Towards a Full Integration of Physics and Math Concepts: Words versus Meanings

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Abstract

Mathematics and physics concepts have been closely interrelated since their formal beginnings in ancient times. Moreover, from a wide variety of perspectives, it is possible to identify that the understanding of physics progressed as more complex mathematical ideas became available. In pedagogical practice, there are many instances where the teaching of one of these disciplines might obstruct the understanding of the other; this problem, combined with the difficulty of teaching them inside or outside a classroom, produces a ripe opportunity for educative improvement. After a significant experience of teaching an integrated physics-math course for freshmen undergraduate students, a number of inconsistencies were identified and previously reported. One of those inconsistencies is a trap rooted in language, and it creates worrying cognitive conflicts that interfere with students' learning. Specifically, the use by teachers of different names for the same concepts or ideas (perhaps because they look to relate specific concepts to everyday language) might be helping misconceptions to prevail. In this work, the authors focused on the analysis of terms like *mass*, *force,* and *torque*. To do this, they analyzed various research sources and texts to identify the roots of different names for similar concepts and their uses, and they considered the consequences of differing terminology and meaning to the construction of complex thinking.

Raising awareness about the inconsistencies of terminology in mathematics and physics and the resulting consequences is the primary objective of this study. This work was motivated by an authentic concern to facilitate the learning and comprehension of these subjects by students. Accordingly, the authors issue a call for action for a transformation in the teaching and learning of physics and mathematics through reflection on better use of terminology in these fields, so that the terms are negotiated between the disciplines, which results in precise descriptions of what is being taught, free of inconsistencies, confusion, and conflict.

Keywords: Physics and math integration, educational innovation, words and meaning, conflicts in physics and math terminologies, interdisciplinary negotiation, language of math and physics.

Introduction

Language is a very complex transit across a bridge of symbols that connect communication and meaning. Chomsky [1] would call language, "the core of the distinctive qualities of mind that are, so far as we know, unique to man and that are inseparable from any critical phase of human existence, personal or social" [1]. It also allows human beings to solve complex functions, plan their actions and solutions, and have control over their own behavior [2]. Nevertheless, the

uniqueness and usefulness of these qualities and tools might be arguable if subjected to in-depth study; i.e., language can also be troublesome and plagued by relativeness, depending on its context.
In this present work, the authors present some of the recollections and findings from an integrated Physics and Math course [3]-[4]. Our teamwork resulted in discussions of different approaches to the classes in physics and math and the discovery of discrepancies in them. In previous work, the authors presented some lines of study related to such findings [5]. This work seeks to expand one of the themes mentioned in said research.

Theoretical Framework

Linguistics, the study of the historical trace of languages, its laws, limitations, and definitions, deal with symbols and their meanings [5]. Semantics and semiotics have given particular order to the linkage between the signifier and the signified, the exponent and its signification, the *word versus the meaning*. The nature of these linguistic networks opens the door to reflect on how we use and transform language and how meaning derives from this process. This reflection reveals the importance of a wide range of critical understanding of meaning, especially important when designing teaching and learning activities.

Furthermore, meaning is internalized through an inherently collective constitution of knowledge. Learning is constructed and reconstructed from social and environmental interactions; it is transmitted in the same way [2]. Distinguishing which symbols and which meanings have direct associations, and which are mainly social constructions helps in organizing language and defining its central role in education. We want our students to understand us, to understand each other, and, most of all, to understand the language of knowledge, including that of mathematics and physics.

With this in mind, it is not surprising that several frameworks account for the understanding of the complex systems of cognition in which language is implicated. The *Theory of Legitimate Peripheral Participation* [6] and the *Systemic Functional Theory* [7] propose that the relationships between learning and language are inextricably linked to each other and to social interactions and that these connections depend on the particularities of their backgrounds. On the one hand, Systemic Functional Theory is used as a point of departure for the description and systematization of semiotic resources [8], supporting a comprehensive and formalized contextual symbolism in interaction with the particularities of its language and giving unique functionality to its discourse. On the other hand, the Theory of Legitimate Peripheral Participation proposes that the cognitive activities of human beings are linked to embodied interactions with materials and tools, to the mental representations that these interactions promote, and to social interactions that are sustained among the participants. It is through these bonds that patterns in learning and knowledge can be observed as situated activities that are contextually dependent and can be studied through sociolinguistic approaches [9].

If learning is a situated activity and language is directly linked to it, then it is logical to propose that contextually there would exist different *disciplinary discourses* [10]; i.e., complex sets of representations, tools and activities; semiotic resources that have been paradigmatically designed and that transcend both oral and written language and the physical means by which it is possible to perform; and the mechanisms of assimilation and meaning comprehension for learners of a particular discipline. Even when two disciplinary discourses might appear perspicuously linked by syntaxis, such as the case of physics and mathematics [11]-[12]-[3]-[4], evidence shows that they "may need to be considered as separate languages" [13] when integrally analyzing their uses and meanings.

Understanding the differences between the disciplinary discourses of mathematics and physics requires a detailed and holistic comprehension of the "full range of resources involved; i.e., language, mathematics, and images, as well as gesture, demonstration apparatus, various symbolic formalisms, and numerous others" [8]. Contrasting linguistic differences may cause severe cognitive conflicts during learning processes if they are not adequately addressed as a whole array of semiotic resources; nevertheless, these conflicts can also be managed purposely to stimulate complex thinking in students, the discovery and testing of newly acquired knowledge, and the learning from mistakes [14].

According to Redish and Kuo [13], conflicts between mathematics' and physics' languages may be classified according to cause: a) specific physical meanings given to symbols, b) symbolic hiding of functions depicting physical interpretations, and c) the portrayal of mathematical relationships as dependent upon "the physics they go through." This points out that the "use of equations in physics goes beyond interpreting and processing the formal mathematical syntax," mainly because of the physical meaning given to symbols that transmit information from the pure mathematical structure of an equation. Although the disciplinary discourse of mathematics could indeed promote the automatic discovery of the physics discourse, it is in their semantic and semiotic roots that one hinders the other.

Even though an integrated experience in the learning of physics and mathematics enriches knowledge, other semiotic resources need to be sustained by *appresentation* [8], an "ability to spontaneously infer the presence of further facets of a disciplinary way of knowing over and above those made available through the mode a student has been presented... before they can appropriately and holistically experience the disciplinary way of knowing." How is disciplinary science education contributing to widening the gap and uncertainty [15] among its shared ideas and meanings? What are the consequences of a symbol or word having several meanings that sometimes represent contrasting and conflicting ideas? The objective of this present work is to understand how and why meaning is accomplished within the context in which a shared concept is used in both physics and mathematics education.

Problems addressed in this paper

All the comparisons between the physics and statics concepts presented below come from the following five books: for physics, Giancoli [16],Young [17] and Mazur [18];and for statics, Hibbeler [19] and Meriam & Kraige [20]. The authors have chosen those books conveniently by familiarity, considering those books representative of the wide variety of textbooks available for the purposes of this present work. Here we discuss the conflicts of terminology for *mass, force,* and *torque* in the disciplines of physics and mathematics:

Mass

One of the most basic concepts required for study in our physics courses is *mass*. This term is also one of the primary sources of confusion among our students, mostly because of the words associated and used when introducing its meaning. It is often common in the classroom to hear students talk about the law of inertia, saying that it has “something to do” with forces. They might even be capable of correctly reciting Newton's first law, never realizing that the conceptual idea to which they refer is the same as the one used for mass. Maybe these semantic differences seem insignificant at first, but when compounded with the concepts about the migration of translational motion to rotational movement, the once-apparently-subtle-language differences become significant problems inside the students’ minds.

Although it may seem simple to relate the concepts of *angle* and *position* as well as *angular velocity* with *linear velocity* and *angular acceleration* with *linear acceleration*, the linguistic relationships left to be established become harder when a word, the signifier, has a different meaning, the signified. These differences interfere with the access to relevant cognitive processes embedded in learning not only the ordinary language of mechanics but also the complex meaning of physics-motion-phenomena. Even simple ideas like *mass*, *inertia*, *rotational inertia* and *moment of inertia* evoke very different meanings in the students’ vocabulary versus textbook lexicology.

In the sample of the five chosen books, the definitions given by Giancoli and by Young are very similar: “The tendency of an object to maintain its state of rest or uniform velocity in a straight line is called inertia. As a result, Newton’s first law is often called the law of inertia” [16]. “Newton used the term *mass* as a synonym for 'quantity of matter'… more precisely, we can say that mass is a measure of the inertia of an object. The more mass an object has, the greater the force needed to give it a particular acceleration” [16]. However, after mentioning that mass is the *measure of inertia*, they never use that idea again until they present the concept of rotational motion.

Marking a stark difference, Mazur [18] always refers to *inertia* without linking the concept to *mass*, at least not until very late in the book when he introduces the notions about relativity. He defines *inertia* as “a measure of an object's tendency to resist any change in its velocity,” [18] and he states that the resistance is proportional to the amount of material of an object, emphasizing that this only depends on the material of the object. In later pages, Mazur defines the standard of inertia as the kilogram and its basic SI unit as *ms* = 1kg.

These differences become highly relevant and problematic when talking about *rotational motion*. Both Giancoli [16] and Young and Lewis [17] present a very similar definition for this concept: “The quantity mr^2 represents the *rotational inertia* of the particle and is called its moment of inertia.” They later recall it by stating, “This product is called the moment of inertia (or rotational inertia)” [16]. Furthermore, Mazur [18] closely relates a new concept to the one previously analyzed: “Apparently an object's tendency to resist a change in rotational velocity, its *rotational inertia*, is not given only by the object's inertia.” In other words, an object can have more than one reason for resistance.

When physics courses given subsequent to the introductory course are reviewed, it is found that the concepts typically used in the Physics I course are not always congruent with the definitions or applications of terms in subsequent courses. For example, textbooks used in Physics I and a subsequent course both use the term, "moment of inertia," but they refer to different concepts; Hibbeler [19] refers to the *moment of inertia* but expands the idea into the *mass moment of inertia* and the *area moment of inertia.*

Force

Some of the exponent words often used to signify the concept of force in its several manifestations may seem to have fundamental misconceptions. For example, depending on the physics textbook, *gravitational force* and *weight* might convey the same meaning, but they may drastically differ. Some books, like Giancoli's [16], refer to *weight,* expressing it always as *mg* in the free-body diagrams used as illustrations, while Young [17] calls it *Weight* and symbolizes it with a *W*. Mazur [18] calls it *gravitational force,* naming it such according to the object on which force is applied in his discussions.

As most teachers probably have experienced, there is a real-life misconception around the daily use of the words *mass* and *weight*. It is essential to think about how these discrepancies affect students’ learning, as seen when a word evokes an unexpected or unreliable meaning already natural in students’ language. Such is the case of the term, *normal force*; Giancoli [16] defines it as *one* of the *contact forces,* and so does Young [17]. It is then specified as the *perpendicular component* of the *contact force* on the surface, additionally defining the *parallel component* as friction.

In the language of mathematics, there are some other common misconceptions related to the normal force and representations of what *being perpendicular* means with respect to the word *normal*. Students frequently indicate their conceptions of the normal force as a synonym for the "common force," and they may believe that it refers to the reactive force of the *weight*. In these cases, a whole cluster of misconceptions can arise, even if they are not directly associated; these are generally addressed using conventions, language usage that may not always be meaningfully constructive.

Such is the case of the concept of *tension*, which only Mazur [18] describes for each of the forces every time. It may be worth pointing out that when teaching students about the terms *normal*, *tension* and *forces*, there are different meanings and thus different words used, but all go together within the force diagram.

Finally, there is also the use of the terms *net force* in contrast to the *sum of forces*. Sometimes when using *net force*, students ignore the *net* part and only remember the mathematical equation of $f=ma$; this confuses them, because the central meaning of the existence of more than one force acting on the object is lost in the wording. A second issue comes when students confuse $Fg=mg$ with $F=ma$, a problem developing when g is called an acceleration force when instead it is really a *field*.

Torque

The third concept in which language inconsistencies have been frequently found has to do with the effect of a force that makes an object rotate. In most physics books, this is referred to as *torque*. Giancoli calls it the *moment of the force* and then uses the idea of *torque* from then on: "The angular acceleration then is proportional to the product of the force times the lever arm. This product is called the moment of the force about the axis, or, more commonly, it is called the torque and is represented…" [16]. In contrast, Mazur specifies it is analogous to the force and highlights where the differences exist: "This ability to rotate an object about an axis is called the *torque about the axis*. Torque is the rotational analog of force: Forces cause (translational) acceleration; torques cause rotational acceleration about an axis" [18].

It is important to note that the word *moment* is not even found in Mazur's book index [18]. All three physics textbooks focus on the use of the term *torque* while the specialized statics books (mathematics) prefer the term *moment*; for example, "When a force is applied to a body, it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called torque, but most often it is called the moment of a force or simply the moment" [19].

A question then arises from these brief examples: Are we, as educators, making the acquiring of knowledge difficult by ignoring negotiations of congruence in the linguistics of mathematics and physics?

Discussion

As introduced in the theoretical framework of this paper, it is crucial to take into account how the use of words and their meanings affect the higher education of students and the development of their competencies, particularly in the appropriation of knowledge in the fields of physics and mathematics [8]. Some words (the signifiers) might carry fundamental conceptual misconceptions; therefore, careful construction of the meaning (the signified) and its particularities is needed so that the linguistic resources for students help them reduce the inconveniences of differing terminology; i.e., the teaching does not make the acquisition of knowledge more difficult.

In these three cases (mass, force, and torque), in both physics and mathematics, Redish and Kuo found language constraints [13]. The specific meanings given to symbols in the case of the concepts of *mass* and *inertia* symbolically represent different functions depicting physical interpretations for the *moment* and the *torque* and the use of mathematics that depends totally on "the physics they go through" in the case of the *force* and its several manifestations. These language traps have direct consequences on the *appresentation* [10] process of students' understanding and acquisition of knowledge in physics and mathematics, leaving cognitive archipelagos between these two disciplines and students with the responsibility of bridging the gaps without enough semiotic tools.

It is notable that language differs not only in the teaching materials of the mathematics and physics disciplines but also in the levels of abstraction and specialization of the courses. This leads to the establishment of different words for the same physics ideas latent in different mathematical expressions, and vice versa. As a brief example, in the Statics textbook of Meriam and Kraige[20], the use of the word *momentum* is highlighted as having a close relationship between its mathematical meaning and its semantic roots.

When building meaningful structures of physics and mathematics concepts, *participation theory* [6] can also be a helpful guide to understand how students' cognitive activities are linked to embodied interactions with their environment. The chosen examples in this paper concern the properties of bodies that students can witness in experiments. Such interactions could be perceived, for example, through language simplifications in the concepts of *mass* and *weight*.

Conclusion

A fundamental goal of higher education institutions should be transforming knowledge that has been comprehensible by just a few and reframing it with an internal consistency so that the democratization of knowledge acquisition is widened regardless of disciplinary boundaries. Many changes have been made in the pedagogical practice of physics and mathematics teaching, but almost none have been directed to the parallel terminologies in the contents of the two

disciplines, where indeed there would be resulting improvements in the way that knowledge is acquired.
This paper, therefore, is a call to action, founded in a deep understanding and reflection on the significant integration that needs to be made among the science disciplines, particularly between mathematics and physics. We call for the search for a broader understanding of the issues and to discover the options for a more integrated teaching and learning experience for students.

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