AC 2010-778: TWO LEGACY CYCLE MODULES IN MATH AND CHEMISTRY FOR HIGH SCHOOL STUDENTS BASED ON FUEL CELL TECHNOLOGY

Mohamed Abdelrahman, Tennessee Technological University

Cynthia Rice-York, Tennessee Tech University

Dalton York, Tennessee Tech University

Cynthia Stowers, Clark Range High School

Robert Sircy, Tennessee Tech University

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Abstract

The current paper is focused on relaying the experience of two high school teachers that participated in a research experience for teachers (RETainUS) program at Tennessee Tech University during the summer of 2009. The program provided the teachers with the opportunity to experience the full cycle of research from formulating a research question and a research plan, to carrying out the research plan along side mentors who acted as consultants to the teachers. The two of the participants were a high school math teacher and a pre-service high school chemistry teacher. Although the two participants worked in the same fuel cell laboratory and shared to some extent the same mentor, the focus of their research and how they would take back their experience to class was completely different. The math teacher focused on research aimed at trying to identify patterns in the response of a PEM fuel cell under different operating conditions in order to better understand the internal behavior of the fuel cell. The chemistry teacher focused on improving the efficiency of Direct Methanol Fuel Cells by reverse engineering an alternative proton exchange membrane composite presented in a recent journal article. The experience of the two teachers has been challenging but rewarding with great contributions highlighted by high potential for being coauthors on technical refereed publications with their mentors. A component of the RET experience was the development of a Legacy Cycle inquiry lesson unit intended to connect engineering research to high school mathematics and science curriculum standards. This paper describes one of the mentor's and teachers' experiences and the two legacy cycles produced by the teachers.

Introduction

The Center for Manufacturing Research of Tennessee Technological University is currently hosting an NSF supported site to provide thirty high school teachers with a 6-week multi-disciplinary summer experience in manufacturing relevant research with an academic year follow-up plan. This program aims to contribute to the retention and the advancement of the manufacturing base in the US through meaningful changes in the teachers' understanding of manufacturing and how it relates to the Math and Science Curriculum[1,2,3,4]. The program also aims at improving the teachers' comprehension of the research and development process through hands-on experience and real world problems[5,6,7,8] that relate to: a) advancing the state of the art in conventional manufacturing processes such as metalcasting; b) new trends in manufacturing such as rapid prototyping, c) emerging technologies such as nanomaterials and manufacturing of fuel cells and special coating materials, and d) enabling technologies serving manufacturing processes in general such as intelligent optimization. Manufacturing is a field where boundaries between disciplines disappear opening opportunities for multidisciplinary research. The research projects and faculty mentors participating in the program represent 5 different disciplines in the college of engineering. This offers the teachers a multi-perspective view of how underlying mathematical and scientific concepts are integrated in engineering applications. RETainUS last summer targeted math and science high school teachers in the Upper Cumberland area which includes 14 counties, many of which are underserved and economically disadvantaged (60%)[9]. The program will also target Hamilton county school district with 42,000 students and a large percentage of minorities (38%). Support for the program from the Upper Cumberland Study Council incorporating these school districts and from Hamilton County has been secured.

As part of the program, the teachers develop a curriculum learning module based on their research experience and have a budgeted amount ($2000) referred to as the "equipment mini-grant" to spend on resources and equipment for their classrooms to aid in the delivery and implementation of their learning modules.

The proven model for this learning module is the Legacy Cycle Module[11] based on the research findings of the VaNTH project group. The Legacy Cycle lesson format consists of six stages 1) a challenge question, 2) generate ideas, 3) multiple perspectives, 4) research and revise, 5) test your mettle, and 6) go public. The cycle is based on current learning theory presented in How People Learn: Mind, Brain, Experience, and School[6]. During the summer research institute, a one-day Legacy Cycle workshop was provided to the teachers. The workshop provided the framework for the teachers to develop their instructional materials and is delivered early enough in the summer to allow for brief checks of progress during the summer institute. It is anticipated that the teachers will beta test components of their modules during the Fall and Spring following the summer institute. Using feedback from the Evaluation Plan, the teacher participant will present a final Legacy Cycle Module at the scheduled Legacy Cycle Module Conference in June, a calendar year after the summer research institute. Figure 1 depicts the main features of the RETainUS program.

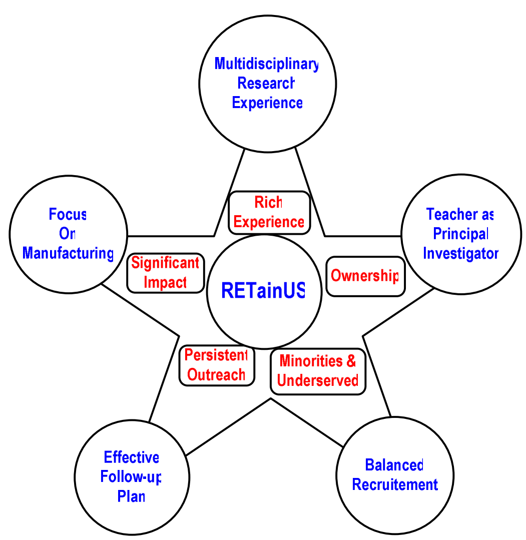

Fig. 1: A Diagram Illustrating Main Features of

Proposed RET Program

Figure 1: Main Features of RETainUS program

The paper is organized as follows: the current section provided an overview of the RETainUS program. Next, a summary of the experience of one of the groups working in the Fuel Cell's laboratory[12] is provided from the mentor's perspective. A summary of the two teachers' experience and legacy cycles is provided in the final two sections of the paper.

Summary of Research Experience in Fuel Cells Laboratory- A Mentor Perspective

The two teachers joined my lab last summer through the NSF RetainUS program, for 6 weeks. They were encouraged to take ownership of their portion of an on-going research project in my lab and tailor a niche that would exploit their individual strengths and interests. Their goals and expertise were vastly different—one was preparing to be a high school science teacher, while the second was an in-service math teacher. My lab is housed in the chemical engineering department at Tennessee Tech University and is very hands on science orientated. The central theme in my lab is fuel cells, promoting green energy—the topic easily lends itself to high school level science questions and demonstrations.

The math teacher was given the challenge to make some portion of an existing fuel cell project relevant to her curriculum. Her project was focused on assessment of freezing phenomena in automotive fuel cell applications. She worked very closely with an undergraduate student in my lab in analyzing the state and amount of water distributed within the active layers of a fuel cell under frozen conditions. Sub-zero cold-start is relevant to automotive fuel cell operation efficiency and durability. Electrochemical impedance spectroscopy (EIS) was used to qualitatively evaluate the changes in charge transfer and diffusional resistance due to the presence of residual water within a frozen fuel cell. Typically, the EIS spectrum is fit using a transmission line model of capacitive and resistive elements. Unfortunately, the resulting numeric output of this type of model is not unique. To avoid miss interpretation of such a transmission line model, I suggested simple error analysis. As the teacher became familiar with the experimental procedures and the EIS output, she made an alternative suggestion of physically measuring the radius of the two semi-circles produced in the EIS spectrum. She found that the output of the EIS measurement provided an excellent example of 'complex variables' and the concept of imaginary numbers, a topic required in her math curriculum.

With the continued use of the math teacher's measurement technique, the freeze project is able to access the changing state and amount of water within our fuel cell under frozen conditions. This tool has been invaluable in accessing relative changes in charge transfer and diffusional resistance upon specific modifications to the pre-freeze conditioning procedure and the freeze profile. We are in the initial stages of preparing a manuscript for publication.

The pre-service science teacher's project helped hone his lab skills and trained him how to operate and evaluate a fuel cell and its related components. He was tasked with reverse engineering a composite proton exchange membrane as loosely described in a recent journal article, for portable direct methanol fuel cell applications. The intent of the composite membrane was to reduce fuel crossover without negatively impacting proton conductivity, thereby enhancing power output of the fuel cell. One testing method that he selected to use to assess the proton conductivity of his membranes in comparison to those commercially available was titration to determine the number of acidic sites with each membrane. The science teacher determined the appropriate testing protocol and measured the ion-exchange capacity based on the number of acidic sites. He plans to further use this protocol in his classroom to help his students visualize the acid/base relationship.

Fuel cell testing was done to assess the quality of the prepared membrane. The science teacher learned to build and assess the characteristics of a single cell fuel cell. He became very adept at evaluating the quality of his fuel cell build. The produced membranes were segregated with the added filler sedimented to the bottom of the Petri dish during recasting, as visualized with scanning electron microscopy. The original intent was to produce a homogeneous composite membrane with the dispersed sulfated zirconia (S-ZrO_2) additive to hinder methanol crossover. Due to the segregated nature of the composite membrane expectations were low for the performance of these membranes. Upon actual fuel cell performance, it was found that his membrane, while not reducing fuel crossover, unexpectedly significantly enhanced fuel cell performance. It appears that the unintentional segregation of the composite membrane enhances performance and has induced a shift in the project focus towards the understanding of the performance enhancement and optimization. We are in the final stages of manuscript preparation, highlighting his direct methanol fuel cell performance.

Both teachers were also involved in a 4 day fuel cell workshop that I was running for high school students in the 'President's Academy' here at Tennessee Tech University. They were put in charge of difference activities designed to educate and excite the students about fuel cells. The performance of the teachers was impressive especially with how fluid their understanding of fuel cells was and how well they communicated it to the students.

The mentor for the teachers' comments: "I speak for my entire group when I say it was a pleasure to have both teachers in our lab. They made truly synergistic contributions to the work, which has had lasting impact. I could see the excitement in their eyes when they found ways to link what they teach to their high school students to relevant physical phenomena."

Legacy Cycles

The teachers prepared two legacy cycles as learning modules, to take back to their classroom, as part of their experience at Tennessee Tech. The first legacy cycle features a particular characterization of a Nafion membrane performed by a classic acid- base titration with one twist. The acid base titration is only effective after a thorough double displacement reaction in the membrane that proceeds according to the activity series. The students are to characterize the membrane and then promote the sale and distribution of their membrane based on their findings via a competitive advertisement. Students' learning and reactions to the cycles will be shared along with samples of their work. The second legacy cycle was focused on teaching concepts of complex numbers to high school students. The summer research experience showed the importance of the complex plane for graphical analysis and provided an interesting context to convey to the students what is otherwise a difficult concept to grasp. The Legacy Cycle is to be implemented part of an Algebra 2 class and would be implemented with 30 students in the 2009–2010 school year. It features several concepts including: imaginary numbers, plotting complex numbers on the complex plane, operations with complex numbers, fractal geometry and uses hands-on activities utilizing miniature fuel cell cars and fractal geometry. During the summer research experience, the teachers had several days spread intermittently throughout the six weeks to be trained in question oriented, project based instruction technique known as a Legacy Cycle. This type of instruction allows for a challenge to be presented to the students, much like an engineering challenge. The responsibility then falls on the student to seek means to answer the challenge. The students are not, however left to their own devices but guided by a series of logical steps that the classroom teacher implements. The first is to generate ideas based on what we know already from previous knowledge. The second is to gain perspective from multiple expert-type sources, which would give insight into the challenge. Then, when the students discover from the expert sources there are topics they do not know enough about to complete the challenge, the students will ask for, and research for the missing information. The last steps are to test the students' ability to complete the challenge, and to present publicly their success.

Nafion Titration Legacy Cycle

The two teachers participated in research in a Lab that was devoted to researching fuel cells, focused on how to overcome the technical limitation that keep them from mass commercialization as an alternative source for energy production. The direct mentor of the science teacher was working on a novel membrane to improve better the performance and durability of Direct Methanol Fuel Cells (DMFCs). There are two problems with DMFCs at this point: (1) is methanol crossover through the membrane, thus limiting power and (2) is the formation of carbon monoxide, a reaction intermediate, blocking the anode catalyst layer of the fuel cell, and thus leading to decreased reaction efficiency. The research topic of the science teacher was to reverse engineer a Nafion based composite membrane that contained Sulfated Zirconium Oxide(S-ZrO_2), to determine if the performance was could be increased due to reduced methanol crossover. Several experimental techniques were implemented to evaluate the performance of the prepared composite membrane vs. a standard Nafion membrane.

The composite membrane was created by the end of the first week of research. The membrane had to be put in a fuel cell to be tested. This required parts of a fuel cell made specifically for DMFC. The performance stand also had to be modified to accommodate the liquid methanol fuel. The process of modifying the test stand, accompanied by a literature review of DMFCs, electrochemistry, and electricity in general, took two more weeks. Testing and retesting performance with more of the S-ZrO_2/Nafion composite membranes, side by side with standard Nafion membranes exceeded the allotted 6 weeks. The teacher was asked to stay for an extra two weeks to continue with the research and gratefully accepted. The performance testing of the membranes outperforming those found in the literature at that time. This led to a continuing of the research by the lab into the fall, and an attempt, in process, to publish the combined efforts in a peer-reviewed journal.

The teacher comments: *"Being an education student graduating the next December, this was a very rewarding opportunity for me. I had some experience with engineering, being a chemical engineering major before I switched to education, however I did not expect to have such great success, most of which came about by the guidance of my mentors and fellow lab associates. More specifically my mentor who is writing the paper we are hoping gets published. I found a love for research and would like to pass that on to my students. I really liked the fact that the research I was doing had a purpose and practical application in everyday life and industry."*

The science teacher further explains: "The Legacy Cycle model, in my opinion, is a very accurate representation of an engineering based problem that can be found in industry and the research lab. The Legacy Cycle also gives instruction and practice as to how to logically and efficiently approach complex problems that one may not have all the information to solve initially. It was the very same progression I went through to complete my research. Therefore I felt the necessity to recreate the exact problems that I faced while doing my research. One of the problems I faced, when I created a new membrane that had not been characterized before, was to identify certain attributes of my new membrane in an acceptable and standardized format. One of the main characteristics of a fuel cell membrane is how many protons it contains that are available for transport across the membrane. While having no procedure to go by, I first assessed my knowledge and then turned to published literature sources to research the standardized method for testing proton capacity in a fuel cell membrane."

The challenge to the students as part of the legacy cycle is based on the premise that the Nafion membrane is the largest contributor to how well the cell performs. If the membrane can allow a large number of protons to cross, then the final reaction will speed up—which makes the electrons flow faster through the circuit, and gives us more electricity. So just how many protons can a Nafion membrane hold? An electrochemical journal article said the answer could be determined by classical titration with strong base, after soaking the membrane in a solution of table salt. Students would be presented with the following challenge: "*You are an employee at a major Fuel Cell membrane plant. You have just made a new membrane that will revolutionize the industry. However first you need to test the membrane, define its characteristics, and market the membrane to potential buyers. Therefore you need to find the amount of protons that can be exchanged through your membrane and develop an advertisement with the data you found."*

Prior to starting the legacy cycle, students will be introduced to fuel cell as a device that converts chemical energy into electricity by utilizing the electrons that participate in the electrochemical chemical reactions. Electrons and protons are separated from a reactant, such as methanol or hydrogen, at the anode via electrooxidation, the protons are directly transported through the membrane. While the electrons are routed through an external circuit, which passes an electronic device (electric motor) on its way to the cathodic side of the fuel cell adjacent to the membrane, to be reunited in the presence of oxygen to form water. The easiest anode reactant to illustrate is Hydrogen gas. A hydrogen molecule can be electrooxidized to two electrons and two protons, reaction (1).

$$H_2 \rightarrow 2H^+ + 2e^- \quad (1)$$

Facile proton transport through the membrane separating the anode and the cathode is required, while restricting electron transport. The best way found to allow only protons to travel across the cell is to use a selectively permeable membrane called Nafion. Nafion is a co-polymer consisting of two distinct chemical structures, a hydrophobic tetrafluoroethylene backbone with perfluorovinyl ether side chains terminated with sulfonate groups (SO_3^-), that phase separate into ionic percolation networks. Nafion is an electronic insulator, forcing the electrons to flow through the external circuit. Nafion will, however, let protons, which are positively charged proceed through the membrane associating with SO_3^- negatively charged end groups, as it makes its way across the membrane. The cathodic reactant is typical oxygen, that in the presence of electrons and protons, is reduced to form water, reaction (2).

$$\tfrac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O \quad (2)$$

Energy is produced through the flow of electrons through an external circuit from the anode to the cathode.

Teacher and students will spend a few days prior to the Legacy cycle learning about fuel cells and discovering the potential of fuel cells by manipulating variables using a hands-on approach with miniature fuel cell cars. The class will form groups and we will find what makes a car perform in different ways, such as heating, cooling, stacking multiple fuel cells in series. This will be tested by having a ‘closest to the line challenge’ and a race to see whose car goes the fastest. The ‘closes to the line challenge’ requires the team to construct a calibration plot to as precisely as possible determine the dependence of distance traveled to the amount of on-board fuel. A summary of the legacy cycle is presented in Table 1. Some of the Tennessee standards[9, 13,14] that are covered by the developed legacy cycle are listed in Appendix A.

Table 1: Summary of a Legacy Cycle to Teach About Titration

Activity	Description
Challenge Question	You are an employee in a major Fuel Cell membrane plant. You have just made a new membrane that will revolutionize the industry. However first you need to test the membrane, define its characteristics, and market the membrane to potential buyers. Therefore you need to find the amount of protons that can be exchanged through your membrane and develop an advertisement with the data you found
Generate Ideas	**What do we know?** Table salt is NaCl, it dissolves in water **What do we need to know more about?** Nafion membrane structure Chemically what happens to Nafion in the presence of salt? Acid-Base Chemistry (Classical) Titrations
Multiple Perspectives	Articles about the chemistry of Nafion membrane Chemist who does titrations daily in industry for paper manufacturing A Chemical engineering student's reaction to the challenge question Acid-Base reaction demonstration
Research & Revise	Acid-Base Chemistry Lecture and demonstration pH lecture and milk of magnesia + vinegar with universal pH indicator demo Acid-Base Lab –pH Testing acidity of different common solutions (soda, orange juice, rain water, coffee,etc.) Titration Lab-Lecture-Lab
Test Your Mettle	Now we can get in the lab and try our own membrane to find how many protons were in it per gram of membrane. The inverse is what is known as equivalent weight, which is a standard to define Nafion by.
Go Public	Students will write a sales publication to advertise your film and detailing your findings.

Fuel Cell Technology to Teach the Complex Number System

This section briefly introduces the legacy cycle of the math teacher. As earlier explained by the mentor, the research project was focused on assessment of freezing phenomena in automotive fuel cell applications where EIS was used to qualitatively evaluate the changes in charge transfer and diffusional resistance within the frozen fuel cell. The teacher found that the output of the EIS measurement provided an excellent example of 'complex variables' and the concept of imaginary numbers, a topic required in her math curriculum. The EIS produced a Nyquist plot, consisting of two semi-circles. In the measurement, an AC current perturbation is applied across the fuel cell over a wide range of frequencies (10 MHz – 0.1 Hz), resulting in a resistive response. The placement and shape of the Nyquist curves describes the state of water contained within the active fuel cell structure—the anode catalyst layer, the Nafion membrane, and the cathode catalyst layer. Changes to the hydration level of the fuel cell prior to freeze are quantified using EIS at subzero temperatures. The left hand intercept, referred to as the high frequency resistant, is dependent on the rate of proton conduction within the membrane—this mechanism is the same as described by the science teacher—and any contact resistances (typically negligible). The 1st arc—the high frequency arc—is mostly dependent upon the charge transfer resistances within the catalyst layers. Based on the imposed experimental conditions of saturating both catalyst layers with 4% H_2 in a N_2 balance, the charge transfer reaction is show in reaction (3)

$$H_2 \leftrightarrow 2H^+ + 2e^- \quad (3)$$

Simply put, this is the resistance of the charged proton (H^+) movement across the electrode interface. The 2nd arc—the low frequency arc—is simply diffusional resistance of the supplied gas (H_2) to the catalyst surface.

The fuel cell was conditioned at several different saturation levels, frozen and then probed with EIS. The math teacher's interpretation technique quantified the relationship between increased saturation level and decreased charge-transfer resistance. This observation is consistent with the facilitated movement of charged ionic species in the presence of supercooled water. Additional, it was confirmed that the water is selectively retained within the membrane and the hydration sphere of the catalyst layer—thus, no additional diffusional resistances were measured.

The legacy cycle for the second teacher is thus intended to be used to teach the relevance of the complex number system in the world of chemical engineering and how that relates to the real world. Throughout this legacy cycle students will focus on the polymer electrolyte membrane fuel cells. Students will research how fuel cells work and read scientific journals to become more familiar with the fuel cell. Some of Tennessee standards for Algebra 2 will be covered [9,13,14]. A list of some of these standards are shown in Appendix B.

The challenge question: "Oil supplies are running low. What are some alternatives to the internal combustion engines currently used in automobiles? How can complex numbers be used to graphically represent the data created by the research?" Students will be asked to generate an individual list of ideas and methods for answering the challenge. Students will then gather in small groups and combine their ideas into a revised list. Students will be given multiple perspectives by evaluating journal articles, watching videos from fuel cell researchers (Dr. Rice-York, Tony Pistono-Chemical Engineering student) and websites. After students have done the research, understanding will be assessed through the use of in-class lectures and activities including building fuel cells and miniature fuel cell cars. Students will prepare a PowerPoint presentation with graphs and pictures to summarize their involvement and understanding of the importance of math in fuel cell research. The local newspaper will be invited to observe the progress made by the students and published in the weekly newspaper. A brief description of the components of the legacy cycle is presented in Table 2.

Table 2: Summary of a Legacy Cycle to Teach Complex Numbers Based on Fuel Cell Technology

Activity	Description
Challenge Question	Oil supplies are running low. What are some alternatives to the internal combustion engines currently used in automobiles? How can complex numbers be used to graphically represent the data created by the research?
Generate Ideas	Students will be asked to generate an individual list of ideas and methods for answering the challenge. Students will then gather in small groups and combine their ideas into a revised list.
Multiple Perspectives	Evaluate journal articles. Watch video of fuel cell researchers.
Research & Revise	✓ In-class lecture • Imaginary numbers • Plotting complex numbers on the complex plane • Operations with complex numbers • Fractal geometry • Finding the absolute value of complex numbers ✓ Hands-on activities (Fuel cell cars, Fractal Geometry) ✓ Academic vocabulary (Fuel Cell and algebraic terminology)
Test Your Mettle	✓ CPS [clickers] (Quick reviews and quizzes) ✓ On-line tests and reviews (Label the parts of the fuel cell, plot points on the complex plane)
Go Public	Students will prepare a PowerPoint presentation with graphs and pictures to summarize their involvement and understanding of the importance of math in fuel cell research. The local newspaper will be invited to observe the progress made by the students and publish in the weekly newspaper.

Conclusion

This paper summarized the experience of a mentor and two teachers who participated in a research experience for teachers program in 2009. One of the teachers was a pre-service science teacher while the other was an in-service math teacher. The teachers participated in research on fuel cells. A summary of the experience is provided from a mentor's perspective. An overview of the research projects is provided along with a perspective of teachers regarding the program. Based on the research experiences, the teachers developed a learning module to be implemented in the following academic year within their classrooms. Learning modules were developed based on the legacy cycle model. The two legacy cycles developed by the teachers are summarized and the standards that are covered by them are listed in the Appendices.

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Appendix A

Tennessee State Standards Addressed by Nafion Membrane Legacy Cycle

Inquiry

CLE 3221.Inq.3 Use appropriate tools and technology to collect precise and accurate data.
CLE 3221.Inq.4 Apply qualitative and quantitative measures to analyze data and draw conclusions that are free of bias.
CLE 3221.Inq.5 Compare experimental evidence and conclusions with those drawn by others.
CLE 3221.Inq.6 Communicate and defend scientific findings

SPI 3221 Inq.2 Analyze the components of a properly designed scientific investigation.
SPI 3221 Inq.3 Determine appropriate tools to gather precise and accurate data.
SPI 3221 Inq.4 Evaluate the accuracy and precision of data.
SPI 3221 Inq.5 Defend a conclusion based on scientific evidence.
SPI 3221 Inq.6 Determine why a conclusion is free of bias.

Mathematics

CLE 3221.Math.1 Understand the mathematical principles associated with the science of chemistry.
CLE 3221.Math.2 Utilize appropriate mathematical equations and processes to solve chemistry problems.
CLE 3221.Math.3 Apply algebraic properties, formulas, and relationships to perform operations on real-world problems (e.g., solve for density, determine the concentration of a solution in a variety of units: ppm, ppb, molarity, molality, and percent composition) calculate heats of reactions and phase changes, and manipulate gas law equations.
CLE 3221.Math.13 Convert among the quantities of a substance: mass, number of moles, number of particles, molar volume at STP.
SPI 3221.Math.4 Apply measurement unit relationships including Avogadro's number, molarity, molality, volume, and mass to balance chemical equations.
SPI 3221.Math.5 Use concepts of mass, length, area, and volume to estimate and solve real-world problems.

Chemistry

CLE 3221.3.3 Explore the mathematics of chemical formulas and equations.
CLE 3221.3.4 Explain the law of conservation of mass/energy

CLE 3221.3.6 Apply information about the molar mass, number of moles, and molar volume to the number of particles of the substance.
CLE 3221.3.7 Balance an equation for a chemical reaction.
CLE 3221.3.8 Classify a chemical reaction as composition, decomposition, single replacement, double replacement, and combustion.
CLE 3221.3.9 Use activity series or solubility product table information to predict the products of a chemical reaction.
CLE 3221.3.10 Predict the products of a neutralization reaction involving inorganic acids and bases.

SPI 3221.3.4 Balance a chemical equation to determine molar ratios.
SPI 3221.3.5 Convert among the following quantities of a substance: mass, number of moles, number of particles, molar volume at STP.
SPI 3221.3.6 Identify and solve stoichiometry problems: volume at STP to mass, moles to mass, and molarity.
SPI 3221.3.7 Classify substances as acids or bases based on their formulas and how they react with various

Appendix B

State Standards Covered by the Legacy Cycle to Teach Complex Numbers Based on Fuel Cell Technology

Standard 1 – Mathematical Processes

CLE 3103.1.1 Use mathematical language, symbols, definitions, proofs and counterexamples correctly and precisely in mathematical reasoning.
CLE 3103.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including testing cases, estimation, and then checking induced errors and the reasonableness of the solution.
CLE 3103.1.3 Develop inductive and deductive reasoning to independently make and evaluate mathematical arguments and construct appropriate proofs; include various types of reasoning, logic, and intuition.
CLE 3103.1.4 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic), to solve problems, to model mathematical ideas, and to communicate solution strategies.
CLE 3103.1.5 Recognize and use mathematical ideas and processes that arise in different settings, with an emphasis on formulating a problem in mathematical terms, interpreting the solutions, mathematical ideas, and communication of solution strategies.
CLE 3103.1.6 Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.
CLE 3103.1.7 Use technologies appropriately to develop understanding of abstract mathematical ideas, to facilitate problem solving, and to produce accurate and reliable models.

a 3103.1.1 Create and analyze scatter-plots of non-linear and transcendental functions.
a3103.1.2 Compare and contrast sampling techniques and identify the best technique for a given situation.

a 3103.1.6 Use graphical representations to perform operations on complex numbers.
a 3103.1.10 Interpret the results of mathematical modeling in various contexts to answer questions.
SPI 3103.1.1 Move flexibly between multiple representations (contextual, physical, written, verbal, iconic/pictorial, graphical, tabular, and symbolic) of non-linear and transcendental functions to solve problems, to model mathematical ideas, and to communicate solution strategies.

Standard 2 – Number & Operations

CLE 3103.2.1 Understand the hierarchy of the complex number system and relationships between the elements, properties and operations.
CLE 3103.2.2 Connect numeric, analytic, graphical and verbal representations of both real and complex numbers.
CLE 3103.2.3 Use appropriate technology (including graphing calculators and computer spreadsheets) to solve problems, recognize patterns and collect and analyze data.
CLE 3103.2.4 Understand the capabilities and limitations of technology when performing operations, graphing, and solving equations involving complex numbers.
a3103.2.1 Understand that to solve certain problems and equations, the real number system needs to be extended from real numbers to complex numbers.
a3103.2.2 Define and give examples of each of the types of numbers in the complex number system.
a 3103.2.3 Identify and apply properties of complex numbers (including simplification and standard form).
a 3103.2.7 Graph complex numbers in the complex plane and recognize differences and similarities with the graphical representations of real numbers graphed on the number line.
a 3103.2.10 Draw conclusions based on number concepts, algebraic properties, and/or relationships between expressions and numbers over complex numbers.
a 3103.2.11 Understand the capabilities and limitations of technology. Make estimations without a calculator to detect potential errors.
a 3103.2.12 Select and use appropriate methods to make estimations without technology when solving contextual problems.

a 3103.2.13 Analyze and evaluate contextual situations involving any type of number from the complex number system.
SPI 3103.2.1 Describe any number in the complex number system.
SPI 3103.2.2 Compute with all real and complex numbers.
SPI 3103.2.3 Use the number system, from real to complex, to solve equations and contextual problems.

Standard 3 – Algebra

CLE 3103.3.5 Use mathematical models involving equations and systems of equations to represent, interpret and analyze quantitative relationships, change in various contexts, and other real-world phenomena.
SPI 3103.3.11 Graph conic sections (circles, parabolas, ellipses and hyperbolas) and understand the relationship between the standard form and the key characteristics of the graph.