AC 2010-269: USING MICROSOFT WINDOWS TO COMPARE THE ENERGY DISSIPATED BY OLD AND NEW TENNIS BALLS

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Using Microsoft Windows to Compare the Energy Dissipated by Old and New Tennis Balls

Abstract
Sound waves produced by bouncing tennis balls are recorded using software commonly available in Microsoft Windows XP and successive times at which the ball impacts a solid surface are extracted from the resulting wave forms. Time intervals between consecutive impacts are related to the energy dissipated by impacts during bouncing. The collected data are analyzed in Microsoft Excel and used to determine the quality of the bounces that can be expected from both new and used tennis balls.

1. Introduction

Experienced tennis players distinguish old tennis balls from new tennis balls by assessing the extent to which the balls dissipate energy during impact with the tennis court. Such bouncing tests relate directly to the concept of the collision of particles, a unit that appears in many curricula that are relevant to the education of physics and engineering majors [3, 9, 10]. In this article, we illustrate how we have used software commonly available in Microsoft Windows XP to demonstrate and analyze energy dissipation that occurs when a tennis ball bounces off the court during a game. We collected data directly from bouncing tennis balls and used them to illustrate a practical application of the coefficient of restitution that students learn in the dynamics of impacts and collisions.

The technique of measuring the coefficient of restitution using the sound produced when a ball strikes a solid surface is not new. It consists of releasing a ball from rest and letting it fall on a rigid surface on which it bounces repeatedly until it stops. The sound produced by successive impacts is recorded and analyzed to give the time intervals separating consecutive impacts. These time intervals are related to the coefficient of restitution. Bernstein used this procedure in1977[13]. Smith, Spencer, and Jones automated this process using a microcomputer in 1981 [14]. Stensgaard and Laegsgaard adapted it to a PC in 2001 [15]. Aguiar and Laudares[16] extended the work of Bernstein [13], Stensgaard, and Laesgaard [15] and used data related to the coefficient of restitution of a bouncing ball to determine of the acceleration of gravity in 2003. Foong, Kiang, Lee, March and Paton [11] applied this work to an examination question aimed at determining how long it took a bouncing ball to bounce an infinite number of times in 2004.

The rest of this paper is organized in the following manner: first we model the mechanics of a bouncing ball and introduce the coefficient of restitution; then, we summarize how this model is combined with the conservation of energy, the coefficient of restitution, and the kinematics of a particle in free fall to yield practical results, which are presented in tabular form; they show how energy dissipation during impact is related to rebound heights, to the time intervals between consecutive impacts, and to the coefficient of restitution; next, we discuss three experimental methods that are suggested by the analytical results obtained; finally, we design and carry out tests that use these methods to determine the energy dissipated by a bouncing tennis ball.

2. Mechanics of a bouncing ball

We consider a tennis ball of mass m. We suppose that, at time $t_0 = 0$, it is dropped from a height $h = h_0$ above a horizontal surface that is both rigid, flat, and located at an elevation $h = h_1$. After free fall, the ball hits a surface that is rigid and flat and bounces a number of times, say, N, before coming to rest. For consistency in notation, we assume that the ball hits the floor at points of elevation $h = h_{2n+1}$ at times time $t = t_{2n+1,}$ respectively, where n = 0, 1, 2, 3, ... Each time, after bouncing, the ball rises towards a corresponding maximum height $h = h_{2n}$, which it reaches at time $t = t_{2n}$. The bouncing cycles, which consist of a free fall, an impact on a rigid surface, and a free rise to a maximum height, keep on repeating themselves until the ball stops bouncing altogether.

When a ball is released from height h_0 and rebounds to height h_2 after impact, the energy dissipated by the impact, denoted by E_{02}, can be expressed as a fraction of the energy that was available at the point of release using

$$\frac{\Delta E_{02}}{mgh_0} = (1 - \frac{h_2}{h_0}). \tag{1}$$

Similar notation is used in the equations found below.

If the ball is allowed to bounce for many consecutive cycles, then, for each cycle, it will attain a new maximum rebound height, h_{2n}, where n = 0,1,2,3,..., N. The energy dissipated by the impact is a fraction of the energy that was available at the start of each drop-and-bounce cycle. These fractions are respectively given by the following energy ratios:

$$\frac{\Delta E_{02}}{mgh_0} = (1 - \frac{h_2}{h_0}), \frac{\Delta E_{24}}{mgh_2} = (1 - \frac{h_4}{h_2}), \frac{\Delta E_{46}}{mgh_4} = (1 - \frac{h_6}{h_4}), \ldots$$

It can be seen that, E, the energy remaining in the system after, say, three bounces is given by

$$E \equiv mgh_0 - \left(\Delta E_{02} + \Delta E_{24} + \Delta E_{46}\right)$$

When this expression is extended to n bounces, and after introducing height ratios, it becomes

$$\frac{E}{mgh_0} \equiv 1 - \left(1 - \frac{h_2}{h_0}\right) - \frac{h_2}{h_0}\left(1 - \frac{h_4}{h_2}\right) - \frac{h_4}{h_0}\left(1 - \frac{h_6}{h_4}\right) + \ldots + \frac{h_{2n}}{h_0}\left(1 - \frac{h_{2(n+1)}}{h_{2n}}\right), n = 0,1,2,3\ldots. \tag{2}$$

2.1 Assumptions. In order to use this analysis to relate time elapsed during bouncing, time intervals between consecutive impacts, heights achieved by the bouncing ball, and the instantaneous velocity of the ball to energy dissipation, we make the following simplifying assumptions:

Assumption 1. The ball is treated as a particle during flight but as a deformable body during impact. The impact process, therefore, involves a change, albeit temporary, in the shape of the ball [3]. A frame-by-frame study of the pictures of bouncing tennis balls obtained using high-speed cameras (2000 frames per second) in our laboratory demonstrated that this process consists of four separate and distinct phases: initial contact, deformation of the original shape, restitution and recovery of the shape of the ball, and separation and takeoff.

Phase 1: Contact. Initial contact between the ball and the surface occurs at one point.

Phase 2: Deformation. Although the lowest point on the ball has been forced to stop moving during initial contact, other parts of the ball continue to move downward. Consequently, a period of continued contact is observed during which the ball is in contact with the surface over an area that increases in size for a short while. The ball accommodates this movement by changing its shape progressively and becoming somewhat flatter and flatter for a very short while. Some of the kinetic energy of the ball becomes stored as elastic potential energy manifested by the deformation of the ball. In this process, the ball is subjected to deformation impulses exerted on it by the supporting surface. They combine with inertial forces to cause the ball to change its shape.

Phase 3: Restitution of the shape. This phase starts immediately after downward and sideways deformations of the ball have stopped and reversed directions. The ball starts to recover the shape it had before impact in a progressive manner; this reversed deformation continues until the original shape of the ball is recovered; during this process, the ball is subjected to restitution impulses.

Phase 4: Separation and takeoff. As time goes on during phase 3, restitution impulses cause the ball to lose contact with the impact surface progressively and to start to rise until the ball loses contact with the surface altogether and takes off.

Assumption 2. Although, in general, the impact surface and the ball can vibrate as a consequence of collisions, our analysis will assume that the speed of the center of mass of the former is zero before, during, and after impact.

Assumption 3. Collision is neither perfectly elastic nor perfectly inelastic. In practice, it is somewhere between these two extremes. Therefore, a fraction of the mechanical energy that is available to the ball is dissipated during impact. Accordingly, energy is not conserved during impact. The concept of the coefficient of restitution (COR) is used to estimate the magnitude of this fraction.

Assumption 4. Aerodynamic effects are neglected, as a first approximation. Thus, the effects of air resistance in the forms of drag, lift, wake, and spin are neglected when the ball is in flight [4]. This assumption allows us to use the conservation of mechanical energy during each free fall of the ball before impact as well as during each free rise of the ball after impact has been completed.

Assumption 5. In each bounce cycle, the time during which the ball is in contact with the bouncing surface, the so-called "dwell time", can be measured. It was shown to be of the order of 5 ms [1]. However, for simplicity, we assume it to be negligible compared to the time during which the ball is in flight. Accordingly, the total duration of a bouncing cycle will be equated to the time during which the ball is in flight.

2.2. The coefficient of restitution. In general, impulses that act on the ball during the deformation phase are different in magnitude and direction from those that arise during the restitution phase of the collision (Assumption 1). It is conventional, therefore, to compare their magnitudes by means of a ratio called the coefficient of restitution.

For two particles A and B that are, say, assumed to be moving in the same direction before as well as after central impact with absolute velocities v_A and v_B, respectively, the linear impulse on the particle during restitution, $\int_{t_d}^{t_i} F_r dt$, divided by that during deformation, $\int_0^{t_d} F_d(t)dt$ is called the coefficient of restitution (COR) and given the symbol e [3].

$$e = \frac{\int_{t_d}^{t_i} F_r(t)dt}{\int_0^{t_d} F_d(t)dt} \tag{3}$$

Here, $F_r(t)$ and $F_d(t)$ are the resultant forces that are applied instantaneously to the ball during the restitution and deformation phases of the impact process, respectively. The symbols t_d and t_i represent the durations of the deformation phase and of the whole impact, respectively. During an actual impact, however, these forces vary with time and are unknown. Thus, it is difficult to determine the COR using Eq.(3) directly. Fortunately, analysis of each of the terms in equation (3) shows that the coefficient of restitution is related to the relative speeds of the particles before and after impact as shown below [3,5,6].

$$e = \frac{(v_B)_{after} - (v_A)_{after}}{(v_A)_{before} - (v_B)_{before}} \tag{4}$$

If particle B represents the ball and particle A the rigid surface, then, using assumption 2, the coefficient of restitution becomes

$$e = -\frac{(v_B)_{after}}{(v_B)_{before}} \tag{4a}$$

In deriving Eq.(4), a sign convention was used: it was assumed that both particles move in the same direction before and after impact (with A following B). Note that the coefficient of restitution will be a positive quantity because the velocities of the ball before and after the collision are in opposite directions due to the fact that the ball reverses directions after impact.

2.3. Results of analysis. The following steps were followed in deriving the results that are summarized and used below.

1) Using the conservation of mechanical energy between the point of maximum height and that of impact (Assumption 4), one gets the speed with which the ball strikes the surface: $\left(v_B\right)_{before}$.

2) Substituting the speed obtained in step (1) into Eq.(4a) and adjusting for the negative sign, one gets the speed of the particle immediately after impact: $\left(v_B\right)_{after} = e\left(v_B\right)_{before}$.

3) Using the conservation of energy again, but this time between the point of impact, where all the energy is in kinetic form, to the point of maximum height, where all of it is in gravitational potential form, one determines the magnitude of the local maximum height, $h = h_{2n}$.

4) Repeating steps1 through 3 over and over again, one gets all the maximum heights achieved by the bouncing ball in terms of the local acceleration of gravity, the initial drop height, and the coefficient of restitution, assumed invariant between consecutive impacts.

5) Using the kinematics of motion of a particle in free fall between the drop height and the first impact and again between consecutive impacts, one gets the duration of the respective time intervals that separate them. The maximum heights and the durations of bounce cycles that were obtained from the application of these steps are summarized in Table 1.

The results that relate successive heights attained by the ball, as shown in Column 2 of Table 1, were used to simplify Eq. (2), that gives the energy that remains in the system after n bounces. Using the results in the last row of Table 1, it can be seen that the total flight time between the nth and the (n+1)th bounces is given by

$$T_n = 2e^n \Delta t_1 = 2e^n \sqrt{\frac{2h_0}{g}} ,$$

which agrees with the expression obtained by Bernstein [13].

If the coefficient of restitution, e, is the same for each successive impact of a given tennis ball, then, the results from Table 1 indicate that the ratio of consecutive heights is a constant given by

$$\frac{h_2}{h_0} = \frac{h_4}{h_2} = \frac{h_6}{h_4} = \ldots = \frac{h_{2(n+1)}}{h_{2n}} \equiv e^2$$

Substituting this result into Eq. (2) gives the following expression for the energy that remains in the ball after n+1 consecutive impacts:

$$\frac{E}{mgh_0} \equiv 1 - \left(1 - e^2\right)\left[1 + \frac{h_2}{h_0} + \frac{h_4}{h_0} + \ldots + \frac{h_{2(n+1)}}{h_0}\right], n = 0,1,2,3\ldots.$$

This expression can be written in compact form as

$$\frac{E}{mgh_0} \equiv 1 - \left(1 - e^2\right)\sum_{n=0}^{N}\left[\frac{h_{2n}}{h_0}\right] ,$$

After introducing the coefficient of restitution, e, this expression becomes

$$\frac{E}{mgh_0} \equiv 1 - \left(1 - e^2\right)\sum_{n=0}^{N}\left[e^{2n}\right].$$

After carrying out the necessary algebra and using mathematical induction, one finds that the fraction of the original mechanical energy that remains in the ball after N consecutive bounces is given by

$$\frac{E}{mgh_0} = e^{(2N+2)}, N = 0,1,2,3,4,\ldots \quad (2a)$$

Since the coefficient of restitution, $0 < e < 1$, Eq. (2a) shows that a bouncing tennis ball dissipates its mechanical energy very fast. Indeed, Eq.(2a) implies that ten consecutive bounces of a tennis ball are not very easy to realize in practice before the ball begins to roll along the bouncing surface.

3. Experimental Methods

The results that are shown in Table 1 indicate that the coefficient of restitution of a tennis ball can be measured using three different experimental methods that are discussed below.

3.1 Experimental Method 1: using height ratios. Using the results in column 2 of Table 1, the coefficient of restitution can be determined by using the initial drop height, h_o, and any maximum height, h_{2n}, achieved during bouncing. Thus,

$$e \equiv \left(\frac{h_{2n}}{h_0}\right)^{1/2n} \quad (5)$$

Table 1. Maximum heights achieved during bounces and the durations of bounce cycles

Peak number	Maximum height	Duration of a bounce cycle	Comments
0 (at time of release)	h_0	$\Delta t_1 = \sqrt{\frac{2h_0}{g}}$	This is a partial bounce cycle. Time interval is from release to the first impact.
1 (after first impact)	$h_2 = e^2 h_0$	$2\Delta t_2 = 2e\Delta t_1$	This is the first bounce cycle. The time interval is between the first and second impacts.
2 (after second impact)	$h_4 = e^4 h_0$	$2\Delta t_4 = 2e^2\Delta t_1$	This is the second bounce cycle. The time interval is between the second and third impacts.
3 (after third impact)	$h_6 = e^6 h_0$	$2\Delta t_6 = 2e^3\Delta t_1$	This is the third bounce cycle. The time interval is between the third and fourth impacts.
4 (after fourth impact)	$h_8 = e^8 h_0$	$2\Delta t_8 = 2e^4\Delta t_1$	This is the fourth bounce cycle. The time interval is between the fourth and fifth impacts.
5 (after fifth impact)	$h_{10} = e^{10} h_0$	$2\Delta t_{10} = 2e^5\Delta t_1$	This is the fifth bounce cycle. The time interval is between the fifth and sixth impacts.
n (after n^{th} impact)	$h_{2n} = e^{2n} h_0$	$2\Delta t_{2n} = 2e^n\Delta t_1$	This is the n^{th} bounce cycle. The time interval is between the n^{th} and $(n+1)^{th}$ impacts.

3.2 Experimental Method 2: using time intervals between impacts. Using the results in column 3 of Table 1, the coefficient of restitution can be determined by computing the ratio between the duration of the time interval between any two consecutive impacts, $2\Delta t_{2n}$, and the initial drop time, Δt_1.

$$e \equiv \left(\frac{\Delta t_{2n}}{\Delta t_1}\right)^{1/n} \tag{6}$$

where

$$\Delta t_1 \equiv \sqrt{\frac{2h_0}{g}} \tag{7}$$

3.3 Experimental Method 3: using the total duration of bouncing. A third way to determine the coefficient of restitution is to use the ratio between the total duration of the bouncing pattern, Δt_{total} , and the initial drop time, Δt_1.

The total time elapsed before the ball stops bouncing is obtained by summing the durations of all the individual intervals given in column 3 of Table 1. Doing so gives

$$\Delta t_{total} = \Delta t_1 + 2\sum_{n=1}^{N} \Delta t_{2n}$$

Using the expression for Δt_{2n} found in Table1 in the above equation results in

$$\Delta t_{total} = (1+e)\Delta t_1 \sum_{n=1}^{n=N} e^{n-1}.$$

Noting that the sum $\sum_{n=1}^{\infty} e^{n-1}$ is a geometric series [7] that converges to $\frac{1}{1-e}, e < 1$, one has

$$\sum_{n=1}^{n=N} e^{n-1} = 1 + e + e^2 + e^3 + e^4 + \ldots + e^{N-1} \approx \frac{1}{1-e}, |e| < 1$$

Thus, the time it takes the ball to stop bouncing altogether can be approximated by

$$\Delta t_{total} = \frac{(1+e)}{(1-e)}\Delta t_1. \tag{8}$$

Solving for the coefficient of restitution from Eq. (8) leads to

$$e \equiv \frac{\alpha - 1}{\alpha + 1}, \tag{9}$$

where

$$\alpha \equiv \frac{\Delta t_{total}}{\Delta t_1}. \tag{10}$$

When a collision is perfectly elastic, no energy is dissipated. Then, the total bouncing time, Δt_{total}, is very large, compared to the drop time, Δt_1, and e approaches unity, as can be seen from Eq. (9). If, on the other hand, the collision is perfectly inelastic, the particles stick to each other and there is no bounce at all; then, the total time, Δt_{total}, is equal to the drop time, Δt_1, and the coefficient of restitution is zero, as can be seen from Eq. (9). However, in general practice, the coefficient of restitution is between these two extremes and must be determined experimentally.

In processing the data collected from the experiments that are described below, we used all three methods outlined above to determine the coefficients of restitution of bouncing tennis balls.

4. Experimental determination of energy dissipation

4.1. Method 1: The standard tennis-ball bounce test. Anyone who plays tennis, or watches it, knows that experienced players test the quality of a tennis ball by assessing its ability to bounce after it hits the playing surface of a court. Indeed, both the International Tennis Federation (ITF) and the United States Tennis Association (USTA) require that tennis balls meet specified bounce requirements before being certified for sale to the public and for use in official tennis competitions [1]. These organizations use the standard test discussed here.

The standard tennis-ball bounce test uses height ratios to assess the quality of a tennis ball. Certification rules require that, when dropped from a height of 100 in (253 cm) onto a concrete floor, the bottom of a sample tennis ball that is to be certified must rebound to a height between 53 in (135 cm) and 58 in (147 cm). When earned, manufacturers imprint such certification on the containers of their tennis balls before marketing. This standard tennis-ball bounce test can be done in class to demonstrate the common expectation that a bouncing tennis ball loses mechanical energy during each impact with the surface of the court.

The standard tennis-ball drop experiment can be done manually in class and it does not take long. Besides a tennis ball, the experiment only requires a ruler long enough and with fine markings to allow the reading of both h_0 and h_2, a wall or some other vertical surface against which the ruler can be taped, and two diligent operators: one to release the ball from a specified height and the other to spot, read, remember, and record the heights to which the ball bounces after impact. It is also possible to videotape the bounces and play them back frame by frame in order to extract the rebound heights from appropriate frames [1].

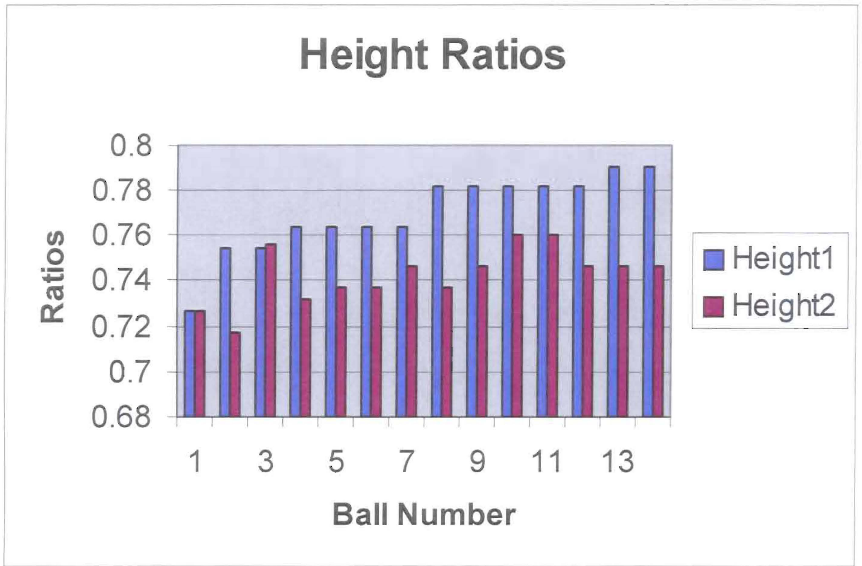

Figure 1. Rebound height divided by the initial drop height.

We tested fourteen used tennis balls this way to determine their respective rebound heights. They were numbered one through fourteen. The first six balls were from group A and the remaining eight from group B. The former had been used in more games than the latter. During the test, we used two different initial drop heights: height1 (h_0 = 91.44 cm or 36 in) and height 2 (h_0 = 177.80 cm or 70 in) and recorded the corresponding first rebound heights (h_2). The ratios consisting of the rebound height divided by the initial drop height were computed, plotted, and shown in Figure 1. For each initial drop height, the plots demonstrate that balls 1 through 6 (group A), which had been used in more games, displayed rebound heights that were smaller than those of balls 7 through 14 (group B), which had been used in fewer games. It can also be seen from that figure that the height ratios decreased as the initial drop height was increased from height 1 to height 2. This agrees with the results reported by Brody [2]. This illustrates the fact that the coefficient of restitution of a ball is a dynamic quantity, for it depends on the speed that the ball has when collision occurs. Figure 2 uses the same data to present different, but related, information. It shows the percentage of the total energy that is dissipated during impact [Eq. (1)], instead of the ratio of heights. As shown in Figures 1 and 2, comparing balls in group A with those in group B indicates that the extent to which a tennis ball has been used affects its rebound height and, hence, the amount of energy that is dissipated during impact.

4.2. Method 2: The revised tennis-ball bounce test. The revised tennis-ball-bounce test uses the time intervals between consecutive impacts to determine the energy dissipated during impact, instead of rebound heights. In carrying out tennis-ball bounce tests as was done in the previous experiment, one discovers that it is difficult to take readings of the maximum heights that are attained by the tennis ball after the first few bounces. This is because the rebound heights become smaller and smaller and the time between them decreases as well. However, Bernstein [13] described another way to collect data during tennis-ball bounce tests that we extended to circumvent this difficulty. He used microphones to record the sound made by the ball as it impacted the floor. Smith, Spencer and Jones [14] computerized the Bernstein procedure; Brody [1] amplified the resulting signal and displayed it on an oscilloscope. He then used the sweep gate outputs of the scope to turn a commercial timer on and off. Stensgaard and Laesgaard [15] used a PC to collect and analyze sound data. All of these authors were able to collect the time elapsed between impacts. Although Brody [1] demonstrated that the sound produced by a tennis ball during impact can be used to determine both its rebound height and the energy dissipated in the process, he did not show the details of the analysis that lead one to relate time intervals between impacts to height ratios and energy dissipation. We did so in Table 1.

We adapted Brody's experimental design to Microsoft Windows XP [1]. A computer microphone (GE, or Radio Shack, or other) was connected to the computer and turned on. Microsoft Sound Recorder was opened and recording was initiated. Then, a tennis ball was dropped onto a hard surface from an initial height of 3 ft (36 in). The sound made by the many consecutive times that the ball hit the floor was recorded as long as bouncing lasted. In order to make it easier to visualize the waveform upon playback, the speed of the recorded sound was decreased. After capturing the waveform, a sound editor (Roxio Sound Editor V5.1.0.104) was used to look at the graph of the sound waves on scope view, which is an amplitude-vs-time graph that simulates the screen of an oscilloscope. Then, using the sound editor repeatedly, the times of impact could be identified from the graph using a curser and extracted for processing.

The collected data were processed in Microsoft Excel. However, students who used other software such as MATLAB, MAPLE, and MATHEMATICA reported results similar to those obtained with Excel.

We tested six different tennis balls (group C) using Microsoft Windows. Three balls were brand new and three had been used in a tennis match by the tennis team of our university. All six balls were from the same manufacturer, and they were different from those discussed earlier (groups A and B). The objectives of these new tests were two-fold: to see if one could detect a difference in the bounce mechanics of the tennis balls after one match and to assess how the coefficient of restitution varied with initial drop heights.

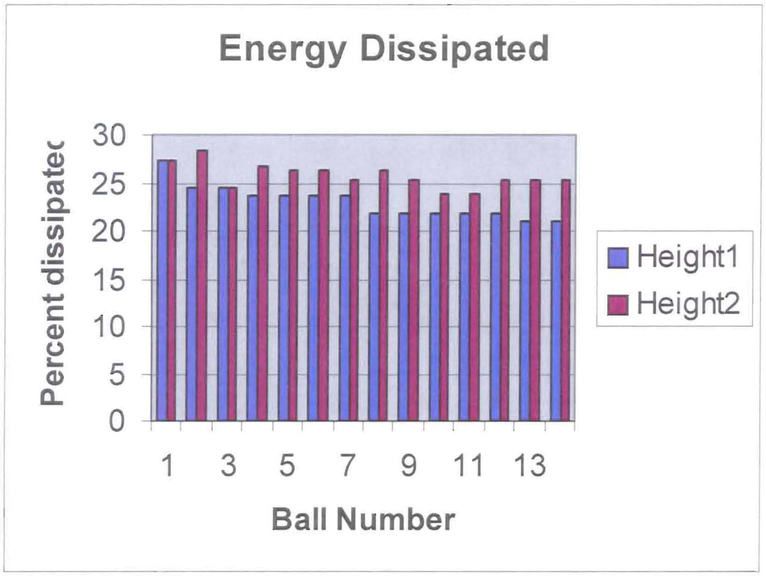

Figure 2. Percent of the initial energy dissipated after the first impact.

Five identical drop-and-bounce tests were done on each tennis ball. And, for each test, we collected data from the first nine consecutive bounces; this yielded the time intervals for the first eight bounce cycles. Thus, for each ball, we had the durations of forty different bounce cycles. A coefficient of restitution was computed for each such cycle using Eqs.(6), (9) and (10). In general, the results were very similar, although using the total time, Eq. (9), one obtained values that were approximately 2 % smaller than those from Eq. (6). For each used ball, both the average coefficient of restitution and the range in which all coefficients fell are shown in Table 2. The corresponding data for new balls are shown in Table 3. In each case, it was observed that the coefficient of restitution increased with the number of bounces, confirming our earlier findings with balls in groups A and B as well as the observation of Berenstein [13] and Brody [1, 8] that the coefficient of restitution decreased with increasing kinetic energy of the ball at impact. Note that the data reported by Aguiar and Laudares [16] did not confirm this observation.

Table 2. Ranges and average values for the coefficients of restitution of used balls.

New Ball no.	Number of data samples	Range of values	Average value
1	40	0.747 to 0.824	0.801
2	40	0.792 to 0.830	0.811
3	40	0.796 to 0.821	0.810
All three	120	0.747 to 0.830	0.8074

Table 3. Ranges and average values for the coefficients of restitution of new balls.

Used Ball no	Number of data samples	Range of values	Average value
1	40	0.780 to 0.834	0.813
2	40	0.795 to 0.829	0.811
3	40	0.773 to 0.825	0.807
All three	120	0.773 to 0.834	0.810

5. Discussion of results

It can be seen from Tables 2 and 3 that the averages of the coefficients of restitution of the balls did not change appreciably after moderate use in one match. However, Eq. (2a) shows that the energy remaining in the ball is very sensitive to the effective coefficient of restitution. Thus, even moderate changes in this coefficient affect the bouncing dynamics of the ball in an appreciable manner. For example, a very lively tennis ball with $e = 0.80$, dissipates 36 % of its energy after the first bounce and 60 % after the second. When, due to usage, that coefficient decreases by, say, 10 % , to $e = 0.72$, the same ball now dissipates 48 % of its energy after the first bounce and 73 % after the second.

In order to verify this effect, we computed and plotted the distribution of the coefficients of restitution within the ranges given in Tables 2 and 3. We found that they varied significantly. This variation is shown in Figure 3. It can be seen that the distribution associated with used balls is different from that obtained from new balls in two ways: their shapes are different and the former shifts to the left, indicating lower bounces, and, thus, higher energy dissipation per impact. This implies that the quality of individual bounces is more variable and, hence, less reliable for used balls than for new ones. In order to confirm these observations, we obtained three balls (group D) that were used in many practice matches (very used). Although the number of actual matches was not known exactly, it was estimated to be between three and six, depending upon the ball. These balls were tested, and the resulting data were plotted together with those shown earlier in Figures 1 and 2.

This new combination of data is shown in Figure 3. It can be seen that both the spread of the distribution and its left-ward shift are confirmed in that figure. This may help explain why many new tennis balls are used during professional matches of tennis as well as why experienced players discard used tennis balls earlier than beginning players. Perhaps, during a given game, the probability of getting reliable individual bounces matters just as much as the magnitude of the average bounce.

Our experiments appeared to suggest that there is a threshold to this energy-dissipation process: a critical number of bounces below which this effect is less appreciable and above which it is. Our data and discussion with tennis players did not allow us to estimate this number precisely. However, it was estimated to be somewhere between the bounces required to complete one and two college tennis matches. In an attempt to have some estimate for this threshold for professional games, we watched the French Open in May 2004, counted the number of bounces during each game that was accessible to us in our TV-viewing area, and compared those numbers to our earlier estimates. The estimates did not agree. Nevertheless, our experiments and our results from the French open indicated that the active life of a new tennis ball is not substantially more than one professional set of tennis games.

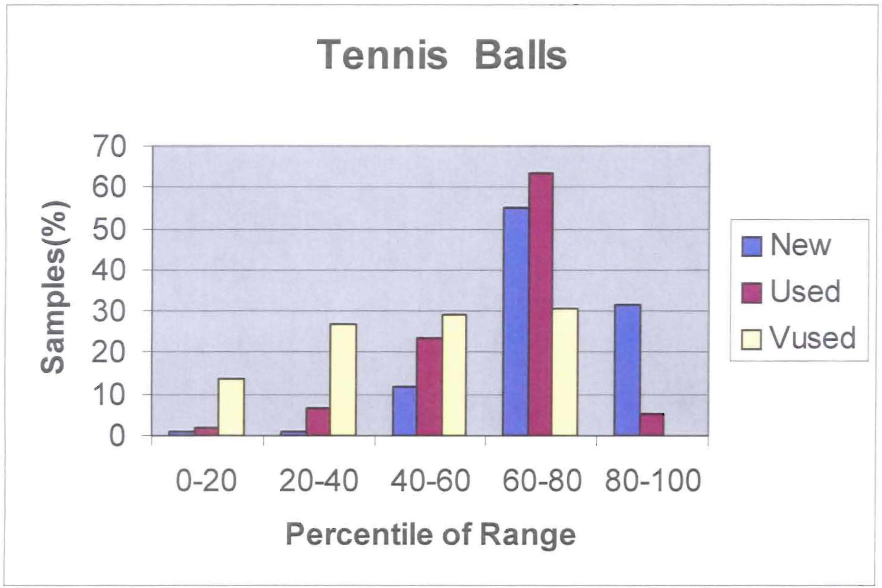

Figure 3: Distributions of the COR of the tennis balls are compared.

6. Conclusions

The data collected in these simple experiments were used in class to illustrate and verify the common experience that usage of a tennis ball increases its ability to dissipate energy during an impact. Hence, the more a tennis ball is used, the more it loses the ability to restore its kinetic energy after a bounce.

Newton observed that the relative velocity after impact is proportional to that before impact. The constant of proportionality is now called the coefficient of restitution (COR). It indicates the extent to which a given impact approaches, the so-called perfectly-elastic collision, the ideal case of collisions that occur without losses of energy. Although many dynamics textbooks do not discuss this fact, experiments show that this coefficient depends on many variables: the elastic properties of the bodies that are involved in the collision, their relative velocities before and after impact, their shapes, their sizes, their masses, the mass density of the medium in which the collisions take place, and the temperature of the environment [9].

During the bouncing process, the tennis ball is subjected to cyclic linear impulses that are alternatively compressive and tensile and the ball becomes less and less elastic while the shapes of the hysteresis loops it traces change and material fatigue sets in slowly but progressively. With repeated usage, then, a new ball loses much of its initial ability to store and restore the energy that is given to it. The explanation for this loss is ultimately related to material properties of the tennis ball as a partially elastic shell that undergoes hysteresis and fatigue [10].

Our principal goal in teaching mechanics is to present the subject as an intellectual exercise in understanding the behavior of phenomena; and one that uses accepted principles of mathematics and physics in getting answers to important questions that are relevant to daily life. Naturally, in the course of doing so, one performs computations, out of necessity. This is particularly important in beginning courses, where the novice tends to view the subject principally as an exercise in computation using calculators, software, and canned formulas. By investigating a commonly encountered problem, such as that of bouncing balls illustrated herein, and doing so with modern tools, we attempt to give life, currency, and relevance to old concepts and principles.

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