. Figures 3 and 4 display the data graphically and shows the torques appear to asymptotically reach constant values for both materials with increase in twist angles. However it was decided to reverse the loading after a certain value of torque. This asymptotic trend was more prominent in Figure 4 for A-36 material. Figure 5 displays the torque twist characteristics for both aluminum and steel specimens.

Figure 2: MTS Axial-Torsion Machine

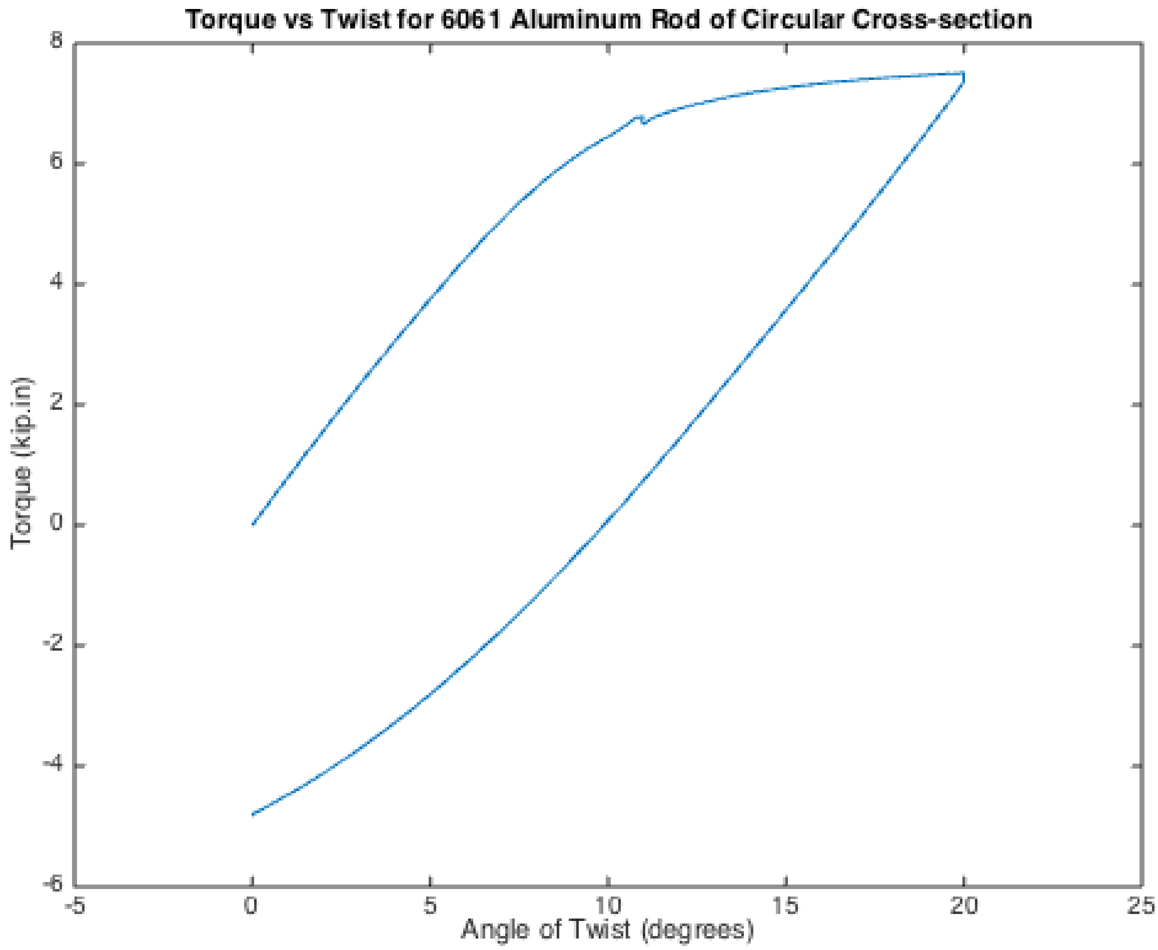

Figure 3: Torque vs Twist for 6061 Aluminum Rod of Circular Cross-section

Figure 4: Torque vs Twist for A-36 Steel Rod of Circular Cross-section

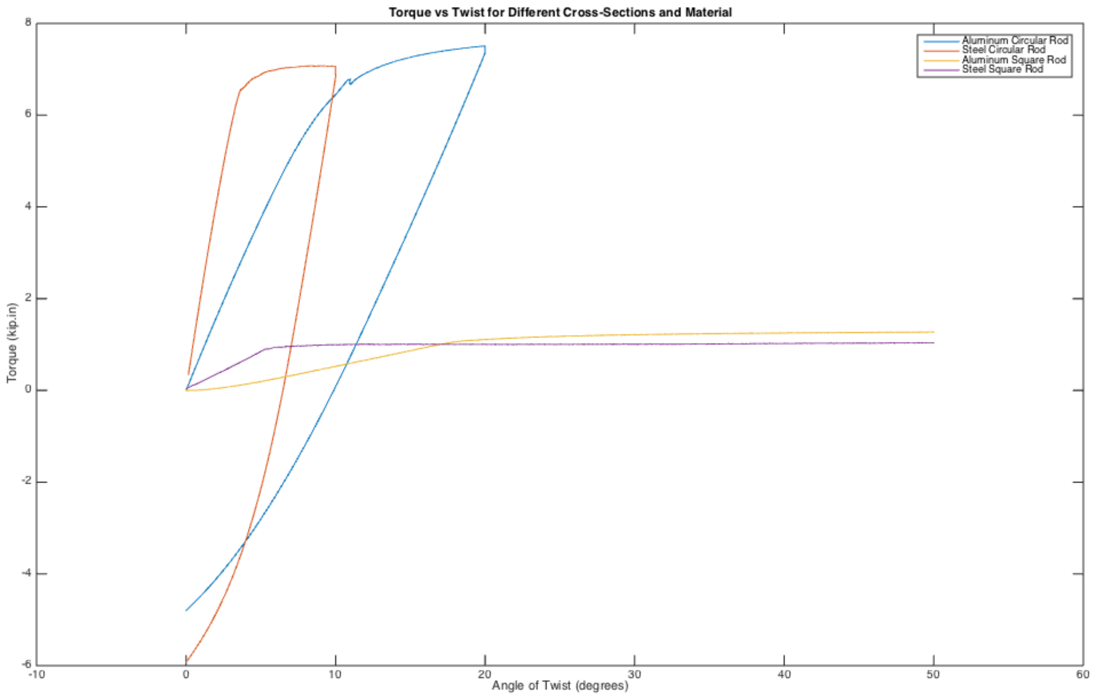

Figure 5: Torque vs Twist for Two Specimens on one plot

BAUSCHINGER EFFECT **OBSERVED IN SHEAR STRESS SHEAR STRAIN CURVES**

The torque twist ($T - \varphi$) curves obtained in Figures 3 and 4 were first converted into shear stress (τ) vs shear strain (γ) curves using equations (1) and (2) (Hibbeler, 2011)

The torsional shear stress is given by

$$\tau = \frac{16\,T}{\pi d^3} \qquad (1)$$

Here T denotes the applied torque, and d is the diameter of the specimen used in the test and equals 1 inch (25.4 mm)

And the shear strain is given by,

$$\gamma = \frac{r}{L}\,\emptyset \qquad (2)$$

Here φ refers to the twist measured in radians, r is the radius of the specimen and equals 0.5 in (12.7 mm) and L is the length of the specimen (distance between the jaws) and equals 8.0 inches (203.2 mm)

The τ- γ curves (as obtained from Figures 3 and 4) are shown in Figures 6 and 7.

One of the uses of the τ- γ plot is to determine the yield strength in shear. The τ- γ curve departs smoothly from the linear law relationship $\tau = G\,\gamma$, where G is the shear modulus. It can be quite difficult to determine the points at which the $T - \varphi$ curves depart from linearity. To deal with the problem, the 0.2% offset has been used for both the direct torsional loading as well as the reversed torsional loading and displayed in Figure 6 and 7. This is the stress level from which an unloading results in a plastic strain of 0.2%, or $\gamma = 0.002$.

Upon re-twisting in the reversed direction, the materials will start to yield in the reverse direction. We observe from both Figures 6 and 7 that both the materials yield at a lower value in the reversed direction. This reduction in yield strength experienced after a reversal of loading is known as the *Bauschinger Effect*. Named after German engineer Johann Bauschinger, the Bauschinger effect is an important property that occurs in most materials, and describes characteristics that changes in them due to stress applied, such as torsion in this experiment. Due to the stress distribution that occurs resulting from plastic deformation in materials, there is possible stress or strain changes that makes one characteristic increase and another decrease. More importantly, this change in property indicates the loss of isotropic behavior that occurs in these metals because of the added deformation by torsion.

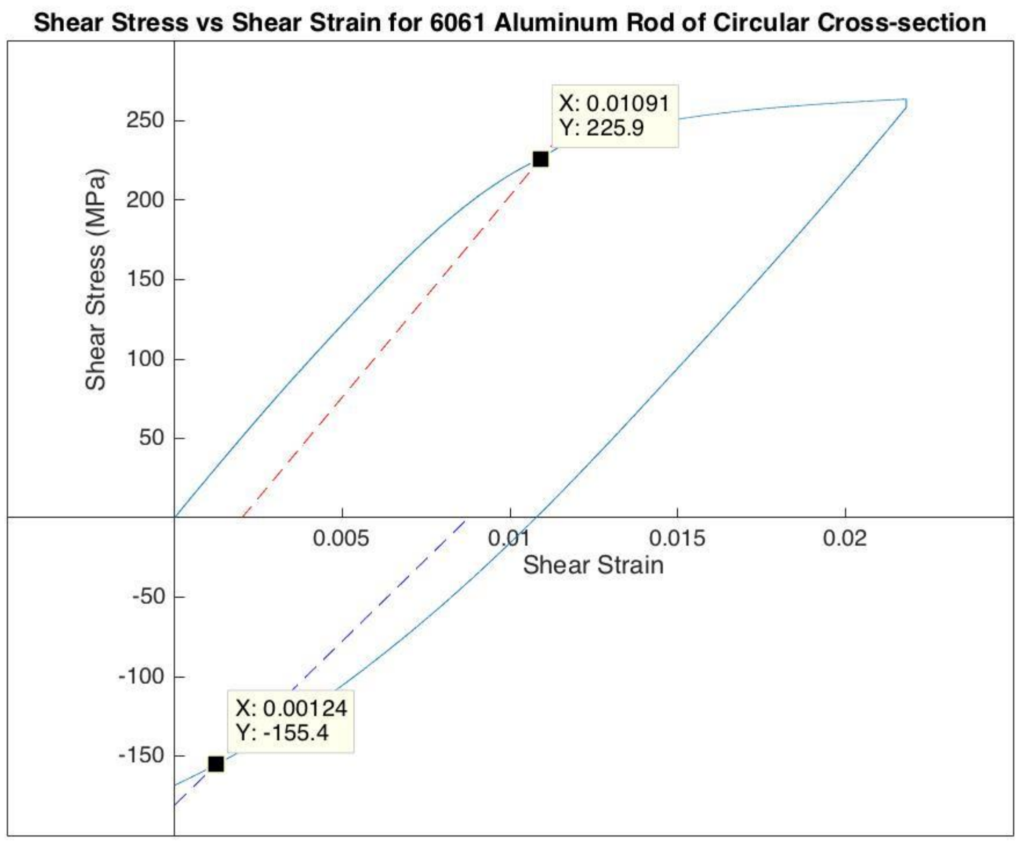

Figure 6: τ- γ curve for 6061 Aluminum

(0.2% offset yield strengths obtained during direct and reversed twisting)

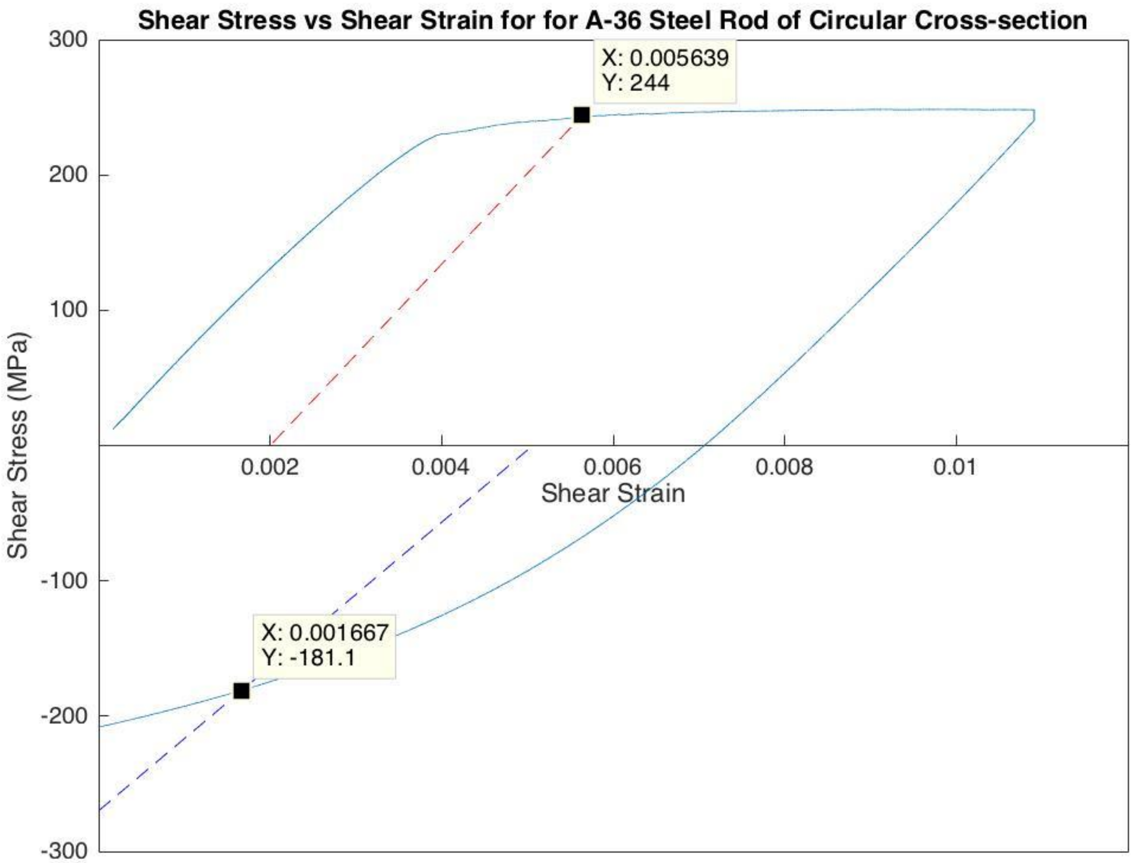

Figure 7: τ- γ curve for A-36 Steel

(0.2% offset yield strengths obtained during direct and reversed twisting)

Table 1 clearly demonstrates the *Bauschinger Effect* in aluminum and steel specimens

Sample	Direct Yield (MPa)	Direct Yield (psi)	Reverse Yield (MPa)	Difference (MPa) (Direct-Reverse)
6061 Aluminum	225.9	32764	155.4	70.5
A-36 Steel	244	35389	181.1	62.9

Table 1: Relation between Direct Yield and Reverse Yield

ESTIMATIONS OF FULLY PLASTIC TORQUES

These estimates are based on Nadia's Sand Heap Analogy:

The fully plastic torques for the aluminum and steel specimens were obtained from Figures 3 and 4, as the curves asymptotically reach the maximum values: For aluminum from Figure 3 we have

$$\boldsymbol{T_p = 7507\ lbin}$$

From Table 1 the 0.2% offset yield strength *k = 32764 psi*. We use this value in the analytical estimate of the fully plastic torque

The volume of a cone of height ka/G and radius (a) = 0.5 in, gives the fully plastic torque

$$T_p = 2G\left[\frac{1}{3}\pi a^2 h\right] = 2G\left[\frac{1}{3}\pi a^2\left(\frac{ka}{G}\right)\right] \quad = \frac{2}{3}\ \boldsymbol{\pi a^3\ k} \qquad (3)$$

$$T_p = \frac{2}{3}\pi(0.5)^3(32764) \Rightarrow \boldsymbol{T_p = 8577\ lbin}$$

For aluminum from Figure 4 we have

$$\boldsymbol{T_p = 7075\ lbin}$$

From Table 1 the 0.2% offset yield strength for A-36 steel *k = 35389 psi*. We use this value in the analytical estimate of the fully plastic torque

Using Equation 3 we get as follows:

$$T_p = \frac{2}{3} \boldsymbol{\pi a^3 k} = \frac{2}{3}\pi(0.5)^3(35389) => \boldsymbol{T_p = 9264\, lbin} \qquad (4)$$

Both experimental values and analytical values were estimated. The tables below show how these values compare with each other.

Sample	Analytical Estimate	Experimental Estimate
6061 Aluminum	8577 lb-in	7507 lb-in
A-36 Steel	9264 lb-in	7075 lb-in

Table 2: Fully Plastic Torque Comparisons

DISCUSSION

This project introduces the students to material plasticity through torsion experiment involving loads that causes the material to yield. As the torque is increased, a plastic region develops around an elastic core. There are errors introduced in the experiment primarily due to setup repeatability.

ASSESSMENT OF STUDENT LEARNING

A number of activities in terms of solving problems, and explaining some concepts will be introduced. Typical questions are:

1. Show that if the yield strength is exceeded, the governing equations for the torsional shear stress and twist do not hold. Why?
2. Calculate the maximum elastic torque when the shear yield is given. Also calculate the maximum elastic twist.
3. Why is the maximum torque different from the value predicted by Sand Heap Analogy?

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