



Using Time-Based Experiences for Explaining the Concept of Discontinuity

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With more than 25 years of combined experience in the high-tech industry, government and academia Dr. Raviv developed fundamentally different approaches to "out-of-the-box" thinking and a breakthrough methodology known as "Eight Keys to Innovation." He has been sharing his contributions with professionals in businesses, academia and institutes nationally and internationally. Most recently he was a visiting professor at the University of Maryland (at Mtech, Maryland Technology Enterprise Institute) and at Johns Hopkins University (at the Center for Leadership Education) where he researched and delivered processes for creative & innovative problem solving.

For his unique contributions he received the prestigious Distinguished Teacher of the Year Award, the Faculty Talon Award, the University Researcher of the Year AEA Abacus Award, and the President's Leadership Award. Dr. Raviv has published in the areas of vision-based driverless cars, green innovation, and innovative thinking. He is a co-holder of a Guinness World Record. His new book is titled: "Everyone Loves Speed Bumps, Don't You? A Guide to Innovative Thinking."

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Using Experiences for Explaining the Concept of Discontinuity

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Abstract

This paper is about real-life, experience-based examples that can help instructors teach, and students understand, the concept of discontinuity. There are, of course, sophisticated examples such as energy levels of electrons and phase transitions where a quantity changes discontinuously. However, there is a need for visualizing the concept in a more fundamental way using examples that are outside the mathematical domain so that students can comprehend the idea prior to delving into purely mathematical proofs and explanations of the different types of discontinuity. This is particularly true since the new generation of students comprehend material in more visual, intuitive, and engaging ways, while at the same time most textbooks pay little attention to connecting the concept to reality.

The paper focuses on two sets of examples:

1. Examples that are unrelated to time. These include (a) discontinuity in space, for example water levels at different sides of the locks in Panama Canal, sharp change in elevation of sidewalks (known as curbs), length of unused paper towel or toilet paper, change in brightness level from light to shadow and between intensity level of pixels in a digital image, (b) numerical displays, such as an abrupt change in the numerical display of an elevator's floor, change in digital display of radio frequencies, (c) switch-based devices such as light switches, (d) audio frequencies, such as audio frequencies of piano keys, and (e) cartoon-based and non-STEM examples of discontinuity.

2. Examples that are related to time. These include: (a) time regions, e.g., time zones, daylight saving time, and the International Date Line, (b) devices that operate based on using discontinuously, e.g., A/C control, ABS, digital clock, stroboscope and Zoetrope, and animation videos, (c) activities, such as "exploding" rubber watermelon, flip book, and (d) experiences that can illustrate potential loss of information due to discontinuity, e.g., the "Heidi" football story, and of course discontinuous sensing while texting and driving. We use anecdotes and stories that can help connect the students to the topic. These include, for example the Samoa new time zone, and the lost day, and Jules Verne's story *Around the World in 80 Days*.

We refer to the material in this paper as a work in progress. It has not yet been tested in a classroom setting. However, we intend to assess the effectiveness of the ideas in the near future. In the past, when we used visual and intuitive examples, students have demonstrated clearer understanding of difficult concepts. Based on similar experiences that were gained and assessed multiple times by the author in other engineering related subjects (Control Systems, Digital

Signal Processing, Computer Algorithms, Mechanical engineering, Calculus, and Physics), it is believed that the approach has great potential. In addition, the information and ideas for enhancing teaching and learning are not meant to replace existing textbooks or other teaching tools. They should be viewed as supplemental to existing, well established and tested educational material.

I. Introduction

Learning preferences of “Gen Z” and “Gen Y” (millennial) students who make up more than one third of the US population are very different from those born a decade or two earlier. As they are exposed to web-based videos, animations, blogs and texts, they make very little use of information from traditional textbooks, and expect more visual, intuitive and engaging, fun, and tech-based information from their instructors. With shorter attention span students expect brief explanations on the relevance of the topics they learn, as well as on potential utility. This expectation is most pertinent in math courses that are usually taught with little or no connection to real life or disciplines (in particular engineering).

At Florida Atlantic University we have also observed that there are students who have already taken calculus courses but are still having trouble understanding key concepts in mathematics, such as discontinuity, limits, integration, and differentiation. In order to adapt to students’ new learning preferences, efforts must be made to further modify teaching methods. Instructors must add value to classrooms and on-line lessons, in particular by introducing different approaches to concept comprehension.

This paper focuses on introducing the concept of discontinuity by linking it to daily experiences using relevant and analogy-based examples. The examples can be used prior to digging into purely mathematical details, and are meant to help in conceptual understanding of the material by developing intuition and basic comprehension, i.e., getting the “Aha!” moment, a moment of sudden inspiration or insight. This is desired since when it comes to grasping concepts in regular classroom settings (that are based on traditional old-style textbooks) students repeatedly miss the eureka effect.

The paper uses multiple examples of discontinuity, as related to: (a) discontinuity in space, (b) numerical displays, (c) switch-based devices, (d) audio frequencies, and (e) cartoons and non-STEM examples. It also uses time-based examples of discontinuity, including: (a) time regions, e.g., the International Date Line, (b) devices that operate based on using discontinuously, e.g., A/C control, (c) activities, such as “exploding” rubber watermelon, and (d) experiences, such as discontinuous sensing while texting and driving.

The examples in this paper are meant to provide additional material for introductory purposes only to allow students to comprehend concepts, develop some intuition, and to see the relevance of math to their daily lives. In addition to helping instructors in enhancing students’ understanding of the material, this approach may keep students more interested and engaged. The examples are meant to be add-ons, and are not meant to suggest replacement of existing course material.

The material is referred to as work in progress and is designed to be shared and discussed with multiple audiences. When these kinds of examples were used, students demonstrated better understanding of difficult, abstract concepts and praised the approach. Based on similar experience that was gained and assessed by the author in other STEM subjects (Control Systems, Digital Signal Processing, Computer Algorithms, Algebra, Calculus, Statics, Thermodynamics, Statistics, and Physics), it is believed that the approach has promising potential.

The planned assessment will focus on the effect of the new content on learning outcomes. It will emphasize concept comprehension and student interest in the topic. Clearly, using a variety of assessment methods provides a more complete picture of students' learning/proficiency, so assessment method will include, at a minimum, individual and group quizzes/tests, authentic performance tasks, portfolios (and possibly projects), observations and interviews. The most relevant types of assessment in this case are formative (multiple times during the instruction process), summative (to be focused on student's comprehension), and interim assessments.

The idea of explaining STEM material in visual and intuitive ways is not new. For example, Tyler DeWitt [1] taught high school students the topic of isotopes. He explained that isotopes are basically the same atom using an analogy involving similar cars with minor changes. There are a few calculus textbooks that include visual explanations (see for example [2-10]). Apostol and Mamikon from Caltech [11,12] explained integration of functions without mathematical formulas. A paper that explains integration using multiple visual and intuitive examples provides specific examples that instructors can readily use [13]. Additional interesting problems and examples for discontinuity are detailed in [14]. Some excellent visual examples can also be found in [15,16]. Recently, Ben Orlin [17] has taken a very refreshing and insightful visual approach to math including the concept of discontinuity.

II. Examples

The following are two sets of discontinuity examples. One set describes examples that are unrelated to time, and the second set describes examples that are related to time.

Set 1: Examples that are unrelated to time

(a) Discontinuity in space

(a1) Steps

A basic way to introduce discontinuity in space is by using daily experiences examples such as sharp elevation changes of a sidewalk, (i.e. a curb) that is almost a step function (Figure 1, left). Similarly, we can look at steps as a set of consecutive spatial discontinuities that can be described as a collection of zero order polynomials (Figure 1, right).

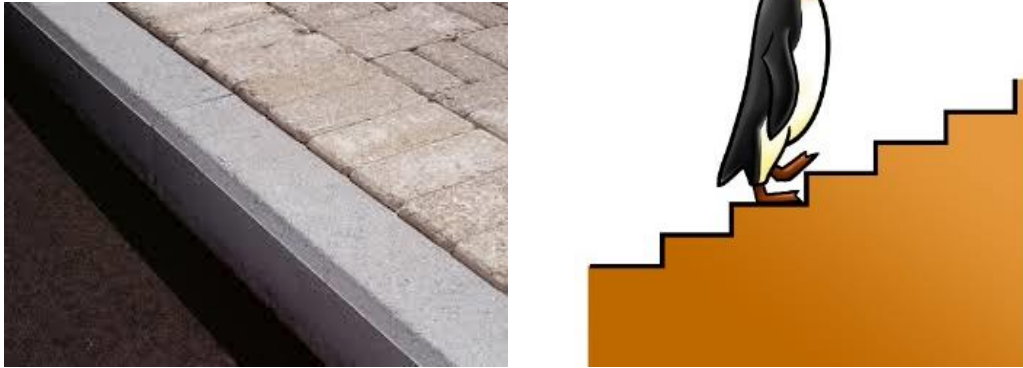


Figure 1: Steps as discontinuities

Figure 2 shows a discontinuity in a function (the “step part” of it) and discontinuities in its derivative. A person walks from left to right on a horizontal flat surface (zero slope), followed by abrupt change to an uphill positive slope, and followed by another change in the slope back to a horizontal surface. The person then encounters a discontinuity in the function itself (a step downward), followed by a sequence of changes in the derivative.

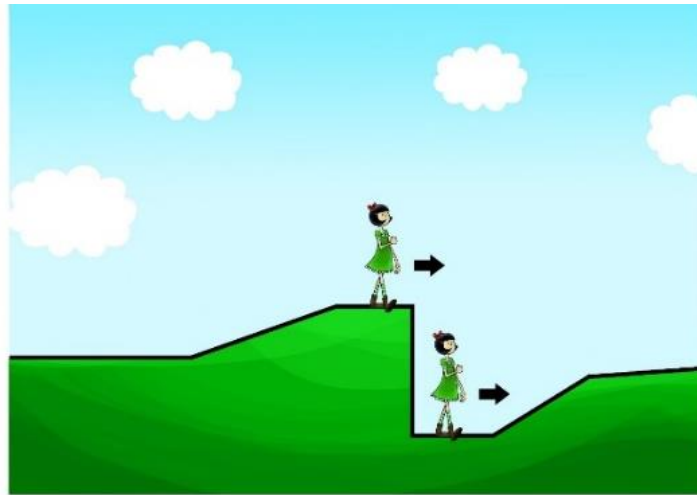
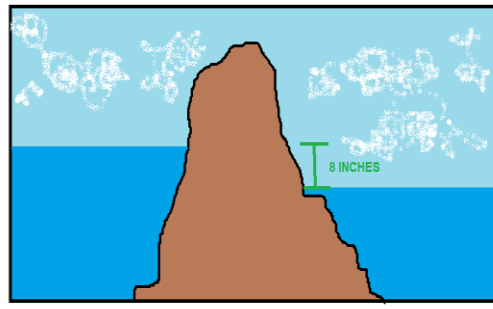


Figure 2: Discontinuities and slopes

(a2) Water level differences in Panama Canal

The Panama Canal (Figure 3, left) consists of a lock system that allows ships to cross between the Atlantic and Pacific Oceans. Even though the height difference between the Atlantic Ocean and the Pacific Ocean is only 20 centimeters (about 8 inches) (Figure 3, right), the lock system of the original canal lifts the ships up 26 meters (about 85 feet), and then down. The original Canal had a total of six lock-based steps to complete the move. The water level between the locks vary during operation, and most of the time there is a sharp change in height level of the water between the two sides of each lock (Figure 4).



PACIFIC OCEAN

ATLANTIC OCEAN

Figure 3: Original Panama Canal (left) and the overall height difference (right)

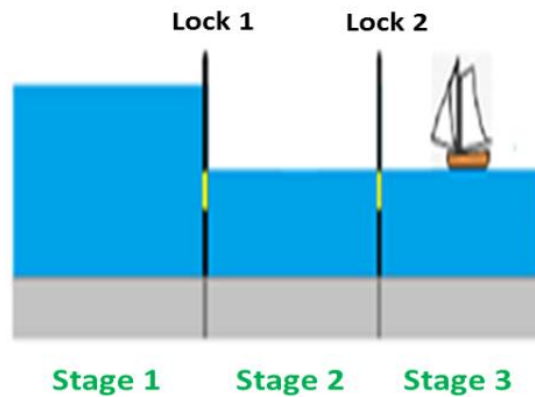


Figure 4: Illustration of locks. Note the discontinuity between stage 1 and stage 2

To make the example more memorable, an instructor can also share some facts about the canal, such as: (1) The Canal is considered one of the seven wonders of the modern world, (2) The Canal took about 34 years to be completed and is one of the most difficult engineering projects ever carried out, and (3) On August 23, 1928 Richard Halliburton transited the Canal swimming, paying a toll of 36 cents, calculated based on his weight of 150 pounds.

(a3) Shadow's edge

Edges of shadows can be used to explain discontinuity by referring to an abrupt change in image brightness. In Figure 5 we can see this effect by observing the shadow edge of the walking surfer as projected on a bright flat surface.



Figure 5: Shadow edge: an abrupt brightness change

Source: <https://expertphotography.com/shadow-photography-tips/>

(a4) Discontinuity due to pixilation

In computer vision, a branch of computer science, black and white images are displayed as a collection of pixels each of which has its own brightness level. The change in brightness between two adjacent pixels is abrupt and can be clearly seen when zooming in on digital images. Analog images can be digitized and pixelated causing multiple abrupt changes in brightness levels. For example (Figure 6) is a pixelated monochromatic photograph of Abraham Lincoln from a \$5 bill [18].

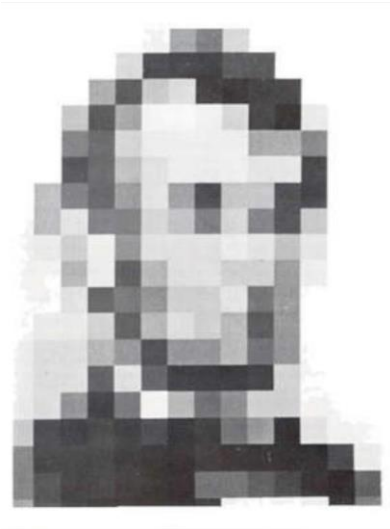


Figure 6: Pixelated photograph of Abraham Lincoln by Leon Harmon [18]

Source: <http://mombartz.com/leon-harmon-pixel-lincoln/>

A few years after Harmon's paper [18] was published, Spanish surrealist painter Salvador Dali created an iconic piece based on Harmon's paper. Depending on the resolution of the pixelated image, the range in brightness discontinuity varies. The higher the resolution the lower the range. Try to squint your eyes to better see such images. By squinting, we see a smeared image that allows us to recognize the person.

(a5) Length of unused of paper towel and toilet paper

The invention of paper towel or toilet paper perforation made it easy to tear one or more sheets off. So even though the paper towel or toilet paper is a continuous roll of paper, the perforation helps with tearing it in a discrete manner, which leads to discontinuity in its length when used, as can be seen in (Figure 7).

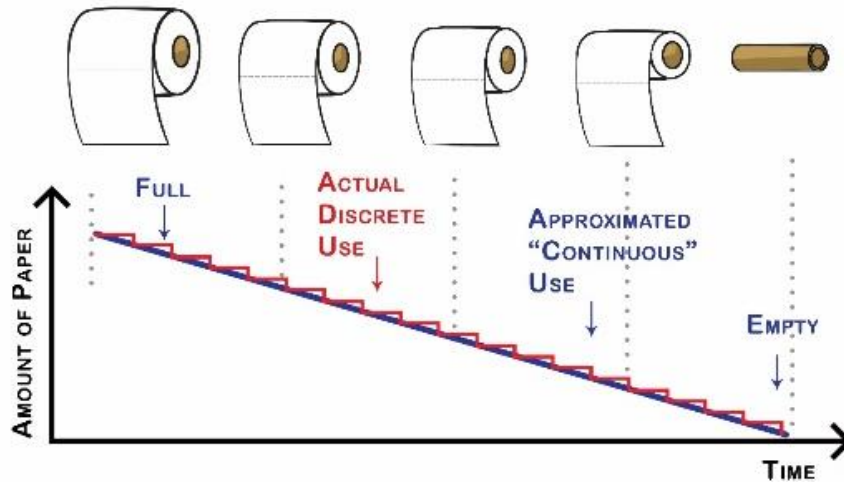


Figure 7: Discontinuity in length of toilet paper due to perforation

(b) Numerical displays

(b1) Display of an elevator's location

Imagine waiting for an elevator to reach a certain floor. Even though the elevator's physical location at any time during motion is a continuous function, you are presented only with a numerical display that tells the approximate floor location of the elevator (Figure 8). This display consists of a set of integers that change abruptly and no fraction is visible. This is a clear discontinuity.



Figure 8: Discrete display of elevator's location

(b2) Display of radio frequencies

Older radios required analog tuning, allowing the user – at least theoretically – to choose any frequency within a certain range. A dial was used to tune the radio until a desired station was clearly heard (Figure 9, left). In today’s radio display, the frequency of a radio station can be chosen and displayed in increments of 0.1 MHz. You may choose and display, for example, 89.8MHz or 89.9MHz but not an in between frequency, say 89.84MHz (Figure 9, right).



Figure 9: Analog radio (left) allows for tuning to a continuous range of frequencies. Digital display (right) shows only finite discontinuous set of frequencies

Source: <http://www.agtannenbaum.com/images/Zenith5S319.jptg.jpg>

(c) Switch-based devices

(c1) Changing position of a light switch

Turning on the light in a flashlight, a room, cell phone or other device, requires a mechanical or electronic switch (Figure 10). The binary switch is either “on” or “off.” Consequently changing positions in a switch causes a discontinuous change in light intensity (at least theoretically). Theoretically the switch is a binary device: it is either open or closed (zero or one).



Figure 10: Light switch (left) and effect of switch-based flash light on ambient brightness

(c2) Output of digital to analog converter

When using digital devices we often use an analog-to-digital converter (ADC). The ADC converts an analog voltage (within a certain range) to its corresponding discrete level. However, since each ADC has a resolution defined by its number of bits, some analog values are truncated to a certain output levels. There are jumps in the values of the discrete signal. For example an 8-bit ADC has $2^8 = 256$ possible output levels. If the input range of the analog signal is 0-10 volts, then the discontinuity between two consecutive discrete vales is $10/256 = 0.039$ volt or 39 millivolt.

(c3) Digital display of an air conditioning thermostat

Digital thermostats accept whole numbers as a desired temperature. One can only enter a whole number, say 70°F or 71°F but not 70.5°F. The display of the actual temperature is also digital and it is hard to tell how close it is to the real physical temperature (Figure 11).



Figure 11: Display of temperature

(d) Audio frequencies

(d1) Audio frequencies of piano keys

Piano keys (Figure 12) produce a set of discrete frequencies out of endless audio frequencies. Each key on the keyboard (including the dark keys) represents a note with a frequency of approximately $2^{1/12} \approx 1.059$ times the frequency of the adjacent lower key. No frequency in between two consecutive notes is available.



Figure 12: A piano key corresponds to a specific audio frequency

Unlike the piano, the violin (Figure 13) produces a continuous range of audio frequencies. It has an unlimited number of notes to choose from a certain frequency range.

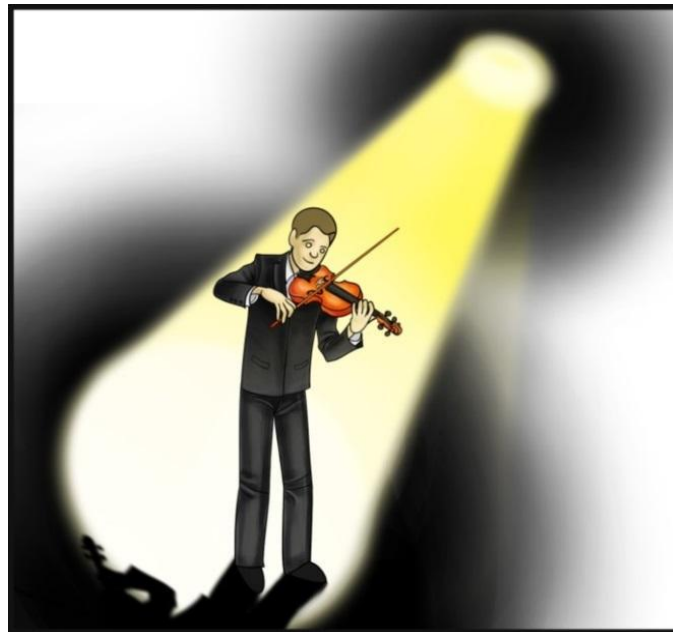


Figure 13: Violin can produce continuous range of frequencies

(e) Cartoon-based and more examples of discontinuity

(e1) End of road (Figure 14)



Figure 14: Discontinuity in road [19]

(e2) Reading a newspaper (Figure 15)

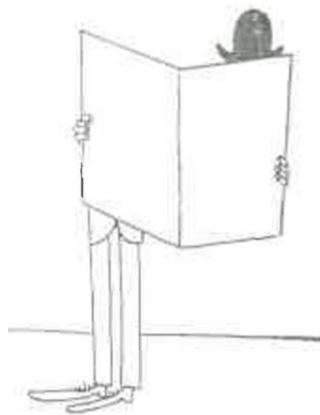


Figure 15: Discontinuity [20]

(e3) Pain assessment

What is your pain level? Here is a simple way to numerically assess pain level using a discontinuous display. (Figure 16).

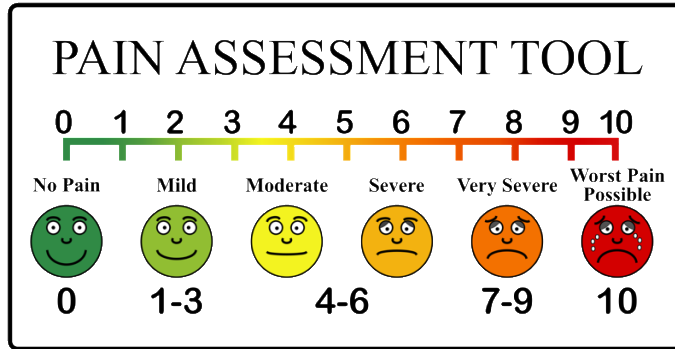


Figure 16: Discrete pain assessment

(e4) Book organization

Assume that you have many books that were put in a certain order on a shelf. A practical way to make sure that they are in a specific order is to draw a diagonal line from one extreme end to the other. Any discontinuity will show up immediately and tell you to re-order them. (Credit: The original idea is credited to my dear colleague N. Shneydor.) (Figure 17).

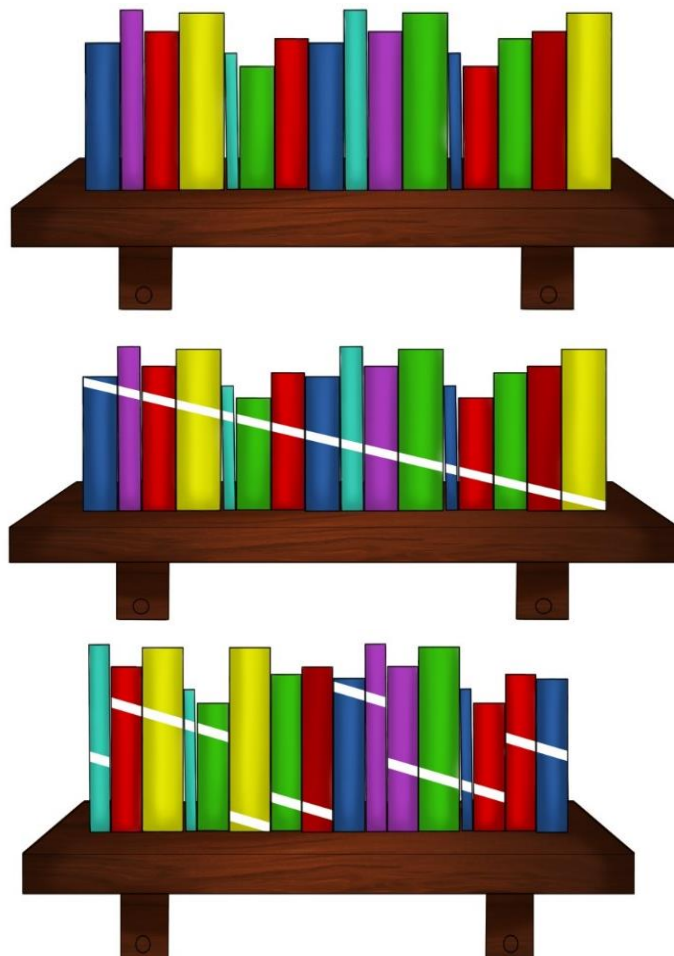


Figure 17: Book organizing

Set 2: Examples that are related to time

We start with an introductory cartoon that visualizes the idiom “The straw that broke the Camel’s back” (Figure 18). This well-known idiom that can give students a glimpse of time-related discontinuity without using an official definition.



Figure 18: The straw that broke the Camel’s back

Source: <https://windowthroughtime.wordpress.com/2016/12/16/what-is-the-origin-of-108/>

(a) Time regions

(a1) The idea of “noon” time

Although time is continuous, we display it in ways that are discontinuous. A basic example is noon time. The time just before “12 noon” is represented as 11:59:59AM and the time immediately after “12 noon” is represented as 12:00:01PM (of course we can increase the resolution of the display to add portions of a second, i.e., tenths and hundredths, etc.). In addition to the abrupt change in all digits from 11:59:59 to 12:00:01, at noon, there is a momentarily change in the display (happens literally at no time) from AM to PM: this is a discontinuity. At an infinitesimally small time ϵ before Noon it is still considered AM and at infinitesimally small time ϵ after Noon it is PM. At some point in time, 12 noon, there is a discontinuity agreed to be part of the PM time region, so noon is 12PM (Figure 19).

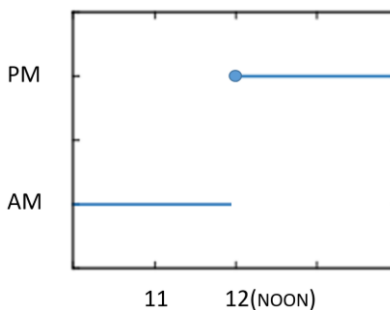


Figure 19: Step function: note the value “PM” at 12 (noon)

(a2) Time zones: Pacific, mountain, Central, Eastern:

Imagine driving from the West Coast to the East Coast. At some point, towards the end of your long trip, you may encounter a road sign that reads “Leaving Central Time Zone” and “Entering Eastern time Zone” (Figure 20). This means that at a certain point on the road the official time is forwarded by one hour, i.e., if while driving at the Central Time Zone the time is 3:12 PM you switch “abruptly” (discontinuously) to the Eastern Time Zone, i.e., to 4:12 PM.



Figure 20: Road signs notifying drivers of change in time zones

Source: <https://twitter.com/FPCSITRIPS/status/898285015447912448/photo/1>

Excluding Alaska and Hawaii, there are four time zones in the US (Figure 21). If for example, the time in California (Pacific Time) is 4:15AM, then in Colorado (Mountain Time) it is 5:15AM, in Kansas (Central Time) it is 6:15AM, and in Florida (Eastern Time) it is 7:15AM. Each change is a discontinuity in the agreed-upon time.

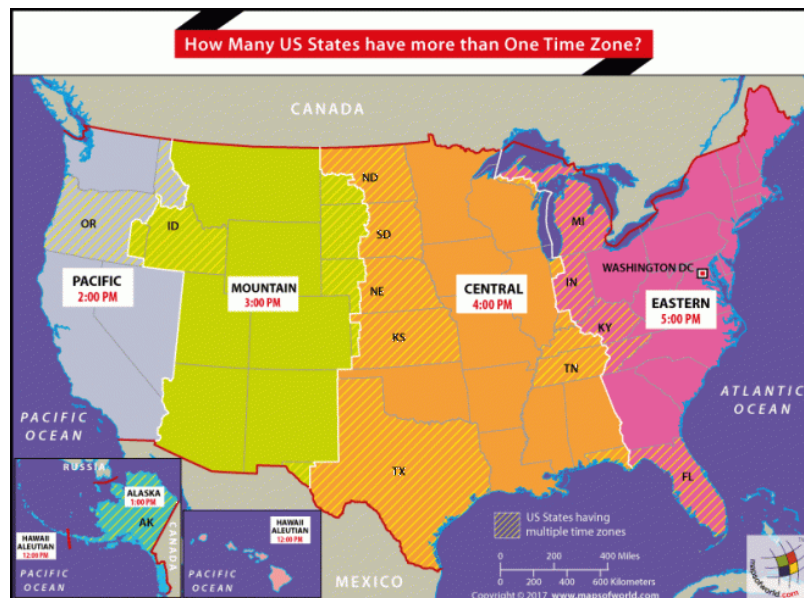


Figure 21: Time zones in the US (excluding Alaska and Hawaii)

Source: <https://www.mapsofworld.com/answers/united-states/us-states-with-more-than-one-timezone/attachment/us-map-states-having-multiple-time-zones/>

(a3) Daylight saving time

Daylight saving time is the practice of advancing clocks usually in March or April so that evening daylight lasts an hour longer, while sunrise times are delayed by one hour. The practice is based on advancing the clocks in the spring on a specific day at 1AM (switching to 2AM, and losing one sleeping hour...) and retreating them at a specific day at 2AM in the fall (moving to 1AM, gaining one sleeping hour...) (Figure 22).

These changes are examples of discontinuities. Most people do not bother with “what happens at exactly 1AM in the spring: Is it 1AM or 2AM? And what happens at exactly 2AM in the fall: Is it 2AM or 1AM?

So to answer the question: “where were you at 1:30am on those 2 days?” Obviously in the spring there was no 1:30AM and in the fall there were two 1:30AM on the same day.



Figure 22: Daylight saving time: “Fall back” (left) and “spring forward” (right)

(a4) International Date Line

The International Date Line (IDL) is an imaginary and arbitrary (but agreed upon) line on earth’s surface somewhere in the pacific ocean, connecting the North Pole to the South Pole. When you cross it travelling west side (opposite the direction Earth rotates), the day increases by one, and if you cross it travelling east side the day decrease by one. In both cases the time of the day stays the same.

The IDL is a standard that is accepted and embraced globally. It is unavoidable (i.e., a line that must exist) if we want to keep time. It is crucial for communication, commerce, travel, etc. Why did the International Date Line come to be?

Due to the rotation of the Earth the time “to the right” of the line is designated as a day earlier than the “left side” of the line (Figure 23).

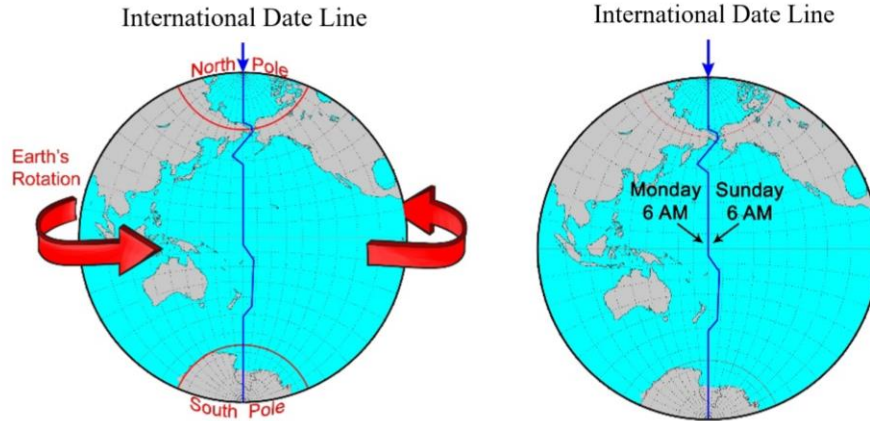


Figure 23: Discontinuity due to the International Date Line

Clearly there is a time discontinuity. To understand the need for the line, observe the following illustration: By looking at the Earth from top-down (above the North Pole) we can see the day and night regions (Figure 24). When it is 18:00 (sunset) on Sunday, it is Midnight at another point (exactly on the borderline between Sunday and Monday), 6:00 (dawn) somewhere on Monday, 12:00 (noon) on Monday at another location and 16:00 on Monday at another location. This exact location is also 16:00 on Sunday! This is where there is a discontinuity from Monday to Sunday.

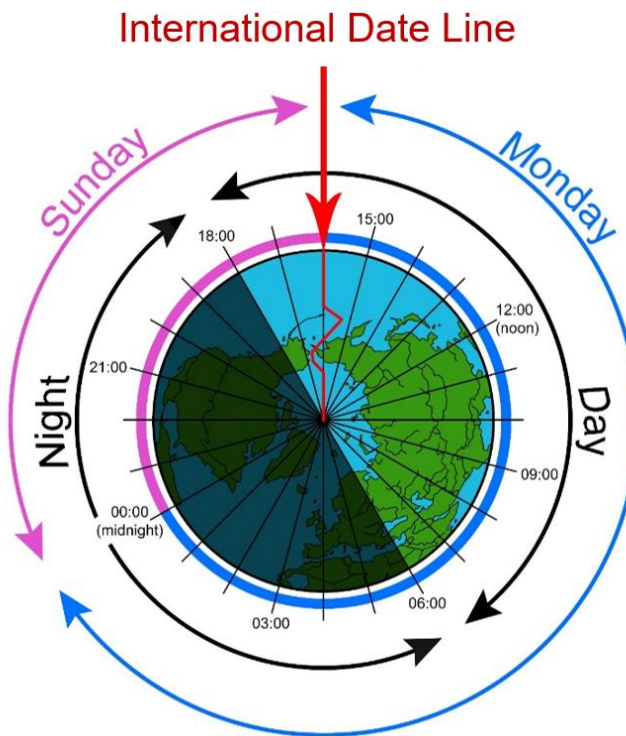


Figure 24: Illustration of global time and International Date Line

Refer to the illustration in Figure 25: if you are standing east of the IDL and meet someone who is on the west side of it you can definitely say “Let’s meet yesterday” and still be correct... another reason why it can be called an International Date Line. If you fly from Beijing to Los Angeles you can say that you “arrived before you left.”

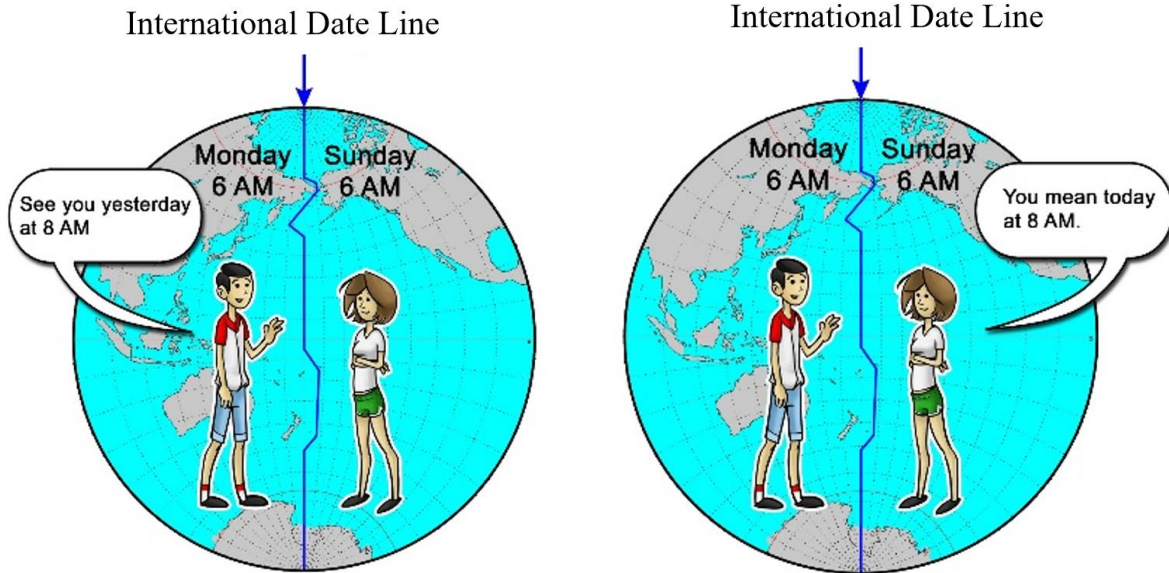


Figure 25: A strange conversation due to International Date Line

(a5) Two related stories

Story I: Jules Verne’s story (*Around the World in 80 days*)

When you travel eastward (in the direction Earth rotates), each day will be slightly shorter than 24 hours, assuming you measure your "day" as the time between two successive "noons" or midnights.

This is one of the great ideas in the 1872 Jules Verne novel *Around the World in Eighty Days*: thanks to the rotation of the Earth and Jules’ travel Eastward, each of his days were slightly shorter than 24-hour, allowing him to make the trip in the planned full original 80 days (and not 81 days as he mistakenly thought). By the way, each day was shortened on average by $(24\text{hours}) \times (60\text{minutes}/\text{hour}) / (80\text{day}) = 18\text{minutes}$

Story II: The island of Samoa skipped a day: an ingenious idea

At Midnight at the end of Thursday, December 29, 2011, Samoa celebrated hopping to the other side of the International Day Line, therefore skipping Friday, December 30, 2011 entirely. Samoa officially shifted to the west side of the International Date Line. By doing so, Samoa moved to the same time zone as Australia and New Zealand, allowing for flourishing trading. Prior to the change, it was difficult to do business because weekends did not fully overlap, leaving only four days of business days. A costless ingenious idea!

(b) Devices that operate based on using discontinuously

(b1) A/C control

Air-conditioning systems are controlled in a non-linear way. The A/C unit gets a binary signal – a discontinuity in time - just like the output of a switch.

(b2) Anti-lock braking system

Anti-lock braking system (ABS) is an anti-skid system designed to help the driver by restoring traction to the tires, and preventing the wheels from locking up. It works by applying automatic on/off signals (on/off discontinuities) to the braking system, despite continuous push by the driver on the braking pedal (Figure 26).

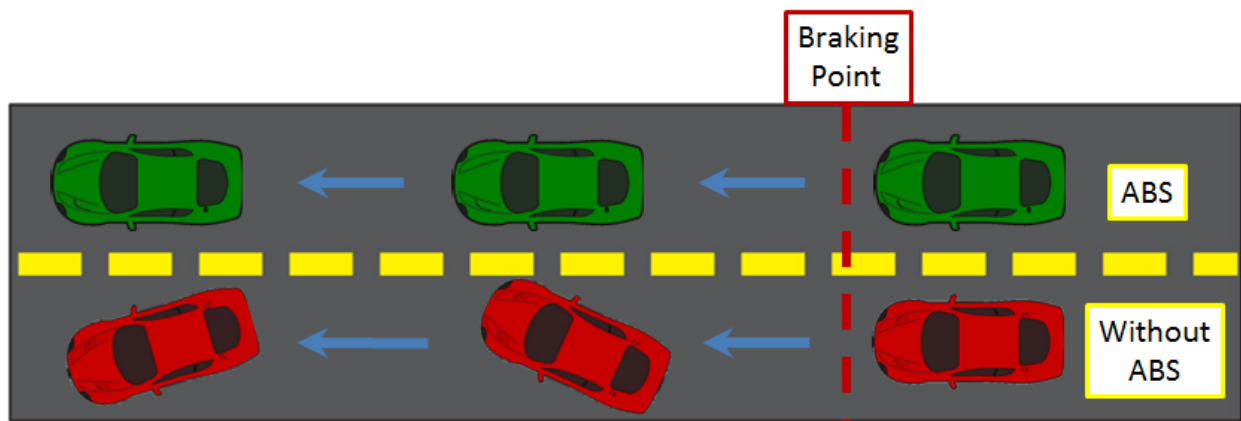


Figure 26: Cars behavior with and without anti-lock braking system (ABS)

(b3) Discontinuity in time display

Refer to Figure 27.

Is it possible to tell the exact time when looking at a clock such as the green clock below on the left? Theoretically, yes. If you had the ability to zoom in more and more, you could tell the time down to the milliseconds, microseconds, and so on. This is called an analog clock; it displays a continuous quantity, known as time. (Some analog clocks do not have fully continuous motion but instead “tick” once per second.)

Now, observe the blue clock below on the right. What time is it? It's 5:53, no questions asked. This is because the clock on the right is a digital clock. The time displayed by the clock is a discrete quantity updated at intervals of one minute. The next display will be 5:54. With this digital display, we have no way to know the actual time between 5:53 and 5:54. The digital display only shows samples of time, clearly a discontinuity in displaying the time.

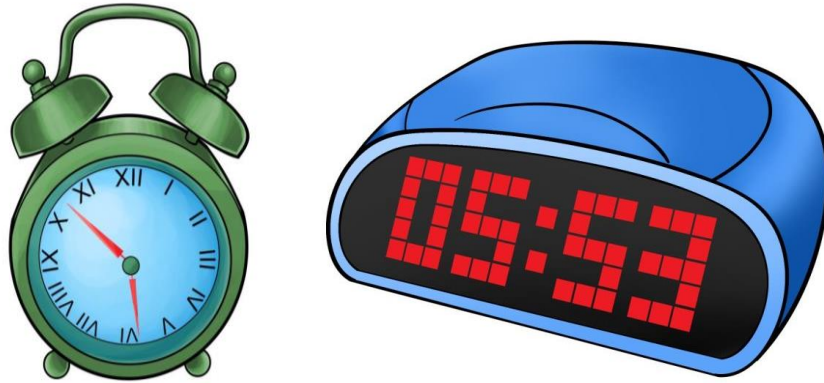


Figure 27: Analog and Digital displays of time

(b4) Stroboscope and Zoetrope

Zoetrope

Zoetropes can be thought of as mechanical movies. A zoetrope is a cylindrical device with images on it portraying consecutive scenes of a story (Figure 28). Slots are used to cover all of the other scenes. The cylinder is rotated and the scenes can be seen in rapid succession. This seems like a continuous story if we get the sampling rate high enough by rotating the cylinder faster. Here, discontinuity between images are seen as continuous signal.



Figure 28: Zoetrope

Source: <http://sciphile.org/lessons/zoetropes-phenakistoscopes-and-other-animation-toys>

Stroboscope

The following video shows how the use of a flashing light (stroboscope) forces our vision to sample at a certain rate. Discontinuous images appear as moving continuously if the sampling is fast enough. <http://www.youtube.com/watch?v=A-19SxqZ8Qs>

(b5) Animation video

The following video may seem like a continuous video but is in fact a stop-motion animation that has a sampling rate high enough so we do not notice the discontinuities between samples. <https://vimeo.com/26877221>. As seen in the next set of images in Figure 29, keeping the background constant but making slight changes to one or more objects in an image can make the changes appear continuous if they are projected fast enough. For illustration purposes, the images are shown between long intervals.



Figure 29: Sample images from an animated video

(c) Activities

(c1) Flip book

To explain Digital to Analog Conversion, we can perform an experiment using a pad of sticky notes, a pen or pencil, and coloring items. Think of a story and continue to draw the scenes on the sticky pad page per page similar to Figure 30.



Figure 30: Sticky pad story
Source: <https://vimeo.com/23673802>

Afterwards, flip through the sticky pad to watch the story. If each scene looks close enough to the previous scene, the resulting story will play smoothly, as if watching a movie. However, if consecutive images appear too different from each other, there will be noticeable jumps in between pages and the resulting story will be perceived as discontinuous.

(c2) “Exploding” rubber watermelon

The following is a set of snapshots from a YouTube movie that shows a discontinuity experiment: <https://www.youtube.com/watch?v=Q1VoK0Ga36w#t=29>

The discontinuity is visual and clear. Don’t try it at home. We refer to Figure 31 as “before”, Figure 32 as “during” and Figure 33 as “after.”

Preparation:



Figure 31: Watermelon – “before”

Adding rubber bands:



Figure 32: Watermelon – “during”

“Explosion:”



Figure 33: Watermelon – “after”

(c3) The bench wet paint example

The following sequence of images (Figure 34) can make the concept of function and its inverse very clear to students. Although there are many examples that can explain this concept (such as “negative” and “positive” images in analog “old fashioned” photographic film processing), this one is experience-based with “low-tech” comical flavor: the “function” is an unintentional act, but its inverse is an intentional act with a creative taste.

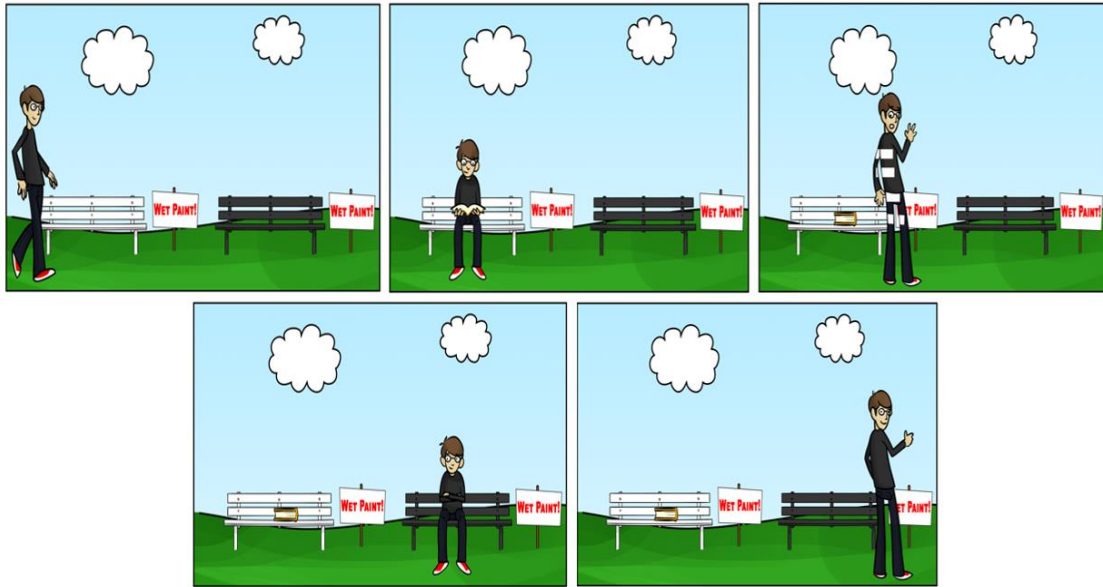


Figure 34: Inverting a “wet paint” discontinuity effect

The example is inspired by an early 20th century cartoon by Starke, an England cartoonist [19] (Figure 35).

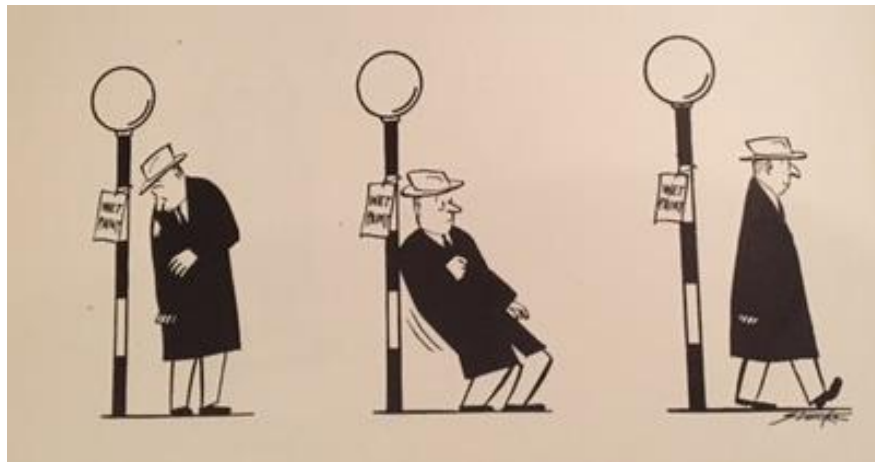


Figure 35: Wet paint example: an inspiring cartoon

(d) Experiences that illustrate potential loss of information due to discontinuity

(d1) The “Heidi” football story

On November 17, 1968, a football game between the New York Jets and the Oakland Raiders ran longer than expected. The movie *Heidi* was aired on time, and the viewing audience missed the end of the game. When the film started airing, the Jets were winning 32-29 with 65 seconds remaining. During the next few minutes (missed by the viewers), the Raiders pulled an impressive comeback to win 43-32. This game is now referred to as the “Heidi Game” and changed the way television networks scheduled showings due to negative fan reception. Here, discontinuity in airing the game caused fans to be totally disappointed.

(d2) Texting and driving



Figure 36: Texting and driving

Texting while driving is an example of loss of information due to visual discontinuity. The texting driver is clearly distracted for a few seconds by not watching the nearby environment, leading to unsafe driving. Unfortunately, many accidents have been documented as texting related. It is the same as driving blindfolded for a few seconds (Figure 36).

III. Conclusion

The examples in this paper are being shared to help instructors *to* better teach, and students to better comprehend, the concept of discontinuity, by relating it to real life examples. The examples are visual, intuitive and engaging, thereby allowing the instructor to introduce the concept in ways that relate to students’ learning styles. The idea is to provide some introductory examples prior to delving into detailed mathematical definitions of discontinuity to better adapt to students’ learning preferences. This is most relevant in math courses that are usually taught with little or no connection to daily experiences.

We emphasize that the examples in this paper are not meant to replace any existing textbook chapters. Based on prior experiences in different STEM topics we believe that these add-on examples will be useful, and we hope that they will be used by Calculus instructors.

IV. Acknowledgment

Special thanks to Juan Ramirez who worked with me on explaining concepts in Digital Signal Processing (DSP) some of which are described here, and to Pamela Noguera for the many great illustrations. Dr. N. Shneydor came up with the creative idea of book organization.

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