



Unified approach to teaching uncertainty across a three-course mechanical engineering laboratory sequence

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Abstract

The ability to analyze and make sense of large volumes of experimental data is critical to prepare engineering graduates for the modern workplace. While a number of students take an elective statistics course, students' main exposure to data analysis in our program comes from a three course in-major laboratory sequence. These courses each target different technical content while emphasizing common skills, including writing (technical memos, lab reports, design reports), formal presentations (oral and poster), and statistical analysis techniques to quantify uncertainty in measured data.

The first laboratory (sophomore year) targets instrumentation and measurement techniques and introduces the concept of bias and precision uncertainty. The second laboratory (junior year) focuses on experiments related to the mechanics of materials and structures and introduces the concept of error propagation. The third laboratory (senior year) includes experiments related to thermo-fluids and heat transfer and is the culmination of experimental uncertainty analysis in preparation for students' capstone design projects. All three labs heavily emphasize digital data acquisition so students are able to apply the learned analysis techniques on large amounts of real-world data.

This paper details the framework of the uncertainty analysis across the 3-course sequence. Impacts are examined through data collected from the students in each lab, as well as a review of experimental data presentation from the summative Capstone Project presentations. Observations and lessons learned are being used to inform further changes in content and lab-to-lab knowledge recall.

1 Introduction

The act of taking a measurement is ubiquitous in engineering practice and in the collection of experimental data. With measurement comes *uncertainty* – the variability of the difference between the measurement taken and its true value. The ability to analyze and make sense of large volumes of experimental data is critical to prepare engineering graduates for the modern workplace. The topics of *error*, *precision*, and *uncertainty* are frequently introduced in Physics and Chemistry courses [1], [2], [3]. Many engineering programs include an Experimental Methods course in their curriculum where students focus on *calibration*, *data acquisition*, and *propagated uncertainty*. The available literature depicts a picture of *uncertainty* being integrated in a variety of courses [4], [5], but indicative of the struggles and misconceptions students face when working with data [6], [7]. This paper details the unified approach to teaching *uncertainty* in the Mechanical Engineering program at the University of New Haven.

2 Uncertainty analysis throughout the Mechanical Engineering lab sequence

At the University of New Haven, the undergraduate lab sequence for Mechanical Engineering majors is designed to provide a hands-on experience in engineering instrumentation and measurement, allowing students to encounter and expand upon theoretical concepts learned in lecture-based courses through experimentation. These three courses include a Instrumentation Lab (sophomore), Mechanics Lab (junior), and Thermo/Fluids Labs (senior). Each course is a standalone lab, separate from any co-requisites. All three labs emphasize digital data acquisition so students are able to apply the learned analysis techniques on large amounts of real-world data.

Each course in this sequence targets different technical content while emphasizing common skills – technical communication and uncertainty analysis – in a scaffolded approach. Technical communication is implemented through a structured approach to writing technical memos, lab reports, and design reports as well as oral and poster presentations described elsewhere [8], while the scaffolding of statistical analysis techniques to quantify uncertainty in measured data is the focus of this paper.

2.1 Instrumentation Lab

The students' first encounter with the Mechanical Engineering labs comes in their sophomore year in Instrumentation Lab, which targets instrumentation and measurement techniques and introduces the concept of bias and precision uncertainty. Any number of introductory experimental measurement textbooks describe these concepts [9, 10] which we will briefly discuss here.

The uncertainty in a measured variable x , written u_x , can be split into two components – bias uncertainty u_x^b and precision uncertainty u_x^p . These are combined together by adding in quadrature to obtain an estimate of the uncertainty

$$u_x = \sqrt{(u_x^b)^2 + (u_x^p)^2}. \quad (1)$$

Bias uncertainty quantifies the accuracy of a measurement, which is affected by systematic errors, while precision uncertainty quantifies the random errors. These concepts are introduced to students using a histogram where the position of the center of the histogram relative to a fixed point reflects the bias uncertainty and the extent of the data scatter (histogram width) is a measure of the precision (Figure 1).

Previous exposure that students have with this concept have generally been framed in terms of "accuracy" and "precision", so a connection is made that bias is just the inverse of "accuracy". Students can then see that bias represents a physical distance from the "right" answer, with low bias

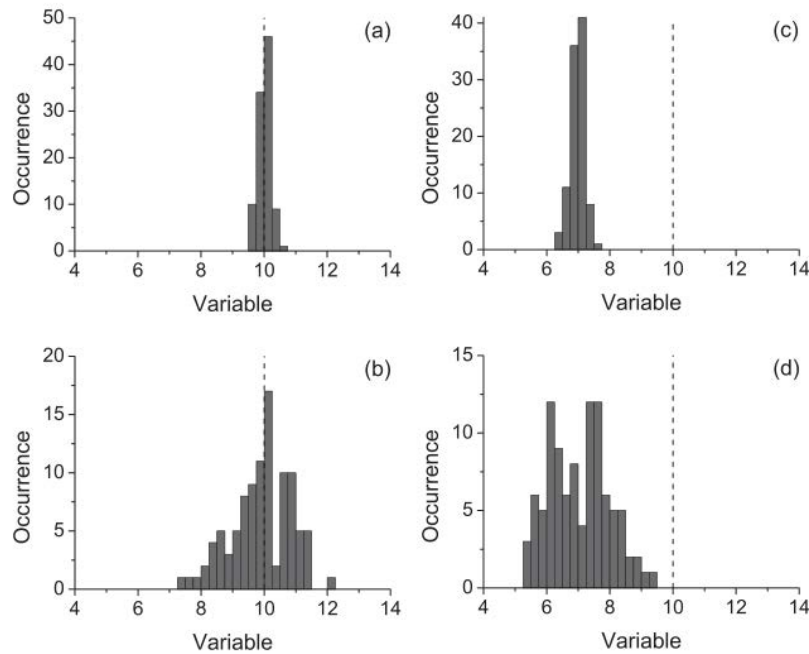


Figure 1: Histograms of the occurrence of 100 measurements illustrate distributions with (a) high precision and low bias, (b) low precision and low bias, (c) high precision and high bias, and (d) low precision and high bias (Image modified from [10])

correlated to high accuracy and vice versa. Methods of estimating bias uncertainty are discussed, including the "half the smallest division" rule-of-thumb for analog instruments and where those assumptions break down (precise machinists scales where the spacing is too close for the naked eye, instruments with digital displays, damaged/unreliable instruments). This is currently taught through an interactive activity where students must measure the physical dimensions of a simple block to see if it meets the specifications of a given engineering drawing with tolerances [11]. The main concept students should come away with is that bias uncertainty is their estimate of how good a single measurement is, bounded by the limitations of their instrument.

The choice of a histogram leads naturally into the next lesson about quantifying the precision uncertainty. The histogram is redrawn as a Gaussian curve and the standard deviation is discussed as a measure of the width of the distribution. The difference between sample σ_{N-1} and population σ_N standard deviations is introduced, as is relationship between the confidence interval and the z-score. Methods for calculating z-score for a given confidence interval by direct integration, use of the error function, and a standard normal table are demonstrated. Finally, the standard error is introduced as a better representation of the variation of repeated measurements to create a formula for the precision uncertainty, $u_x^p = \frac{z\sigma}{\sqrt{N}}$.

The initial exposure to uncertainty wraps up with the correct form for expressing uncertainty in

measured variables at a given confidence interval CI,

$$\bar{x} \pm u_x \text{ units } [\text{CI}\%], \quad (2)$$

where there is one significant digit in the uncertainty and the answer is rounded to match its decimal place. It is reiterated that all reported measured values must follow this format.

As a note, an approach to precision uncertainty that uses the Student t-distribution $u_x^p = \frac{t\sigma}{\sqrt{N}}$ is more accurate for small sample sizes. However sophomores with little previous statistics background were not able to fully grasp these nuances and had issues calculating $t(\bar{x}, \sigma, N)$.

At approximately the 2/3 point in the semester students are introduced to Chauvenet's criterion as a method to remove outliers. Staging the uncertainty coverage within the course in this manner means that students are less likely to be overwhelmed with concepts, while still giving them exposure to additional statistical tools that are useful in their analysis.

2.2 Mechanics Lab

The second laboratory (junior year) focuses on experiments related to the mechanics of materials and structures and introduces the concept of error propagation. This is done over the course of six experiments and a multi-week design project where students must develop and apply an understanding of statistical methods to select the best experimental approach to satisfy given requirements of accuracy.

The uncertainty in calculations involving measured quantities can be expressed as

$$u_{f(x,y,\dots,n)} = \sqrt{\left(\frac{\partial f}{\partial x} u_x\right)^2 + \left(\frac{\partial f}{\partial y} u_y\right)^2 + \dots + \left(\frac{\partial f}{\partial n} u_n\right)^2}, \quad (3)$$

where $f(x, y, \dots, n)$ is a function of the measured variables x, y, \dots, n . A functional approach can also be used for uncorrelated independent variables

$$\begin{aligned} u_{f(x,y,\dots,n)} = & \left\{ [f(\bar{x} + u_x, \bar{y}, \dots, \bar{n}) - f(\bar{x}, \bar{y}, \dots, \bar{n})]^2 \right. \\ & + [f(\bar{x}, \bar{y} + u_y, \dots, \bar{n}) - f(\bar{x}, \bar{y}, \dots, \bar{n})]^2 \\ & + \dots \\ & \left. + [f(\bar{x}, \bar{y}, \dots, \bar{n} + u_n) - f(\bar{x}, \bar{y}, \dots, \bar{n})]^2 \right\}^{\frac{1}{2}}, \quad (4) \end{aligned}$$

which is well suited for analysis of uncertainty using spreadsheets or computer scripts. Both methods are introduced to students early in the course and used for analysis throughout.

As an example, one experiment conducted has students determine the mechanical properties of materials using a torsion test. Students performing the test acquire measurements of torque and

twist angle from a motorized apparatus but must also measure dimensions of the material under test in order to calculate the shear stress and shear strain. Each of these measurements has an associated uncertainty which the students must propagate through to find an uncertainty in the final results.

2.3 Thermo-Fluids Lab

The third lab in the sequence is the Thermo-Fluids Laboratory; a two-credit course taken by 4th-year students. The experiments in this lab focus on setups involving the measurement of thermo-fluids related quantities, e.g., temperatures, pressures, flow rates, and heat-transfer coefficients. Students complete five guided experiments. Traditionally, the lab has served as the evaluation point for ABET's Outcome: previously (b) an ability to design and conduct experiments, as well as to analyze and interpret data; now (6) an ability to develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgment to draw conclusions. To accomplish this, the latter half of the course is dedicated to designing, building, and testing an experimental setup.

At the start of the semester, students are reminded that accounting for uncertainty is expected and that it is not new to them. A quick review of concepts is provided emphasizing notation, as well as a handout with practice exercises focused primarily on propagated uncertainty. This was a change once the faculty from the three lab courses started sharing what was taught and emphasized in each course; prior to that, a whole class period was dedicated to teaching uncertainty propagation.

Data analysis in this course is framed from the perspective that the students' role is to assess if the experimental setups are working effectively given the data collected versus the results expected. Based on their analysis, they are to recommend changes to the experimental setup and the experimental procedure. The analysis, of which uncertainty analysis is integral, is 35% of each lab report. Data plots must include error bars in both horizontal and vertical directions and comparison to published or predicted data. If the analysis omits uncertainty, the report is graded as 'unacceptable' and returned to the students to complete and resubmit.

3 Findings

Overall, students struggle with the concept of uncertainty and how it fits into their design decisions and recommendations. Given the choice, students omit uncertainty analysis from their work, even if it means losing points. Students often provide a percent difference between two quantities based solely on the nominal values.

To assess students' understanding of the value of 'uncertainty', some students in the Thermo-Fluids lab are asked to submit technical memos as opposed to full detailed lab reports for certain

experiments. While students were instructed to provide an overview of the results, uncertainty analysis was not a required component of the memos. Of 15 memos collected during Fall 2019, only 1 student made a statement qualifying that a full analysis factoring in uncertainty was needed to determine if results were of value. In all other cases, students made statements based solely on the nominal results. This is for the summative course in the lab sequence.

To better understand how much uncertainty knowledge students are carrying throughout the lab sequence and how we can improve student recall of uncertainty throughout the lab sequence we have recently started giving students a simple uncertainty question as a 20 minute unannounced quiz. The question is a variation of the following, where the numbers are changed but the goals remain the same:

The sensitivity of a photodiode circuit is measured as: 5.33, 4.98, 4.93, 5.08, 4.95, 4.96, 5.02, 4.99, 5.24, 5.25, 5.23, and 5.01 A/W. Calculate the mean, appropriate standard deviation, standard error, and precision uncertainty at a 95% confidence interval. Assuming a bias uncertainty of 0.1 A/W calculate the total uncertainty and report your final answer in the correct format.

While we have only begun this assessment and do not yet have data to track performance of a cohort of students, we can note two interesting things from our limited subset of data from the Fall 2019 Instrumentation Lab and Spring 2020 Mechanics Lab and Thermo-Fluids lab. The question was administered twice in the Instrumentation Lab, once about halfway through the semester and again on the final exam. Data was collected from the Spring 2020 offerings on the first day of classes, though students in the Mechanics Lab were warned via course announcement to review uncertainty before the first meeting.

Students currently in the Instrumentation Lab had an average score of 40% (N = 32) on the question in mid-semester with 7/32 receiving a passing grade. When the same question was asked on the final exam the average score was 70% with 27/32 receiving a passing grade and 9/32 receiving an exemplary "A" grade. This indicates that students are learning the uncertainty content in the Instrumentation Lab. However, the comparison of recall between the lab courses is more distressing. Average performance in the Mechanics Lab was 19% (N = 18) and 26% in the Thermo-Fluids Lab (N = 13). This correlates to the average student being able to calculate the mean and recall that the total uncertainty is comprised of bias and precision components (but not necessarily how) and nothing else.

4 Next Steps

The results observed are far from ideal. We will continue to collect data to track the performance of a known cohort of students, but clearly more is needed to help students retain uncertainty knowledge from one lab to another. This may include:

- reexamining how the methodologies are introduced, reinforced, and reviewed in each of the labs;
- looking for opportunities to integrate uncertainty in other Mechanical Engineering courses;
- exploring additional methods for uncertainty analysis such as sequential perturbation and Monte Carlo methods; and
- additional resources for students such as online, self-paced modules which students can access frequently.

We are curious to investigate if the results seen in the unannounced quiz correlate with content/performance in final project reports. Systematic assessment of final reports from both the Thermo-fluids Lab and the Capstone Project course may provide insight into how and if students incorporate concepts of uncertainty into their data analysis and results presented.

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