Session 3662

Using Commercially Available Finite Element Software for Fatigue Analysis

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I: Introduction

Fatigue analysis is a branch of the science of fracture mechanics. It is widely known that a metal subjected to a repetitive fluctuating load will eventually fail at a load much lower than that required to cause fracture on a single application of the load.

Fatigue failures can be grouped into two broad categories of “Low Cycle Fatigue” and “High Cycle Fatigue”.

The primary cause of failure in “Low Cycle Fatigue” is large strain fluctuations. For example, extreme temperature fluctuations can cause large strain fluctuations. Another possible cause of large strain fluctuations is the large variation in centripetal forces in a high speed rotating machine during acceleration and deceleration.

The primary cause of failure in ‘High Cycle Fatigue” is fluctuating stress levels. For example, fluctuating stresses can be caused by vibration caused by a minor imbalance in a high speed rotating part.

The primary analytical technique for analyzing “Low Cycle Fatigue” is the strain-life technique, and the primary analytical technique for analyzing “High Cycle Fatigue” is the stress-life technique. The finite element technique enables one to determine the stress and strain levels for complicated geometries and loading conditions.

This article contains the background information and introduces concepts that illustrate the use of commercial finite element software as an aid in teaching fatigue analysis. The use of the finite element technique will enable an instructor to move beyond presenting simple geometries and loading conditions, and will thereby allow the teaching of fatigue analysis techniques involving real world geometries, boundary and loading conditions.

II: Cyclic Loading

Fatigue failure is a byproduct of cyclic loading. Figures 1, 2 and 3 illustrate typical fatigue loading (stress) cycles.

Figure 1 is an idealized situation which is produced by a rotating-beam fatigue machine and which is approached in service by a rotating shaft operating at constant speed without overloads. For this type of stress cycle the maximum and minimum stresses are equal.[1]

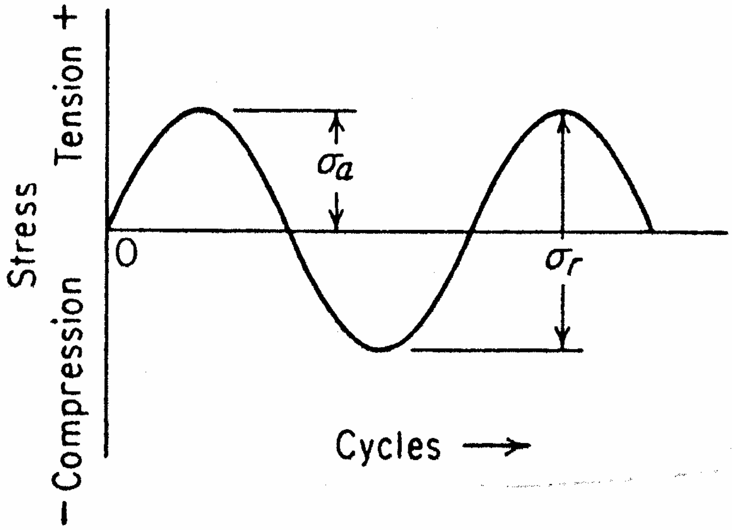

Figure 1: An illustration of a reversed stress fatigue cycle[1]

Figure 2 illustrates a repeated stress fatigue cycle in which the maximum stress and the minimum stress are not equal. For this type of stress cycle the maximum and minimum stresses can be both tension, both compression or one tension and one compression. These types of stress cycles are present in rotating shafts with overloads(s).[1]

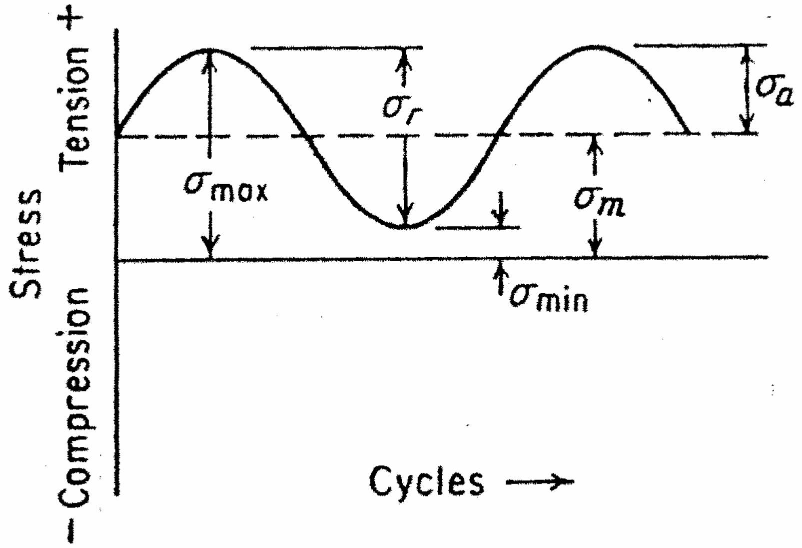

Figure 2: An illustration of a repeated stress fatigue cycle[1]

Figure 3 illustrates an irregular or random stress cycle. This type of stress cycles can be encountered in a part such as an aircraft wing which is subjected to periodic unpredictable overloads.[1]

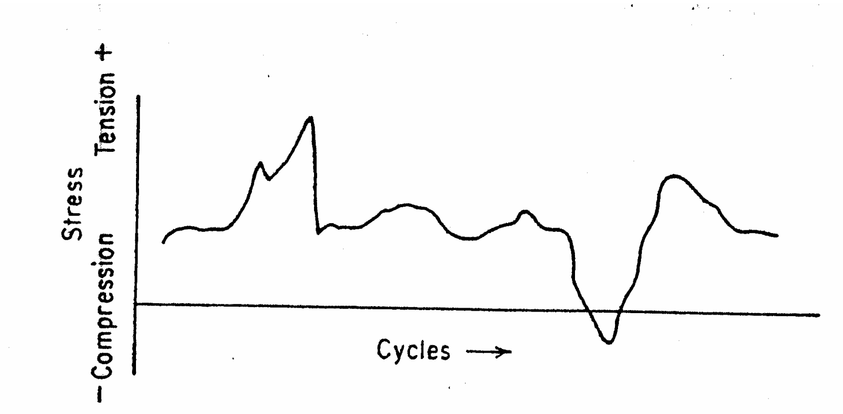

Figure 3: An illustration of an irregular or random stress cycle[1]

III: High Cycle Fatigue Description and Analysis Techniques

High cycle fatigue is referred to situations where the fatigue failure occurs at more than 10^4 loading cycles. A high cycle fatigue failure scenario exists when stresses are below the plastic limit of the material. By definition, when the stresses are below the plastic limit, the material is operating within its elastic range. When the stresses are within the materials' elastic range, stress analysis and stress cycles are the same as strain analysis and strain cycles. Therefore, high cycle fatigue is analyzed by stress-life techniques.

The most common method of presenting fatigue data is by means of an *S-N* curve, which is a plot of stress *S* against the number of cycles to failure. Figure 4 is an illustration of a typical *S-N* curve.[2]

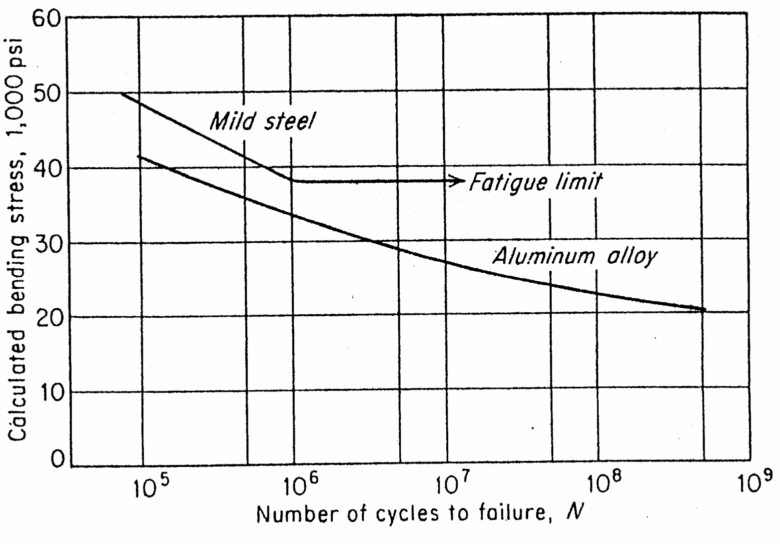

Figure 4: Typical fatigue curves for ferrous and nonferrous metals[2]

A conventional *S-N* curve such as the one shown in figure 4 provides a certain number of cycles *N* to failure for a given completely reversed stress level. However, for situations where the stress level is fluctuating, the simplest method of analysis is the linear cumulative damage rule.[3]

According to the linear cumulative damage rule, if $n_1, n_2, \ldots\ldots, n_k$ represent the number of cycles of operations at specific stress levels and $N_1, N_2, \ldots\ldots, N_k$ represent the life (in cycles) at these stress levels, then, the following formula applies.[3]

$$(n_1/N_1) + (n_2/N_2) + \ldots\ldots + (n_k/N_k) = 1 \qquad (1)$$

The plot shown in figure 4 assumes the mean cyclic stress is zero. Figure 5 is a possible method of showing an *S-N* curve when the mean stress is not zero.[4]

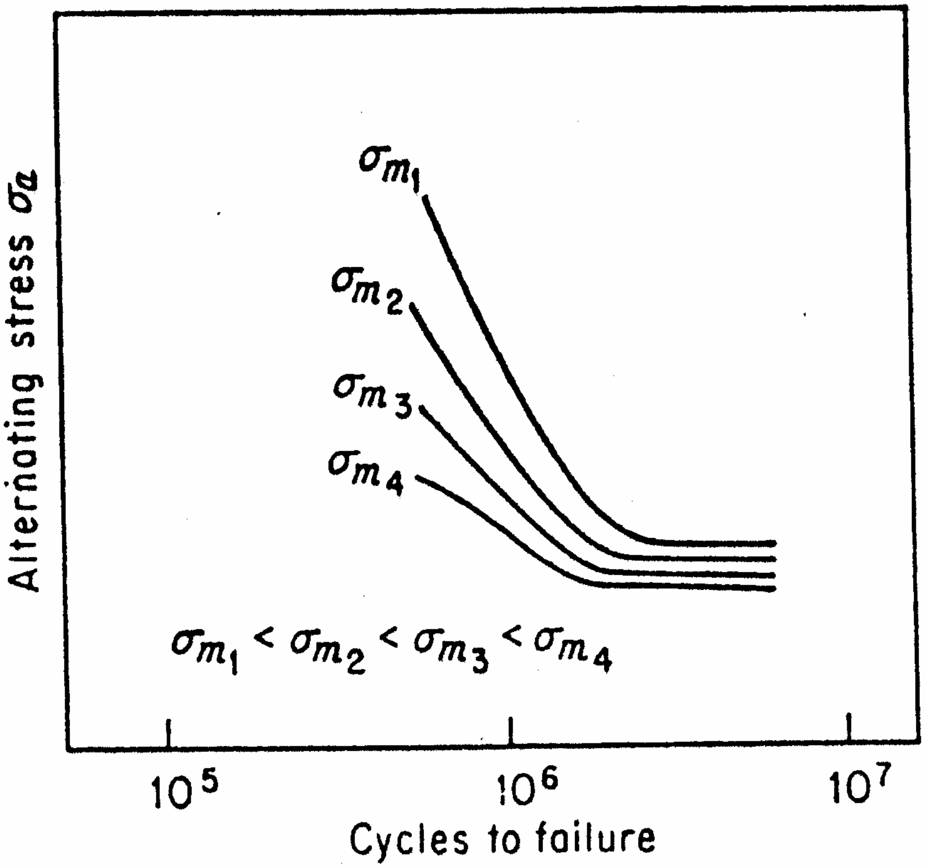

Figure 5: A method of plotting fatigue data when the mean stress is not zero[4]

One of the most common sources of high cycle fatigue is vibration. Determining an accurate vibration response for a complex structure subjected to varying forces is not possible without the use of the finite element technique. The use of the Finite Element technique for analyzing the vibration characteristics of a structure enables one to analyze complex geometries, boundary conditions and external forcing functions. The results of such analyses can then be used to analyze the fatigue life of a structure. Examples 1, 2 and 3 illustrate the different types of vibration analyses possible by using a finite element analysis software such as ANSYS. Each of these examples contains a discussion of how the vibration analysis results can then be used for analyzing the fatigue characteristics of a structure.

Example 1: Modal analysis of a model plane wing.[5]
A model plane wing is fixed at one end. The geometry and material properties are shown in figure 6. Figure 7 is the finite element model of the wing. The first 5 modes of the structure are calculated. The natural frequencies for modes 1 through 5 are calculated to be (3.6 Hz, 17.2 Hz, 22.9 Hz, 36.4 Hz & 65.9 Hz). The ANSYS software produces mode shapes for all calculated modes. To illustrate the concept of mode shapes, the mode shape for mode 5 is shown in figure 8.

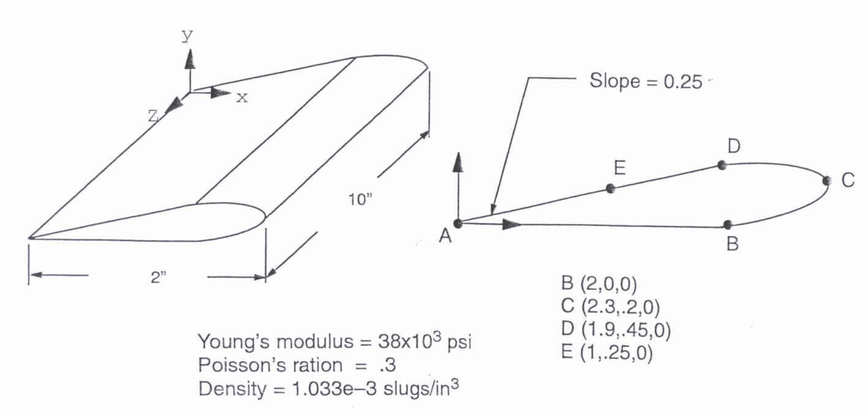

Figure 6: Geometry and material properties for the model plane wing[5]

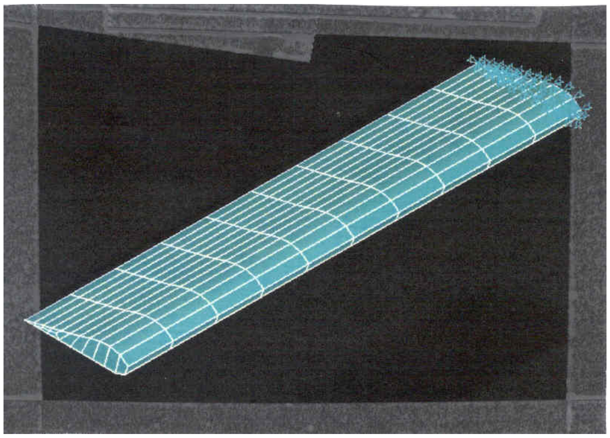

Figure 7: ANSYS finite element model of a model plane wing, fixed at one end

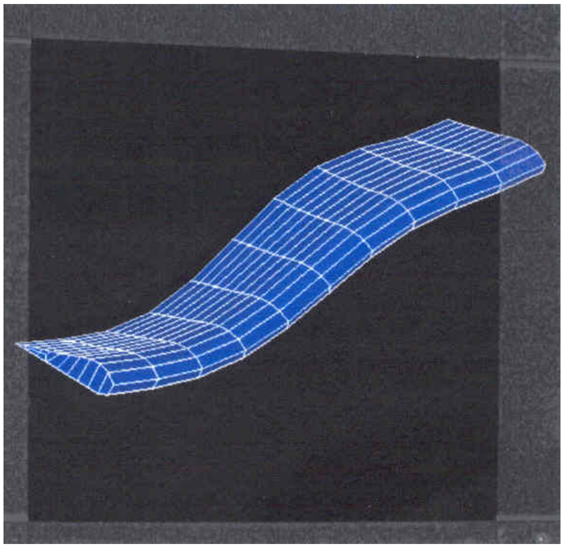

Figure 8: Mode shape 5 for the model plane wing

A modal analysis such as shown above can be used in fatigue prevention. It is known that if a structure is subjected to external forces that are sinusoidal in nature, and the frequency of the external forces are close to the lower mode vibration frequencies, the structure will vibrate at or close to a resonance condition. Excessive vibration at large amplitudes will have the effect of subjecting the structure to a large number of N's at high stresses at the deflection points. As shown in figure 4, higher stress levels and/or higher N's will lead to an earlier fatigue failure. The knowledge gained by a modal analysis of the structure will enable a designer to take the necessary steps to design against a premature fatigue failure.

Example 2: A Harmonic Response Analysis Example.[6]
Figure 9 is a sketch for a harmonic analysis problem. Material properties are $m_1 = m_2 = 0.5$ lb-sec^2/in, and $k_1 = k_2 = 200$ lb/in. The spring lengths are arbitrarily selected and are used only to define the spring direction. Two masters degree of freedom are selected at the masses in the spring direction. A frequency range of zero to 7.5 Hz with a solution at 7.7/30 = 0.25 Hz intervals is chosen to give an adequate response curve. Figure 10 shows the representative finite element model.

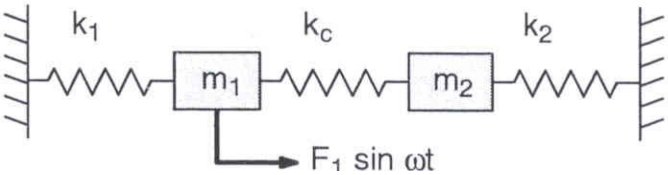

Figure 9: Sketch of the system for example 2[6]

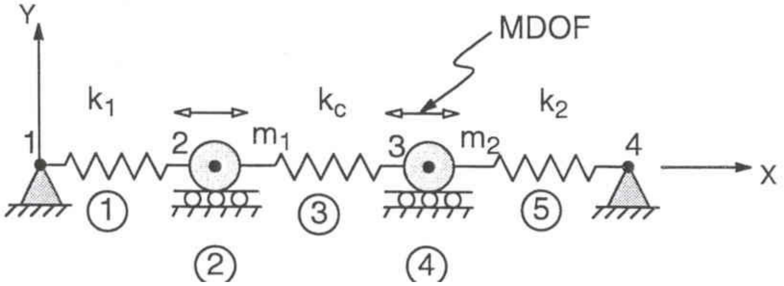

Figure 10: Finite element model for system of figure 9[6]

Figure 11 is the harmonic response of the system shown in figure 10.[6]

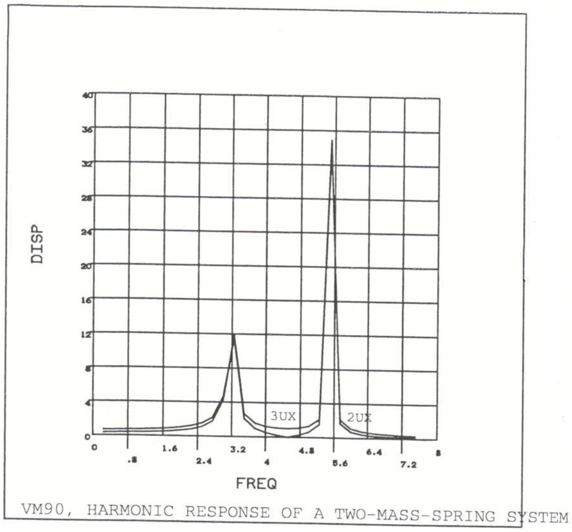

Figure 11: Amplitude vs. frequency plot showing the
response for the system of figure 18[6]

The knowledge gained from plots such as shown in figure 11, and combining these results with *N-S* plots such as shown in figures 4 and 5, and using equation (1) will enable one to conduct practical fatigue analyses. This approach can be extended to study the effects of random stress cycles such as shown in figure 3 on the fatigue life of complicated structures. The vertical axis in figure 11 is the displacement. The displacement values along with the modulus of elasticity of the material provide the stress values. Alternatively, the static stress analysis capabilities of a typical finite element software can be used for stress value determination(s).

Example 3: Illustration of the use of the finite element analysis technique in a real world gas turbine engine fatigue analysis.
Figure 12 is an illustration of the components for a gas turbine engine.[7]
Figure 13 illustrates more details regarding the assembly of a gas turbine engine.[8]

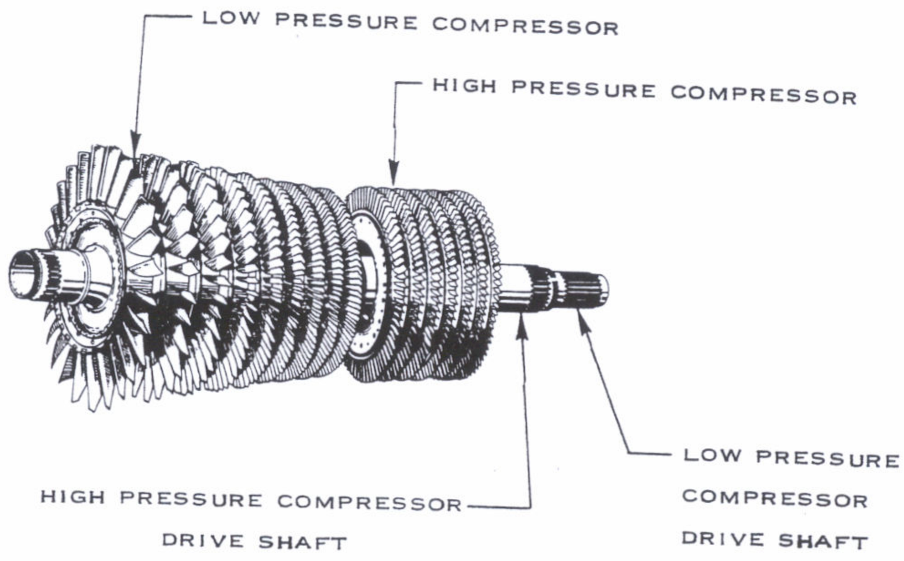

Figure 12: Components of a gas turbine engine[7]

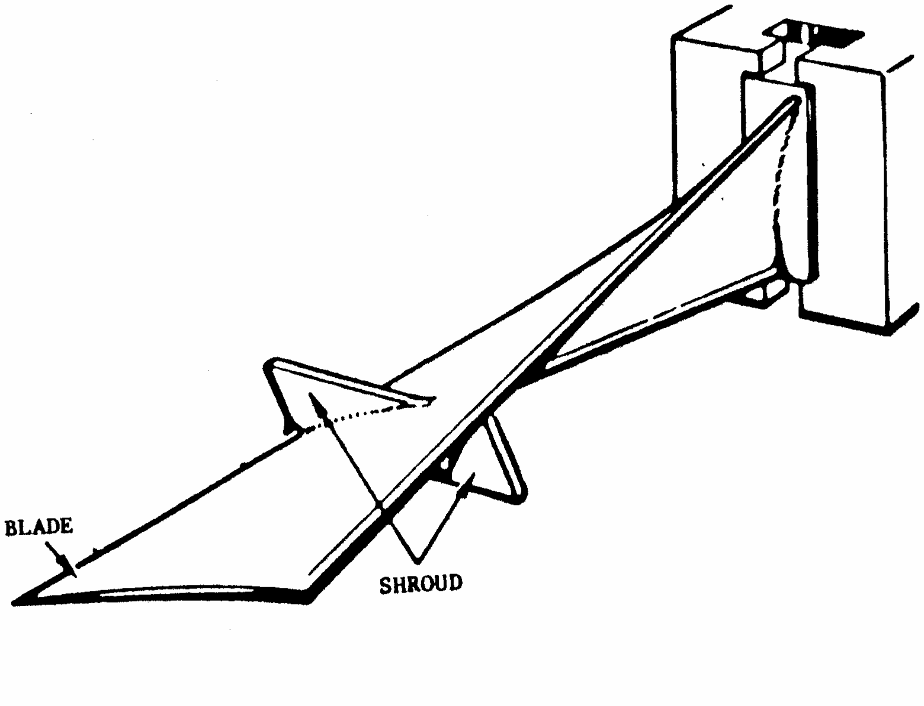

Figure 13: A shrouded fan blade in a gas turbine engine[8]

Figure 14 contains the frequencies and mode shapes obtained for a fan blade by using the ANSYS finite element software.[8]

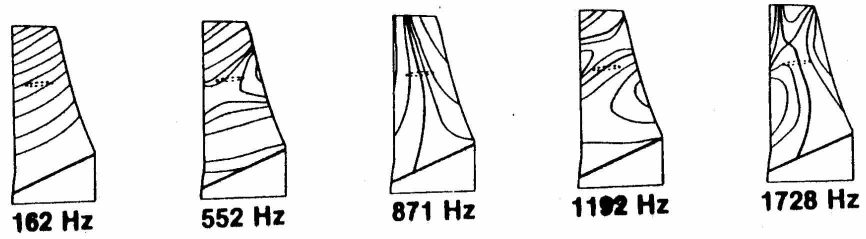

Figure 14: The first five vibration modes of a fan blade[8]

The type of information shown in figure 14, along with the operating modes of a gas turbine engine and the *S-N* curves for the blade material, will enable an engineer to perform an accurate fatigue analysis for a shrouded fan blade in a gas turbine engine.

IV: Low Cycle Fatigue Description and Analysis Techniques

Low cycle fatigue is referred to situations where the fatigue failure occurs at less than 10^4 loading cycles. Low cycle fatigue is present where the elastic limit of a material is exceeded. Once the elastic limit of a material is exceeded, the material properties are changed due to the resulting plastic deformation. The plasticity effects are determined by studying the strain history of the material. Therefore, low cycle fatigue is analyzed by strain-life techniques.

From a fatigue point of view, the changes in material properties due to plastic deformation are reflected in a material's *S-N* curve. In the case of plastic deformation, *S* in an *S-N* curve stands for strain and not stress. Figure 15 is an illustration of a typical *S-N* curve where plastic deformation has taken place. In figure 15, the vertical axis is defined as half the strain.

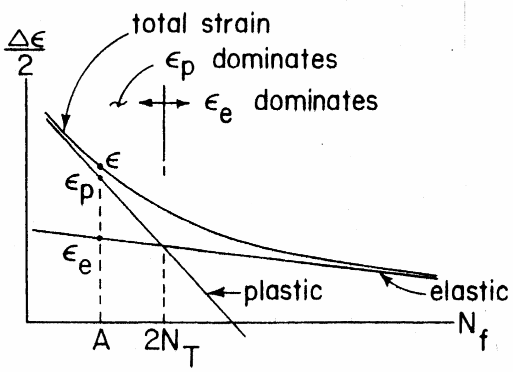

Figure 15: Outline of a typical *S-N* curve with plastic deformation[9]

Figures 16 and 17 are *S-N* curves for 7075-T6 aluminum alloy and for TI-8a1-1 Mo-1V titanium alloy.

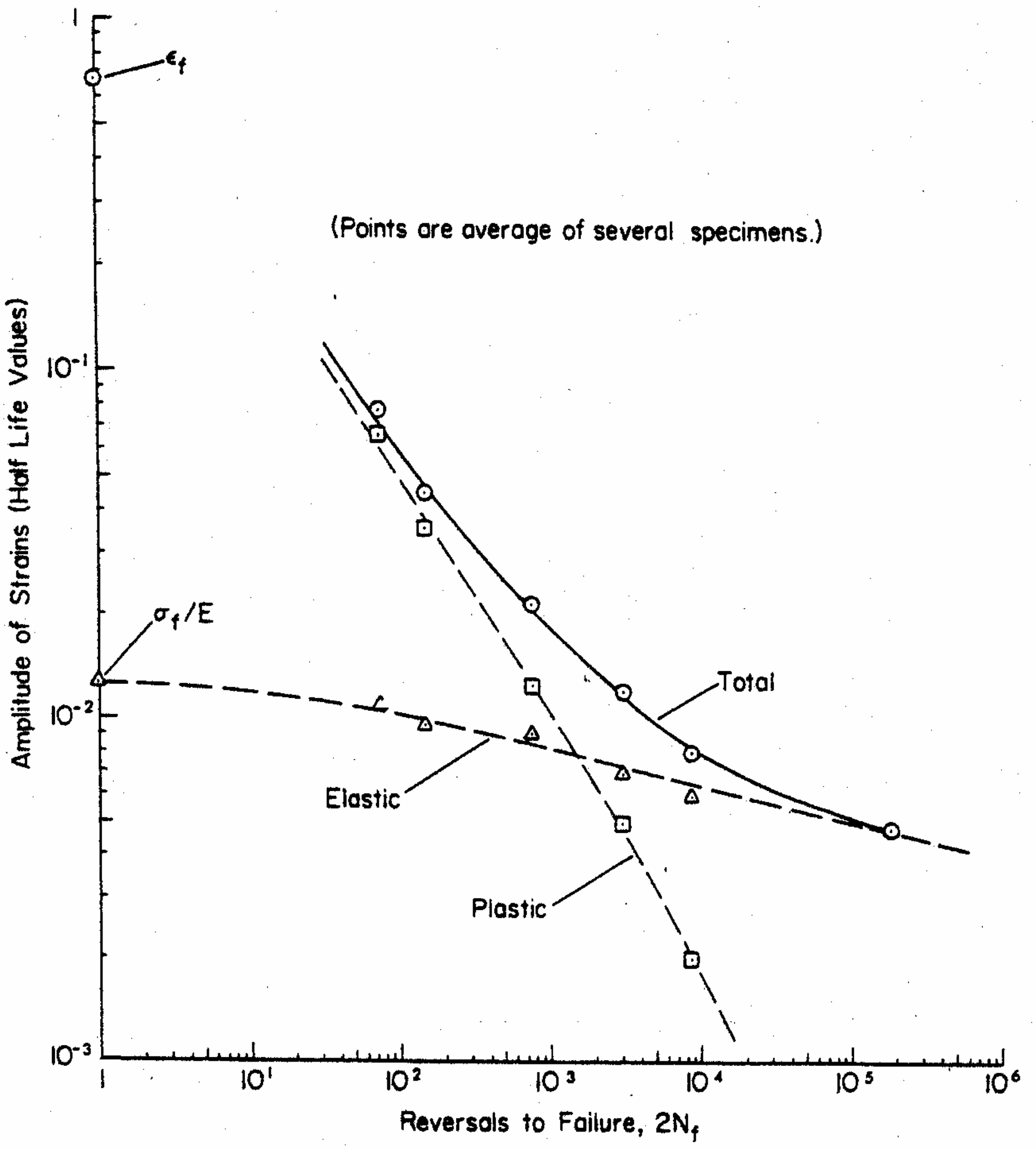

Figure 16: Fatigue properties of 7075-T6 aluminum alloy[10]

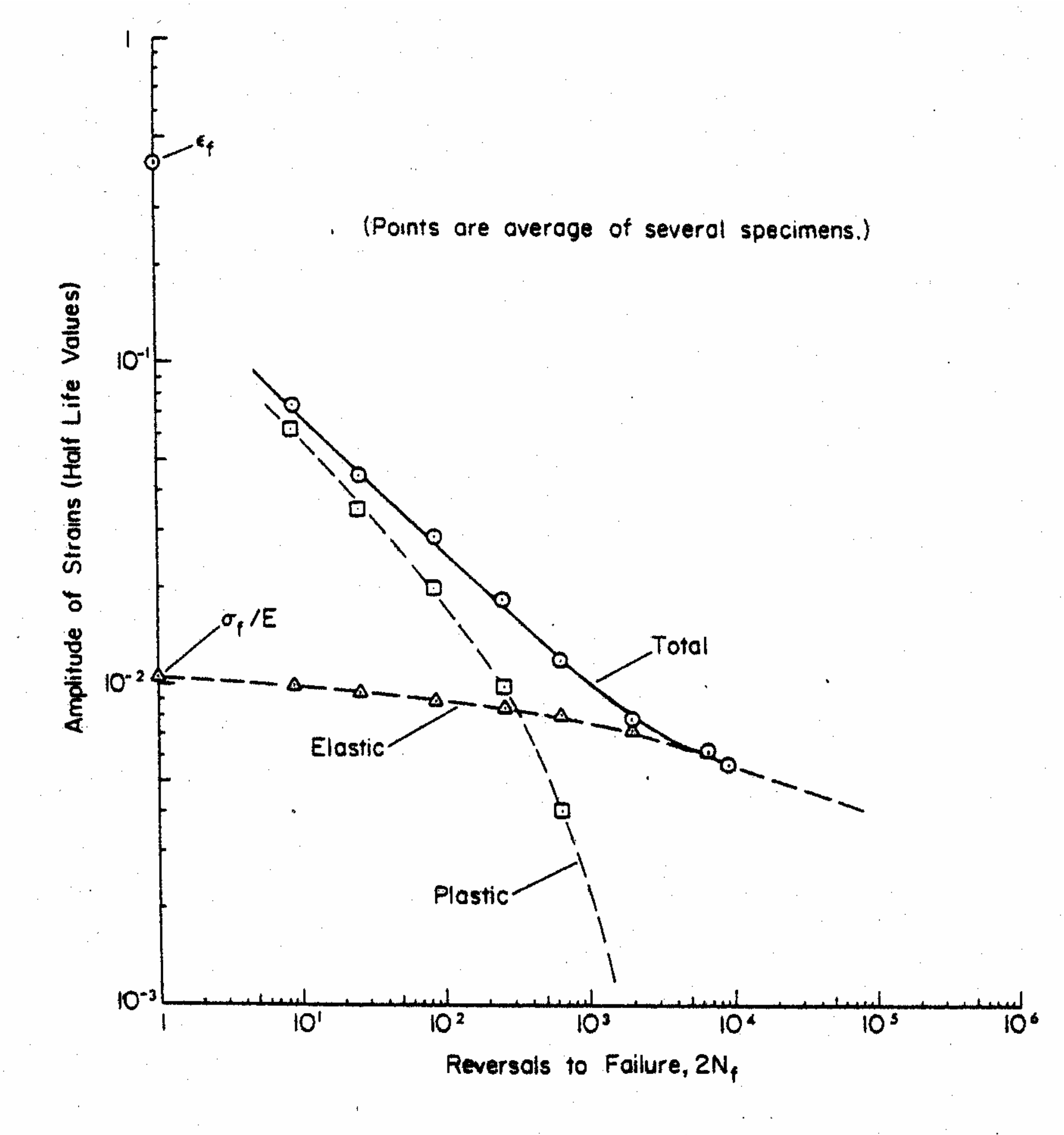

Figure 17: Fatigue properties of Ti-8A1-1 Mo-1V Titanium alloy[11]

In order to be able to accurately determine the resulting strain after a plastic deformation has taken place, a nonlinear stress and strain analysis is required. Most commercial finite element software are capable of performing a nonlinear static stress and strain analysis. The ANSYS software nonlinear approaches and capabilities are discussed below for the purpose of illustrating the concept.

Once the strain is determined by the use of the nonlinear capabilities of the finite element software, curves such as shown in figures 16 and 17 can be used.

V: ANSYS Nonlinear Capabilities

The nonlinear capabilities of ANSYS that are of interest in a low cycle fatigue analysis scenario are the geometric nonlinearities and the material nonlinearities.

If a structure experiences large deformations, its changing geometric configuration can cause the structure to respond nonlinearly. The ANSYS program can account for the geometric nonlinearities.[12]

Nonlinear stress-strain relationships are a common cause of nonlinear structural behavior. Many factors can influence a material's stress-strain properties, including load history (as in elasto-plastic response), environmental conditions (such as temperature), and the amount of time that a load is applied (as in creep response). The ANSYS program can account for nonlinear stress-strain relationships.[12]

The ANSYS program's equation solver operates on a set of simultaneous linear equations to predict the response of an engineering system. However, a nonlinear structure's behavior cannot be represented directly with such a set of linear equations. A series of linear approximations with corrections are needed to solve nonlinear problems.[12]

ANSYS performs a nonlinear analysis at three levels as shown below.[12]

- The load steps are defined over a "time" span. The loads are assumed to vary linearly within load steps.
- Within each load step, the program can be directed to perform several substeps to apply the load gradually. At each load step or substep, the geometry and stress-strain relationship are updated.
- At each load step or substep, the program will perform a number of equilibrium iterations to obtain a converged solution.

Figure 18 illustrates a typical load history for a nonlinear analysis in ANSYS.[12]

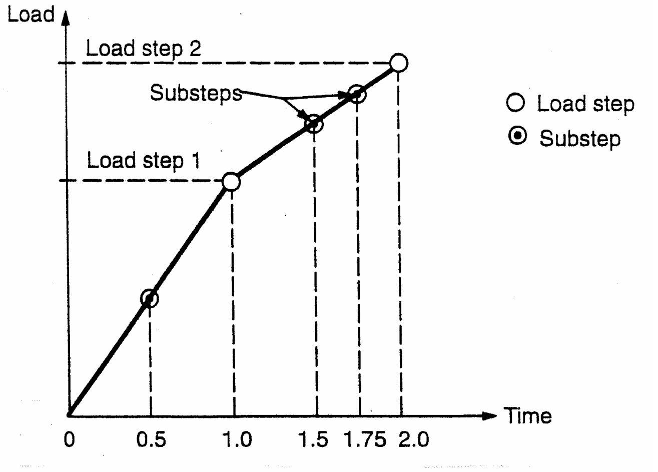

Figure 18: A typical load history for a nonlinear analysis in ANSYS[12]

VI: Built In Fatigue and Fracture Mechanics Capabilities in Commercial Finite Element Software

Most commercial finite element software have built in capabilities for fatigue and fracture mechanics analyses. However, from an educational point of view, it is recommended that these capabilities not be used initially. After a student understands the concepts by going through the steps discussed in this article, he/she can then use the additional capabilities of the software correctly.

A lack of knowledge of the theory behind the more advanced capabilities of the software can lead to the incorrect use of the software.

VII: Conclusion and Summary

It is widely known that a metal component subjected to a repetitive fluctuating load will eventually fail at a much lower load that the load required for generating the maximum allowable static stress. The science of determining the life of a metal component subjected to cyclic loading is called metal fatigue analysis.

This article illustrates the use of commercially available finite element software as a supplemental teaching tool for teaching metal fatigue analysis.

Metal fatigue can be classified into two broad categories of “High Cycle Fatigue” and “Low Cycle Fatigue”. When the stresses in the part are below the metal’s elastic limit the fatigue is usually of the “High Cycle Fatigue” type. When the elastic limit of the metal is exceeded, the fatigue is usually of the “Low Cycle Fatigue” type.

This article discusses the use of the static stress analysis and the vibration analysis capabilities of a typical finite element software for analyzing “High Cycle Fatigue” scenarios. This article also discusses the use of the nonlinear static analysis capabilities of a typical finite element software for analyzing “Low Cycle Fatigue” scenarios.

The use of the finite element technique will enable an instructor to move beyond presenting simple geometries and loading conditions, and will thereby allow the teaching of fatigue analysis techniques involving real world geometries, boundary and loading conditions.

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