# Utilization of EXCEL in Solving Structural Analysis Problems 

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#### Abstract

Engineering educators are always searching for new and effective tools to speed up their computations, enhance their teaching, and elevate student learning. Currently, several different approaches in teaching structural engineering concepts are being investigated by the author. In this paper, the use of EXCEL spreadsheet software in analyzing structures is discussed employing several classical structural engineering methods. Using the developed spreadsheets in this project, the instructor can quickly generate and analyze a variety of problems with ease during the lecture to enhance student course comprehension. These spreadsheets can also be provided for the students to allow them verify the accuracy of their developed hand-solutions. Note that it is much more difficult to achieve the objectives described above using other commercial structural engineering software, since these packages are often quite expensive, more complex, and harder to work with. Many civil engineering and civil engineering technology faculty can neither afford to acquire this type of software in their institutions, nor they have time to teach the students how to use the software. The use of EXCEL spreadsheet package might be one easy solution to remedy these problems. This relatively inexpensive software is a component of the Microsoft Office suite which is already preinstalled in many computers when purchased, and is already available in many educational institutions through a site-license. Using this software many of the classical approaches associated with analyzing structurally determinate as well as indeterminate structures can be formulated with ease. The purpose of the presented paper is to illustrate some of the capabilities of this software package related to solving structural engineering problems through presenting and discussing the solution for two sample problems.


## I. Introduction

In an earlier work of the author the utilization of MATLAB software in complementing a structural engineering course was investigated. The paper based on this investigation was published in the Proceeding of the 2002 ASEE Annual Conference ${ }^{1}$, and was presented at the ASEE/SEFI/TUB International Colloquium in Berlin, Germany in October of 2002. Although this method of complementing a course offers a number of great advantages in terms of enhancing the understanding and capabilities of the students in analyzing structures, this method has two main drawbacks. Familiarizing students with the MATLAB programming principles, concepts, and tools requires additional time and effort. The instructors teaching the course may not have enough time to adequately teach the students about all the needed skills in formulating the solutions for the problems. The other main shortcoming of this approach has to do with the availability of the MATLAB software to the students. The students may not afford to purchase
this software or readily have access to it on their campuses. At Georgia Southern University, where the author is currently teaching, only limited licenses for MATLAB are available in three computer labs on campus. These labs have restricted access and are accessible to the students only during certain hours. Utilization of the Excel software circumvents the shortcomings pointed out above. Excel is an affordable software that can run on inexpensive computers and is readily accessible to the students. Another great advantage of using Excel is the relative ease with which the tools and features of this software can be utilized in solving a variety of problems. There are a number of publications that describe the general utility of Excel. One particular publication which focuses on the use of this software in solving engineering related problems is listed in the bibliography of the paper ${ }^{2}$.

The author of the submitted paper has developed the instructional material needed to teach the students about all the important features and tools of Excel in two 2-hour lab periods. The developed material has been successfully utilized in the last few semesters in teaching a course entitled as the Computing for Engineers course (ENGR 1631) at the Engineering Studies Program at Georgia Southern University. The instructions provided in these labs are deemed sufficient in enabling the students to use the tools in Excel to analyze a variety of structures.

In this paper the procedure for utilization of Excel in solving structural analysis problems are discussed through presenting the sample solutions for a statically determinate, as well as, a statically indeterminate beam. This method of analysis can be used as an effective tool in enhancing and complementing the traditional approaches for teaching structural analysis courses. The Excel Workbook files similar to the ones presented in the paper can be utilized in enhancing the teaching effectiveness of the instructors and elevating the students learning in the following fashion. The instructor can utilize these workbooks during lectures to demonstrate and discuss the behavior of different beams subjected to various loading conditions. This can be accomplished by simply changing the beam and load parameters placed in appropriate cells. The developed Excel workbooks can also be placed on a web server to allow students access to these files. The students can further enhance their understanding of the course topics by exploring these files themselves at a time and pace suitable for them. The students can also modify these files to analyze other classes of problems, and use the files to perform a check on the validity and accurateness of their developed hand solutions.

## II. Analysis of a Statically Determinate Beam

In this section of the paper the procedure for determining the shear and moment diagrams and plotting the variation of the slope and deflection of the beam using Excel is discussed. This procedure is illustrated through presenting the solution for a sample beam and loading condition shown in Figure 1(a).


Figure 1. Free Body Diagrams for the Beam
Writing the force and moment equilibrium equations for the free body diagrams of the two sections of the beam shown at the bottom of Figure 1, the following expressions for the shear force $V$ and bending moment $M$ can be established.
$0 \leq x \leq L$
$L \leq x \leq L+a$
$V_{1}=\frac{-w}{2 L}\left(a^{2}-L^{2}\right)-w x$
$V_{2}=w(L+a-x)$
$M_{1}=\frac{-w x^{2}}{2}-\frac{w}{2 L}\left(a^{2}-L^{2}\right) x$

$$
\begin{equation*}
M_{2}=\frac{-w(L+a-x)^{2}}{2} \tag{2}
\end{equation*}
$$

Upon substituting for the moments $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ in the differential equations for the two segments of the beam shown below:
$0 \leq x \leq L$

$$
L \leq x \leq L+a
$$

$E I v_{1}^{\prime \prime}=M_{1}$

$$
\begin{equation*}
E I v_{2}^{\prime \prime}=M_{2} \tag{5}
\end{equation*}
$$

the following two specific equations are obtained for the beam under consideration. In these expressions E, and I are respectively the modulus of elasticity and moment of inertia of the beam. All other parameters in the equations are as defined in Figure 1.

$$
\begin{array}{ll}
0 \leq x \leq L & L \leq x \leq L+a \\
E I v_{1}^{\prime \prime}=\frac{-w x^{2}}{2}-\frac{w}{2 L}\left(a^{2}-L^{2}\right) x & \text { (7) }
\end{array} \quad E I v_{2}^{\prime \prime}=\frac{-w(L+a-x)^{2}}{2}
$$

Upon employing the method of successive integration and enforcing the boundary conditions:
$v_{1}(x=0)=0$

$$
\begin{equation*}
v_{2}(x=L)=0 \tag{9}
\end{equation*}
$$

and the continuity equations:
$v_{1}(x=L)=v_{2}(x=L)$

$$
\begin{equation*}
v_{1}^{\prime}(x=L)=v_{2}^{\prime}(x=L) \tag{11}
\end{equation*}
$$

the following equations are obtained for the slope $v^{\prime}$ and deflection $v$ for the two segments of the beam. Note that the boundary conditions and continuity equations listed above respectively indicate that there is no deflection at the pin and roller supports, and that at the roller support there should only be one single value for the slope and one single value for the deflection.

$$
\begin{align*}
& 0 \leq x \leq L \\
& v_{1}^{\prime}=\frac{-w}{24 E I L}\left[4 L x^{3}+6\left(a^{2}-L^{2}\right) x^{2}-L^{2}\left(2 a^{2}-L^{2}\right)\right]  \tag{13}\\
& v_{1}=\frac{-w x}{24 E I L}\left[L x^{3}+2\left(a^{2}-L^{2}\right) x^{2}-L^{2}\left(2 a^{2}-L^{2}\right)\right] \tag{14}
\end{align*}
$$

$L \leq x \leq L+a$

$$
\begin{align*}
& v_{2}^{\prime}=\frac{-w}{24 E I}\left[-4(L+a-x)^{3}-\left(L^{3}-4 L a^{2}-4 a^{3}\right)\right]  \tag{15}\\
& v_{2}=\frac{-w}{24 E I}\left[(L+a-x)^{4}-\left(L^{3}-4 L a^{2}-4 a^{3}\right) x-\left(a^{4}-L^{4}+4 L^{2} a^{2}+4 L a^{3}\right)\right] \tag{16}
\end{align*}
$$

The Excel workbook developed for solving this problem is presented in Figure 2 for the specific case when $w=0.2 \mathrm{kip} / \mathrm{ft}, \mathrm{E}=29000 \mathrm{ksi}, \mathrm{I}=100 \mathrm{in}^{4}, \mathrm{~L}=20 \mathrm{ft}$, and $\mathrm{a}=10 \mathrm{ft}$. In this spreadsheet to compute the values of shear, moment, slope, and deflection along the length of the beam in columns B through E, the equations 1-4 and equations 13-16 are used in the following fashion. Using these equations, four Excel formulas are first developed in cells B4 through E4 that are capable of computing the correct values of shear, moment, slope and deflection for any particular value of $x$. These formulas are then copied down each of the columns B through E to calculate the values corresponding to all the values of $x$ listed in column A. Note that the developed formulas are capable of detecting for each selected value of $x$ whether to use the set of equations for the first segment or the second segment of the beam. One sample Excel formula developed for computing the deflection in cell E4 corresponding to the case when $\mathrm{x}=0$ is shown inside the formula-bar at the top of the spreadsheet presented in Figure 2. It should be stated that the parameters used in this formula are set up to serve as "named cells". These "named cells" are assigned to point to cells or range of cells that contain the actual values of the parameters used in this problem. By creating and employing this special type of cells, descriptive names rather than cell addresses are used to obtain formulas that are more pleasing to the eye and easier to manage. The procedure for creating "named cells" is discussed in a variety of publications that describe the features of Excel. One such publication is listed in the bibliography of this paper ${ }^{2}$.

Once the spreadsheet is completed using the procedure described above, various powerful Excel plotting tools can be employed to plot the distributions of shear, moment, slope, and deflection along the length of the beam as shown in Figure 2. To further aid in the readability and documentation of the problem, various other Excel formatting and drawing tools can be utilized to sketch the beam and loading conditions and to place gridlines and shading on the spreadsheet. The utility of these tools has also been illustrated in Figure 2.

The method described above for the beam and loading condition depicted in Figure 1 can be utilized to analysis beams subjected to other loading conditions. The results from each of these loadings can also then be added using the superposition method to yield the results for other combined loading cases. In the presented paper, the results for a beam subjected to a concentrated end load, and a beam acted upon by a combined distributed and concentrated end loads are presented in Figures 3 and 4. In computing the results for the concentrated end load in Figure 3 the following equations were utilized. These equations can be derived using the same type of analysis performed previously.

$$
\begin{array}{lll}
0 \leq x \leq L & L \leq x \leq L+a \\
V_{1}=\frac{-P a}{L} & \text { (17) } & V_{2}=P \\
M_{1}=\frac{-P a x}{L} & \text { (18) } & M_{2}=-P(L+\succ \tag{22}
\end{array}
$$

$$
\begin{align*}
& v_{1}^{\prime}=\frac{-P}{6 E I L}\left(3 a x^{2}-a L^{2}\right)(19) \quad v_{2}^{\prime}=\frac{-P}{6 E I}\left[-3(L+a-x)^{2}+a(3 a+2 L)\right]  \tag{23}\\
& v_{1}=\frac{-P a x}{6 E I L}\left(x^{2}-L^{2}\right) \quad \text { (20) } \quad v_{2}=\frac{-P}{6 E I}\left[(L+a-x)^{3}+a(3 a+2 L) x+a\left(-a^{2}-3 a L-2 L^{2}\right)\right] \tag{24}
\end{align*}
$$

One sample Excel formula for computing the value of deflection corresponding to the case when $x=0$ is shown inside the formula-bar at the top of each of the spreadsheets in Figures 3 and 4. Examining Figures 2-4, it can be noted that the results for different loadings are placed on successive sheets of the same Excel Workbook. The results in these sheets can be accessed by clicking on the tabs created at the bottom of the presented workbook. These tabs are appropriately labeled as "Distributed Load", "Concentrated Load", and "Combined Load" to indicate the type of results that these sheets contain. The formula in Figure 4 demonstrates how the results from the "Distributed Load" sheet and "Concentrated Load" sheet can be added to yield the results for the combined loading case.

Additional provisions can easily be made in the developed spreadsheets to also compute the stresses acting on the beam to make sure that these values are not exceeding the allowed limits. The developed workbook can serve as a great tool for the designing beams.

## III. Analysis of a Statically Indeterminate Beam

The development of an Excel Workbook file in regards to analyzing a statically indeterminate beam is described in this section of the paper through providing the solution for a sample problem. This problem is taken from the textbook ${ }^{4}$ of a Structural Analysis course taught at Georgia Southern University. The beam used in this problem is depicted at the top of the spreadsheet in Figure 5. The procedure implemented is based on the "Moment-Distribution Method", a widely used and well-established method developed by Hardy Cross ${ }^{3}$ in 1930. Details in regards to the development and utilization of this method can be found in various structural analysis text ${ }^{4-7}$. A brief description of this method is provided below to enable the reader to understand the basic concepts and the steps involved in developing the Excel solution for the problem.

The application of the moment-distribution method requires that the stiffness factor (K) and the distribution factor (DF) and the fixed-end-moments (FEM) of the beam spans to be computed first. For the sample beam under consideration, these factors and moments are computed next for the specific beam and load data presented at the bottom of the spreadsheet in Figure 5.


Figure 2. Screen-shot of an Excel Workbook Developed for a Beam Subjected to a Uniformly Distributed Load

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Figure 3. Screen-shot of an Excel Workbook Developed for a Beam Subjected to a Concentrated End Load

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| 7 | 36 | 650 | 34200 | $-5.930 \mathrm{E}-04$ | $-2.684 \mathrm{E}-02$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 8 | 48 | 450 | 40800 | $-4.370 \mathrm{E}-04$ | $-3.305 \mathrm{E}-02$ |
| 9 | 60 | 250 | 45000 | $-2.586 \mathrm{E}-04$ | $-3.724 \mathrm{E}-02$ |
|  | 72 | 50 |  | 460 |  |



| 12 | 96 | -350 | 43200 | $3.112 \mathrm{E}-04$ | -3.623E-02 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 108 | -550 | 37800 | 4.796E-04 | -3.146E-02 |
| 14 | 120 | -750 | 30000 | $6.207 \mathrm{E}-04$ | -2.483E-02 |
| 15 | 132 | -950 | 19800 | $7.246 \mathrm{E}-04$ | -1.671E-02 |
| 16 | 144 | -1150 | 7200 | $7.812 \mathrm{E}-04$ | -7.627E-03 |
| 17 | 156 | -1350 | -7800 | $7.808 \mathrm{E}-04$ | 1.807E-03 |
| 18 | 168 | -1550 | -25200 | $7.134 \mathrm{E}-04$ | $1.084 \mathrm{E}-02$ |
| 19 | 180 | -1750 | -45000 | $5.690 \mathrm{E}-04$ | 1.862E-02 |
| 20 | 192 | -1950 | -67200 | 3.377E-04 | $2.415 \mathrm{E}-02$ |
| 21 | 204 | -2150 | -91800 | $9.517 \mathrm{E}-06$ | $2.634 \mathrm{E}-02$ |
| 22 | 216 | -2350 | -118800 | -4.254E-04 | 2.395E-02 |
| 23 | 228 | -2550 | -148200 | -9.770E-04 | $1.566 \mathrm{E}-02$ |
| 24 | 240 | -2750 | -180000 | -1.655E-03 | $0.000 \mathrm{E}+00$ |
| 25 | 252 | 2300 | -151200 | -2.340E-03 | -2.409E-02 |
| 26 | 264 | 2100 | -124800 | -2.910E-03 | -5.569E-02 |
| 27 | 276 | 1900 | -100800 | -3.376E-03 | -9.351E-02 |
| 28 | 288 | 1700 | -79200 | -3.747E-03 | -1.363E-01 |
| 29 | 300 | 1500 | -60000 | -4.034E-03 | -1.831E-01 |
| 30 | 312 | 1300 | -43200 | -4.247E-03 | -2.329E-01 |
| 31 | 324 | 1100 | -28800 | -4.395E-03 | -2.848E-01 |
| 32 | 336 | 900 | -16800 | -4.489E-03 | -3.381E-01 |
| 33 | 348 | 700 | -7200 | -4.538E-03 | -3.923E-01 |
| 34 | 360 | 500 | 0 | -4.552E-03 | -4.469E-01 |
| 36 | Data: |  |  |  |  |
| 38 | $\begin{aligned} & \mathrm{W}(\mathrm{lb} / \mathrm{in})= \\ & \mathrm{E}(\mathrm{psi})= \\ & \mathrm{I}\left(\mathrm{in}^{\mathrm{n}} 4\right)= \end{aligned}$ |  | 16.667 | $L$ (in) $=240$ |  |
| 39 |  |  | 2.90E+07 | $\mathrm{a}(\mathrm{in})=120$ |  |
| 40 |  |  | 100 | $\mathrm{P}(\mathrm{l})=500$ |  |


$\mid 1 \rightarrow M$ Distributed Load $/$ Concentrated Load $\lambda$ Combined Load / | $\mid$


Ready

Figure 4. Screen-shot of an Excel Workbook Developed for a Beam Subjected to a Combined Uniformly Distributed Load and a Concentrated End Load

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$$
\begin{array}{ll}
K_{A B}=\frac{4 E I}{L_{A B}}=\frac{4 E I}{20}=0.2 E I & D F_{A B}=0 \\
K_{B C}=\frac{4 E I}{L_{B C}}=\frac{4 E I}{20}=0.2 E I & (25) \\
D F_{B A}=\frac{K_{B C}}{K_{A B}+K_{B C}}=0.5 \\
F E M_{B C}=\frac{-w L_{B C}^{2}}{12}=\frac{-(0.8)(20)^{2}}{12}=-26.667 \mathrm{k} . f t \\
F E M_{C B}=\frac{w L_{\mathbf{B C}}^{2}}{12}=\frac{(0.8)(20)^{2}}{12}=+26.667 \mathrm{k} . f t & \\
F E M_{C D}=-P L_{C D}=-(0.5)(15)=-7.5 \mathrm{k} . f t
\end{array}
$$

In the above equations $\mathrm{E}, \mathrm{I}, \mathrm{w}$, and P are respectively the modulus of elasticity, moment of inertia, distributed load, and concentrated end load. Also note that $\mathrm{L}_{\mathrm{AB}}, \mathrm{L}_{\mathrm{BC}}$, and $\mathrm{L}_{\mathrm{CD}}$ refer to the length of spans $\mathrm{AB}, \mathrm{BC}$, and CD .

The procedure for determining the moment at the end of each span of the beam using the method of moment-distribution can be summarized in the following four steps ${ }^{4}$. Note that these steps are followed in the Excel spreadsheet provided in Figure 5 to obtained the moments at the joints A, B , and C of the beam.

Moment-Distribution Steps:

1. The ends of the beam spans are first assumed clamped and the fixed-end moments are used to determine counterbalancing moment needed to keep the joint in equilibrium (e.g., the counterbalancing moment for joint B is $26.667 \mathrm{k} . \mathrm{ft}$ ).
2. The ends of the beam spans are then unclamped to allow them to rotate. The counterbalancing moment computed in step 1 is now distributed to the spans of the beam connecting to the same joint using the distributions factors (e.g., distributed moment for span BA is $26.667 \mathrm{x} 0.5=13.333 \mathrm{k} . \mathrm{ft}$. This value is shown in cell D9).
3. The distributed moments computed at each joint of the beam is step 2 is then carried over to the opposite end of each span by multiplying these moments by the corresponding carry-over factors (e.g., carry-over-moment to joint A is $13.333 \times 0.5=6.667 \mathrm{k} . \mathrm{ft}$. This value is shown in cell C10).
4. The process of clamping and unclamping of the joints and distributing and carrying-over of the moments as described in the previous three steps should be repeated over several cycles until sufficiently small values for the distributed moments are yielded. At this point the procedure can be stopped and the tabulated values of fixed-end moment (FEM), distributed moment (DM), and carry-over moment (COM) can be added to yield the net values of the moments at each joint.

When computing the distributed moments (DM) and carry-over-moments (COM) in the spreadsheet in Figure 5, the Excel formulas are only developed once for rows 9 and 10 and then simply copied down each column to produce the moment values for the rest of the remaining cycles. Note that when these formulas are copied, the cell references used in the formulas will be automatically adjusted to refer to the appropriate cell locations to produce the correct moment values. One such formula for computing the distributed moment for member BA (in cell D9) is shown in the formula-bar at the top of the screen-shot of the spreadsheet. Summing the values of the FEM, DM, and COM in each of the columns C through G, the final values of moment at the end of each span are computed. These values are circled on the spreadsheet. Once these moments are obtained, free-body-diagrams can be drawn for each span and the shear and moment diagrams for the beam can easily be established. These diagrams have not been included on the workbook.

The spreadsheet developed in Figure 5 can also easily be modified to analyze the beam subjected to other loading conditions. This can be accomplished by modifying the formulas placed in row 8 to compute the fixed-end-moments (FEMs). The spreadsheets similar to the one presented can serve as a great tool for understanding and predicting the behavior of beams subjected to various loading conditions.

## IV. Summary and Conclusion

In the submitted paper, a procedure for complementing a structural analysis course utilizing Excel was discussed. This method of analysis can be utilized as an added measure to further elevate the students' course comprehension and enhance the instructor's effectiveness. Using the special features and powerful tools of this affordable and easy-to-learn and easy-to-use software, workbooks can be developed based on many of the classical approaches available for analyzing structures. In the presented paper, a solution for a statically determinate, as well as, a statically indeterminate beam were presented to clearly establish the power of this method of analysis. The instructor can effectively utilize the developed workbooks during the lectures to quickly generate different scenarios for in-class discussions. The students can also use these files themselves to further enhance their own understanding of the course topics and learn more about the behavior of structures subjected to various beam, support, and loading conditions. As mentioned previously, the student can also utilize this workbook to perform a check on the validity of their developed hand-solutions.


Figure 5. Screen-shot of an Excel Workbook Developed for a Statically Indeterminate Beam

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## Page 8.1267.12

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