UTILIZING CONSTRAINT GRAPHS IN HIGH SCHOOL PHYSICS

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ABSTRACT

Memorizing equations, recalling them, and then plugging numbers into those equations to obtain answers for test questions. This characterizes in part how novices approach problem solving in a content area such as physics. However, a novice's preoccupation with the mathematics of physics leaves little attention or consideration directed toward the underlying laws and principles. In this paper, we present both an instructional and problem-solving approach in the realm of physics that employs constraint graphs – i.e., a representational convention in which a multitude of variables are combined into a network of mathematical relationships. Specifically, constraint graphs serve to organize and structure the mathematics of physics in such a way that more easily renders tasks of problem-solving and learning. Our hope is that high school students' use of constraint graphs eases the burden associated with mathematics and provides an opportunity to understand better the laws and principles of both physics and engineering.

Keywords: physics, mathematics, constraint graphs, high school outreach, engineering

1.   OUR FRAME OF REFERENCE

Solving physics problems is a difficult, intellectual endeavor for novice students at the high school level. For many of them, solving such problems is merely a process of memorizing equations, evoking the equations, and then plugging numbers into those equations to obtain "the answer." Moreover, their problem-solving strategy is based on the principle of trial-and-error, epitomized by confusion as to where to begin, countless false starts, cursory manipulations of mathematical

expressions, and becoming "lost." However, this strategy proves to be highly inefficient and ineffective for numerous reasons. The number of unique equations and redundant variations that students memorize from a textbook is voluminous; the way in which students recall equations germane to the problem at hand is largely ad hoc and serendipitous; the methods by which students manipulate and solve a system of equations is haphazard; and the compartmentalized nature of how students think about problems obfuscates important linkages between relevant subject matter. Overall, the novice's preoccupation with the mathematics of physics leaves little attention or consideration directed toward the underlying laws and principles.

The literature from the burgeoning area of physics education research is replete with examples of similar observations. Physicists and cognitive scientists alike strive to understand how students learn physical principles and what can be done to improve the efficacy of physics instruction[22, 24].

Several researchers recount their experiences with alternative approaches to physics instruction. Reif and coauthors[25] describe an investigation in which students were taught how to learn relations in physics, in addition to general problem-solving skills; van Weeren, et al.,[35] explain their method of instruction, which involves the use of a compiled list of key relations and a systematic approach to problem-solving; Leonard, et al.,[15] instruct students on a qualitative approach to solving physics problems, implemented through written strategies and concept analysis; and an analysis of survey data by Hake[7] strongly suggests the effectiveness of instructional methods that interactively engage students in physics. Other researchers relate their use of computers to augment physics instruction and experimentation[3, 26, 32].

Through various surveys and interviews, researchers attempt to understand what students think about physics in general and how they go about learning it. It is suggested that the students' epistemological beliefs and expectations about physics affect their behavior, understanding, and subsequent class performance[9, 21, 23]. Consequently, Hammer[10, 11] recommends that teachers consider students' beliefs during instruction, challenge their misconceptions, and instill the appropriate conceptual knowledge. Other researchers endeavor to explain the origins of students' preconceived beliefs about the physical world. DiSessa[4] theorizes that naïve students have a weakly organized framework of intuitive knowledge about physics composed of phenomenological primitives (p-prims) abstracted from everyday experiences. Sherin[29] proposes a similar framework of primitives that comprise an intuitive understanding of the mathematical expressions representing physical principles.

Using protocol analysis, researchers characterize the differences in how experts solve physics problems as compared to novices. It is reported that other than the level of physical intuition and amount of practice acquired solving such problems, experts and novices pursue entirely different solution strategies. Experts "work forward" from the known quantities to the unknown value, while novices "work backward" from the unknown solution to the given values by means of intermediate sub-goals[30, 14]. These different solution strategies have been validated in part by means of computer simulation[13]. Chi and coauthors[2] attribute differences between novice and expert students to how they categorize physics problems into various "types," with which contextual knowledge and representations are associated.

But what is missing from the literature is some indication of how we can assist high school students lacking a strong background in mathematics cope with the systems of equations encountered when they solve physics problems. In terms of a metaphor, our students are too overwhelmed by individual trees (i.e., equations) such that they are unable to see the forest (i.e., physical principles). This behooves us to ask the following question, *how might the mathematics of physics be organized and structured in such a way that more easily renders students' tasks of problem-solving and learning?*

In this paper, we answer the preceding question in the context of our experiences interacting with students in high school physics classes as part of the NSF GK-12-supported Student and Teacher Enhancement Partnership (STEP) program at the Georgia Institute of Technology. Specifically, we introduced students to the representational formalism known as a constraint graph, described in Section 2. Constraint graphs are a type of pictorial information model, particularly amenable to application in a mathematically intensive domain like physics as demonstrated in Section 3. In Section 4, we relate how students utilized constraint graphs to model physics problems and structure their problem-solving efforts. Summary remarks and avenues for future research are given in Section 5.

2.   OUR FOUNDATION: CONSTRAINT GRAPHS

2.1   Historical Overview

Following the advent of computer technology, mathematicians turned their attention toward structuring the solution of very large systems of equations. Due to the limitations of early computing, it was only possible to calculate automatically the values for system variables by means of solution procedures that were not merely effective, but also highly efficient. Researchers like Parter[19], Harary[12], and Steward[33] developed partitioning techniques and alternative representations, which rendered such mathematics tractable. To determine if a very large system of equations was well posed, Friedman and Leondes[5] developed a formal mathematical basis for what they coined *constraint theory*. Work in this area enabled computers to calculate solutions to problems involving *n* equations in *n* unknowns, making possible the computer-based simulation and analysis of complex systems from, e.g., biology, economics, and electronics.

The computer science and artificial intelligence communities recognized the ramifications of constraint theory and soon adopted research thrusts for applications in areas as diverse as planning, decision-making, and vision. Among the first computer programs to employ rigorously the notion of constraints was SketchPad[34], an interactive computer graphics system in which geometric entities were modeled with constraints. ThingLab[1], a computer-based simulation environment, utilized constraints to model the interactions of assorted objects. Steele and Sussman[31] devised a computer language for modeling and computing networks of constraints that represented physical systems such as electrical circuits. But perhaps the most relevant applications arise from the field of engineering design, in which constraints are used to model the various attributes of technical systems[6, 28, 27, 16].

Today a comprehensive theory of constraints includes software systems for building constraint

networks, techniques for managing constraints, algorithms for solving constraints, and methods for ensuring the consistency of constraints. Details about such topics are rooted in computer science and can be found in, e.g., [17, 18, 36]. In this paper we limit our focus to a single aspect of constraint theory; specifically, we consider in the next section the pictorial representation of mathematical relations in the form of constraint graphs.

2.2   Anatomy of a Constraint and its Graph

Constraints have been called functions, requirements, relations, and rules in the literature. Despite the abundance of terms, a constraint is simply an explicit relationship among variables typically stated in the form of a mathematical expression. For example, the constraint represented by the equation $a = b \times c$ relates the variables $\{a, b, c\}$ utilizing the operators $\{=, \times\}$, thereby specifying how allowable values are obtained. Of course, this anatomical view of constraints is not limited to the standard set of arithmetic operators, but may also include operators of types relational, Boolean, integro-differential, exponential, etc. This allows for a diversity of mathematical expression, including constraints that are of nonlinear, discontinuous, partial differential, logical, and inequality forms.

Constraint theory incorporates a unique scheme for representing such expressions. A constraint graph denoting the equation $a = b \times c$ is illustrated in Figure 1. In graph theoretic terms, the variables $\{a, b, c\}$ are each represented by a vertex (known as a "knot") while the relationship itself is represented by another type of vertex (known as a "node"). Knots are connected to a node by lines (known as "edges" or "arcs") if and only if the corresponding variables are involved in the respective constraint. Note that in the graphical representation of a constraint, the number and names of the variables are visually highlighted while the mathematical expression is de-emphasized. In this manner, the valuation of the variables achieves predominance and the constraint is merely a rule for attaining a missing value.
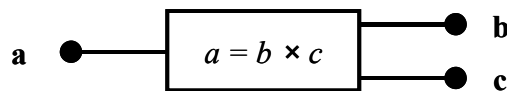

Figure 1. Example of a constraint graph

Now consider the following system of equations: $\{a = b \times c, d = a + c\}$. Following the representational convention outlined previously, constraint graphs are generated as shown in Figure 2. Since the variables $\{a, c\}$ are members of both constraints, lines are drawn connecting the corresponding knots to each node. In this scheme explicit linkages are established among the graphs, thereby allowing one a better feel for topological properties – i.e., how a system of constraints is connected and what pathways the values of variables might traverse in its solution. Such a representation of any number of linked constraint graphs is termed a constraint network.
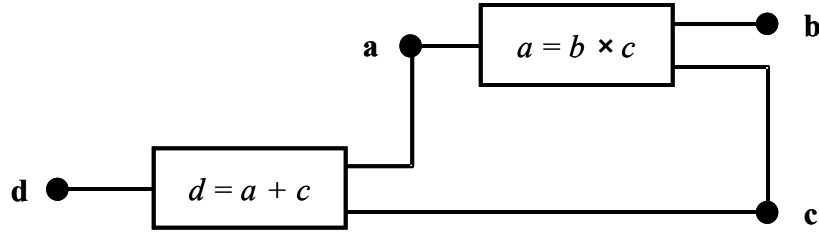
Figure 2. Example of a constraint network

Adopting the technique employed by Peak and coauthors[20], a number of constraints can be bundled together into what is termed a "subsystem view." This is illustrated in Figure 3, featuring the previous system of equations and instantiated values for the variables. Here, the semantic of the representation is akin to a "black box"; the constraints are masked and all that is visible are the variables and their valuations. Although this view is particularly useful when the network of constraints is well known and variable valuations are of prime importance, we find little use for it in the context of this paper. Instead, we focus on applying the representational formalisms for constraint graphs and networks to high school physics problems in the next section.
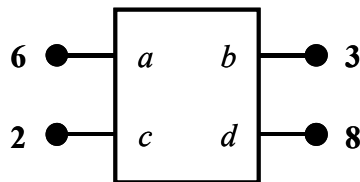
Figure 3. Instantiated subsystem view of the previous constraint network

3. APPLYING CONSTRAINT GRAPHS TO CONCEPTS IN HIGH SCHOOL PHYSICS

It is our belief that constraint graphs are ideally suited for application within a high school physics class. In this section, we demonstrate how constraint networks representing introductory physics concepts can be built. What follows is an example of how constraints and their graphs might be introduced and assembled into a constraint network modeling an inclined plane concept. In this discussion, we presuppose a working knowledge of the concepts involved (cf. [8]). In Section 3.2, brief mention is made regarding the topic of making constraint graphs computable.
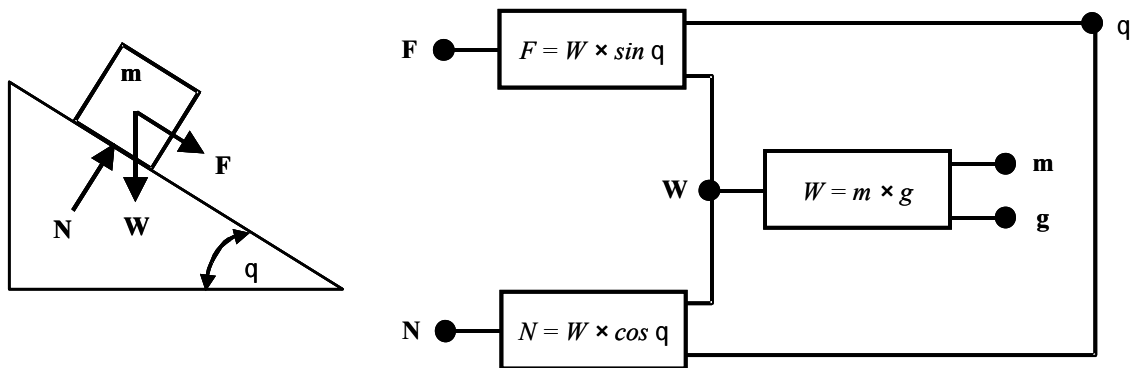
3.1 Inclined Plane Problems

Physics problems involving a mass sliding down an inclined plane are a common fixture of classical mechanics encountered early in the course. Solving such problems requires the application of elementary principles from geometry, trigonometry, and algebra, not to mention Newtonian mechanics.

Consider a block of mass m resting on a horizontal plane. The weight W of the block is defined by the following constraint: $W = m \times g$, where g is the local acceleration of gravity. A free body diagram of this situation is depicted in Figure 4, along with the associated constraint graph.
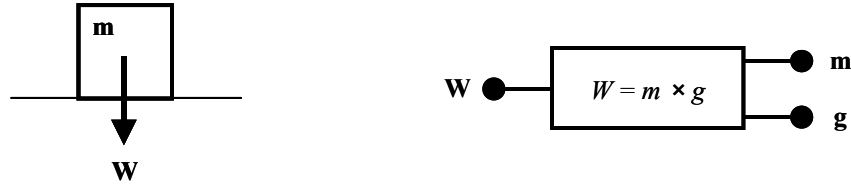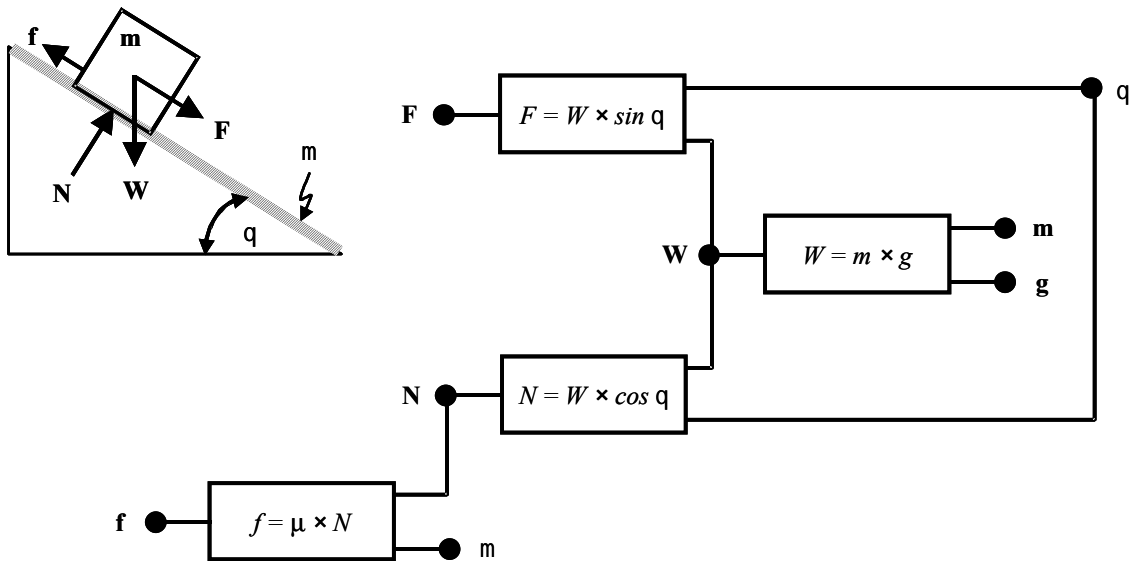
Figure 4. Free body diagram and constraint graph representing weight

Now imagine that the plane upon which the block rests is inclined by angle q. Choosing an appropriate coordinate system, the resultant vector *W* representing the weight of the block can be resolved into two constituent vectors, namely, (1) a vector parallel to the inclined plane (representing the force *F* directed down the ramp), and (2) a vector perpendicular to the inclined plane (representing the normal force *N* exerted on the block by the plane). These vectors are illustrated in the free body diagram of Figure 5. Employing concepts from geometry and trigonometry, the relevant constraints are as follows: $\{F = W \times \sin q, N = W \times \cos q\}$. The corresponding graphs are also depicted in Figure 5, connected into a constraint network.
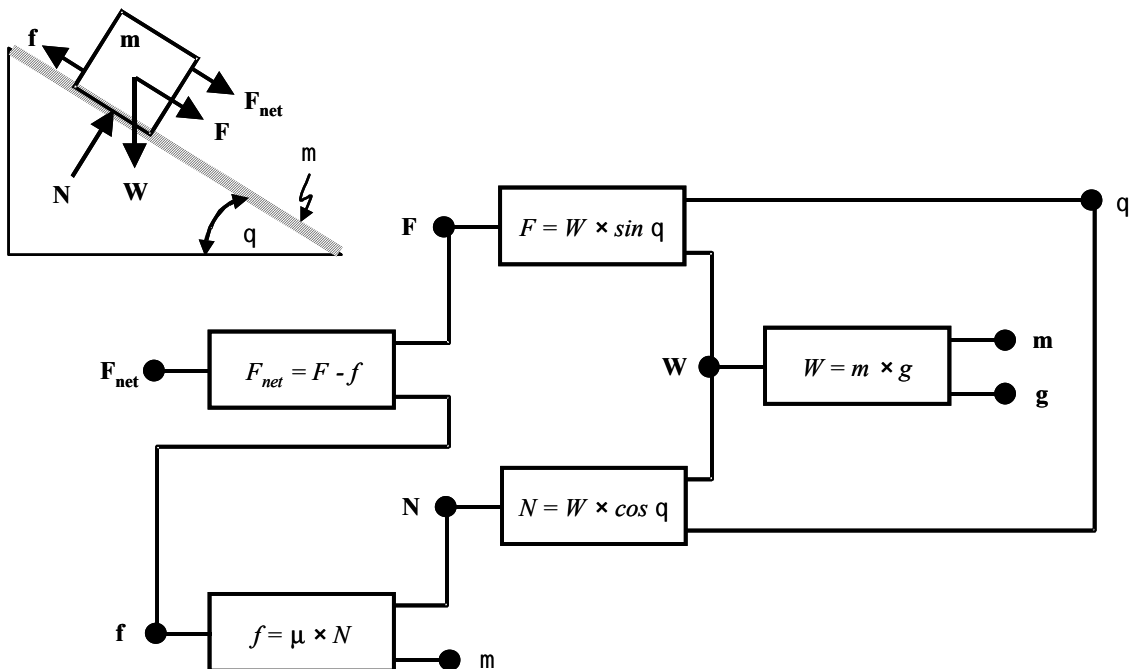

Figure 5. Free body diagram and constraint network for an inclined plane

Previously, it was assumed that the inclined plane was frictionless; but, consider the situation in which the surface of the plane is characterized by a coefficient of static friction m. Consequently, a frictional force *f* is defined by the constraint $f = m \times N$. This new frictional force is shown in both the free body diagram and constraint network of Figure 6.

Figure 6. Free body diagram and constraint network with addition of friction

A final task in this analysis of forces acting upon the block-at-rest on an inclined plane involves computing the net force directed down the ramp, F_{net}. This net force is the vector difference between the force F and the frictional force f, the magnitude of which is defined by the constraint $F_{net} = F - f$. The previous free body diagram and constraint network is augmented with the net force as shown in Figure 7.

Figure 7. Free body diagram and constraint network with addition of net force

3.2   A Note on Making Constraint Graphs Computable
As discussed in Section 2.1, a theory of constraints emerged for the purpose of enabling

computers to solve a very large system of equations. Although the graphical representation of constraint networks serves as an extremely useful information model, its sole consideration in isolation from its computational counterpart is like riding in a carriage without an attached horse. For that reason, brief mention is made on implementing automatically computable constraint graphs; however, it should be realized that anything more than a passing glance falls outside the scope of this paper.

Historically, a significant challenge of programming a computer to solve constraints involves the multidirectional aspect of a mathematical relation given the imperative, procedural nature of traditional computing languages. As such, unless programmers are utilizing a declarative language (e.g., Prolog) then they must provide explicit instructions regarding the solution of a constraint for any permutation of its variable valuations. Here, we demonstrate an approach to implementing an automatically computable constraint graph by embedding JavaScript 1.5 code in a HyperText Markup Language (HTML) encoded webpage. We limit our discussion to aspects of the JavaScript code alone.

Consider the scalar form of the constraint representing Newton's second law of motion, in particular, $F = m \times a$, where F is the force resulting from mass m undergoing an acceleration a. Given valuations for any two of the variables, then the value for the third variable may be computed by means of algebraically manipulating the governing equation. For instance, given values for m and a, then F is computed by $F = m \times a$; but when given values for a and F, then m is computed by $m = F / a$; similarly, when given values for m and F, then a is computed by $a = F / m$.

Allowing users to input any permutation of numerical values (of floating-point type) for two of these variables in text fields (named "force," "mass," and "accel") associated with a HTML-encoded form (named "cgcalc"), a series of conditional tests can be utilized to determine which variable is not valued. This is accomplished with the three successive if-statements shown in Figure 8, in which each variable is tested for a value that is not a number ("isNaN"). In the event that the test returns a true value, then the appropriate form of the constraint is solved and the resulting valuation of the variable is output to its respective text field on the HTML form. Note that additional code should be added in order to perform error-trapping and consistency checking. A more complete version of this function and its associated HTML-encoded form is available (URL: `http://srl.marc.gatech.edu/~scottc/step/physics/secondlaw.html`).

```
<script type="text/javascript">

function calculate()
{
      var force = parseFloat(document.forms.cgcalc.force.value);
      var mass = parseFloat(document.forms.cgcalc.mass.value);
      var accel = parseFloat(document.forms.cgcalc.accel.value);

      if (isNaN(force))
            force = mass * accel;
            document.forms.cgcalc.force.value = force;

      if (isNaN(mass))
```

```
            mass = force / accel;
            document.forms.cgcalc.mass.value = mass;

        if (isNaN(accel))
            accel = force / mass;
            document.forms.cgcalc.accel.value = accel;
}

</script>
```

Figure 8. Solving a constraint graph utilizing JavaScript 1.5, embedded in HTML

4.   OUR EXPERIENCES UTILIZING CONSTRAINT GRAPHS IN HIGH SCHOOL PHYSICS

To determine whether or not the use of constraint graphs organizes and structures the mathematics of physics in such a way that more easily renders tasks of problem-solving and learning, we interacted with students in physics classes at Westlake High School as part of Georgia Tech's NSF GK-12-supported Student and Teacher Enhancement Partnership (STEP) program. It was evident that students in these high school physics classes experienced difficulty mastering the underlying mathematics of physical principles – particularly when solving a collection of equations representing a physical system. Generally, such problems are solved by means of a strategic, logical progression in which the value of a variable obtained through one mathematical relation is substituted into another expression, thereby enabling the valuation of a different variable. However, we observed that students frequently struggled to make sense of these problems, instead becoming mired in identifying the given information, distinguishing missing information, and then utilizing appropriate mathematical expressions to relate one with the other. Their deficiencies in planning and implementing mathematical solution procedures make these physics students ideal subjects for our study.

Initially, our investigation was piloted with two students during one-on-one physics tutorials in an after-school setting. Here, the representational formalism of constraint graphs was explained and demonstrated utilizing web-based forms of the type discussed in Section 3.2. The students were asked to complete their homework independently (e.g., physics problems featuring pendulums) and then to check their answers with a computable constraint graph template (e.g., URL: `http://srl.marc.gatech.edu/~scottc/step/physics/pendulum.html`). In doing so, the students became comfortable with the constraint graph representation in which the mathematics governing pendulums was made explicit in a highly structured manner. Increased proficiency with correctly solving problems was observed as the two students demonstrated an improved understanding of the topics addressed with constraint graphs during these tutorials. Consequently, our qualitative pilot study suggested that high school physics students are able to comprehend constraint graph models and perhaps benefit educationally from instruction utilizing the representational formalism. This served as impetus to initiate a more quantitative study on the matter involving a greater number of participants.

Towards that end, approximately 25 students in each of three different physics-based classes at Westlake High School – i.e., Physics, Honors Physics, and Physical Science – were introduced to the notion of constraint graphs over the course of a one-hour period. Since these classes

represent a broad range of academic ability at the high school level, the presentation of details regarding constraint graphs and their usage was tailored accordingly. For instance, the introduction of constraint graphs to students in Physical Science did not include the concept of multi-graph networking, as this complication was inappropriate to their current academic level; on the other hand, students in Honors Physics received concentrated instruction associated with networking five or more constraint graphs, in keeping with the sorts of problems that they are expected to solve. Further, the students were exposed to various examples of constraint graphs (both hand-drawn and computer-generated) representing a variety of physical principles.

Following this introduction, another period of each class was spent incorporating constraint graphs into the topic at hand. In the case of the Honors Physics class, this involved the application of constraint graphs to problems concerning energy, work, and momentum. The equations $I = F \times t$, $p = m \times v$, $W = F \times s$, and $KE = 0.5 \times m \times v \char`^2$ were formed into a constraint network with which problems were solved. Afterwards, the students were asked to augment the problem by linking another equation, $s = v \times t$, to solve for distance. Throughout the process, we noticed that students seemed to enjoy the graphical representation as it was extremely novel and somewhat challenging compared to the traditional problem-solving format. When practicing computations with the constraint graphs, students used simple integer values initially, filling in the missing value without hesitation. The visual nature of the format seemed to make checking answers easier and more automatic as opposed to what is deemed the tedious routine involving the substitution of computed answers back into equations. Several students reported that the inclusion of units in the graph made the process even clearer to them. Moreover, there were several "ah-ha" moments when students audibly reacted to a new understanding of the concepts encountered.

Serving as a brief check to ensure that the inclusion of constraint graphs in the physics-based curricula was being well received, a short quiz was administered in which students were instructed to solve a few problems utilizing constraint graphs. As with any new concept, the proper application of constraint graphs was on a continuum from careful attention to detail to complete lack of understanding. Interestingly though, some students who had struggled previously showed additional understanding. Samples of Honors Physics students' attempts to generate constraint networks are shown in Figure 9. Note how the constraint graphs in 9(d) remain isolated, compared to the networked versions of 9(a-c); regardless, each of the submissions features reasonably accurate valuations. This suggests that there may be some value in students' use of independent constraint graphs as opposed to constraint networks.

(a)   (b)
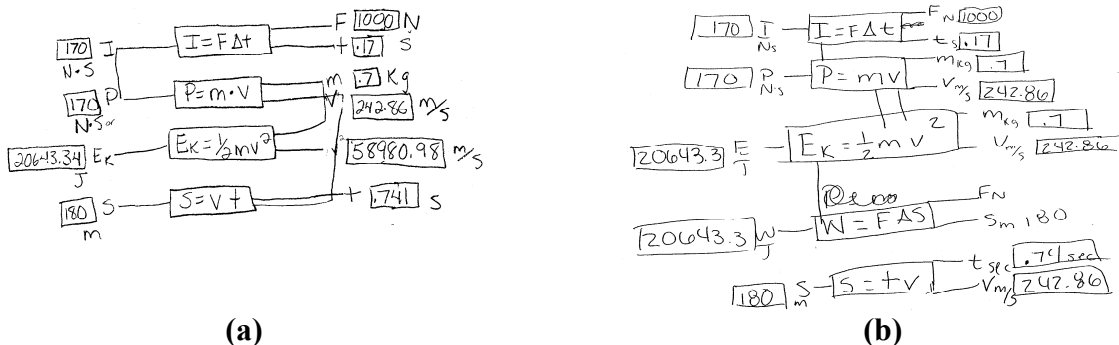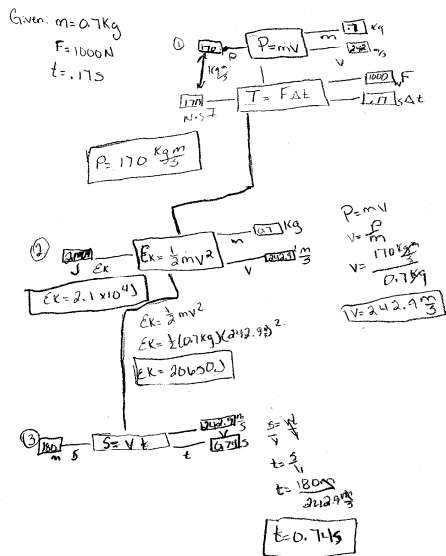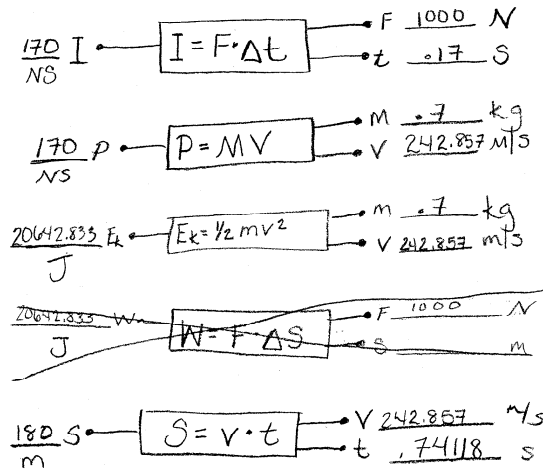
(c)   **(d)**

Figure 9. Samples of student-generated constraint networks

To complete the class unit before proceeding to the next topic, students were asked to write and submit comments regarding their individual feelings (e.g., positive, negative, or neutral) about constraint graphs. Students responded with a range of statements about the perceived impact. Most positive statements revolved around improved organization and better understanding of relationships between equations; several of the responses focused on the linkages and logical progression through a problem.

> *"It's very helpful because it shows how each equation connects to the others."*
> *"Using graphs is helpful for solving word problems."*
> *"I have a better understanding of how these formulas relate to one another."*
> *"This has helped me a little in keeping the numbers and units straight."*
> *"The constraint graphs help me place my numbers in the right place."*

Some comments were more general.

> *"Although I have been absent a few days, this made the lesson quick to learn."*
> *"Yes it helped, I got through the problem a lot quicker."*
> *"I really like doing these graphs, it helps me understand."*

One comment from a consistently excellent student was significant.

> *"I believe that this process will be helpful to others because I have been doing the same thing in my head to organize and understand the different equations and to help me solve the problems successfully."*

There were also negative comments.

> *"No. Did not help. Too much involved."*
> *"This method confuses me, especially since I have to figure out how the constraint graphs are related."*

In all, students made roughly ten positive comments for every one negative comment.

In the units that followed, the instructor presented the mathematics underlying physical principles within the classes in both traditional and constraint graph formats. Ten quizzes were administered during that time, in which students were permitted to solve the problems using any method desired. Roughly 25% (at least six students) in each of the classes chose to utilize constraint graphs when solving those problems, which suggests that some of the students find the technique appealing or preferable to the traditional approach.

In order to quantify and interpret the impact of constraint graphs on student performance, the following three evaluative categories and associated rubrics were defined.

- **Proficiency** – Is the constraint graph representation well formed in terms of boxes, vertices, lines, and labels? The merit of the representational formalism is judged utilizing the following criteria.

| Rating	Descriptive Characteristics
5	Constraints are boxed; 100% of arcs are present; linkages among graphs are correctly established; labels are present; all valuations and units are indicated
4	Constraints are boxed; 100% of arcs are present; linkages among graphs are correctly established; labels are present; some valuations are indicated
3	Constraints are boxed; 75% of arcs are present; linkages might exist among graphs, although they are generally incorrect; labels are present; few valuations indicated
2	Constraints are boxed; 50% of arcs are present; no linkages exist among graphs; labels might be present; valuations are not indicated
1	Constraints are boxed; 25% of arcs are present; no linkages exist among graphs; no labels are present; valuations are not indicated
0	No apparent effort is made to utilize constraint graph representation

- **Correctness** – Are the numerical answers computed with the constraint graph correct? To make assessment of this category simple, the rating scheme is binary with each correct numerical answer receiving one point. We utilized quizzes that feature two problems, each requiring the computation of a single answer; in this case, the rubric is as follows.

Rating	Descriptive Characteristics
2	Two numerical answers are correct
1	One numerical answer is correct
0	None of the numerical answers are correct

- **Likelihood of usage** – Given opportunity for using constraint graphs, will students employ

the technique? This category is assessed by a simple count of those students who utilized the technique to any extent versus those who did not.

Given these means for assessment, two quizzes were administered in each of the physics classes regarding the current topics of wave propagation. On the day of the first quiz, it was requested that the students utilize constraint graphs to solve two problems regarding sound waves; on the day of the second quiz, the students were informed that they could select any method desired to solve two different problems on the subject of electromagnetic waves. Results from the Honors Physics class were compiled and included here. For those students who employed constraint graphs, their proficiency was evaluated utilizing the aforementioned rubric; the mean value of proficiency on Quiz 1 was 3.0, while that of the students who chose to use constraint graphs on Quiz 2 was 3.43. In terms of correctness, the entire class achieved a mean value of 1.2 on Quiz 1; the average measure of correctness for students who utilized constraint graphs on Quiz 2 was 1.14, while the mean value of correctness for students who chose not to use constraint graphs on the same quiz was 0.67. Likelihood of usage, measured as the percentage of the class electing to make use of constraint graphs on Quiz 2, was determined to be 60.9% (i.e., 14 out of 23 students); however, this figure is somewhat inflated compared to the previous observation that roughly 25% of the students consistently employ constraint graphs. At the very least, these preliminary results suggest that it is possible for constraint graphs to play a role in the improvement of high school student physics performance.
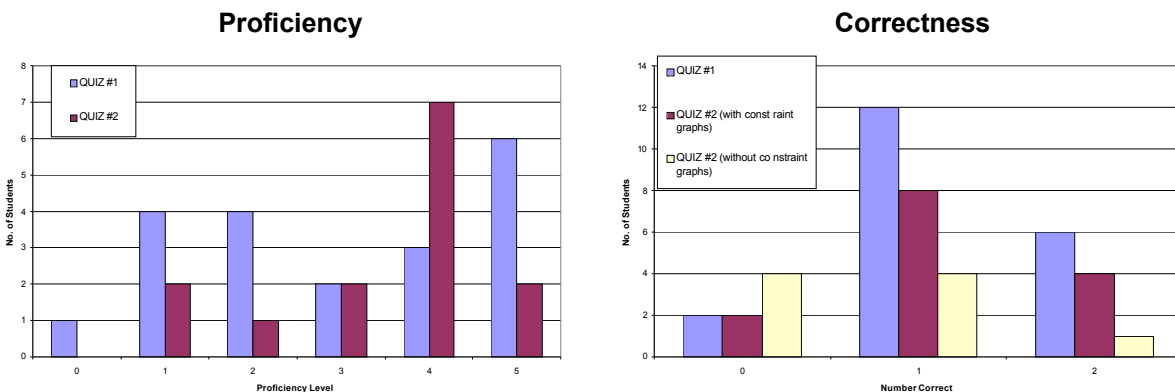
Figure 10. Assessment of constraint graph usage in the Honors Physics class

5.   CLOSING REMARKS

We assert that the representational formalism of constraint graphs provides a number of affordances for novice students solving physics problems at a high school level. The elegant representation of constraint graphs makes explicit the linkages between mathematical relationships, which students might otherwise find disparate or irrelevant; it provides a means for tracking the flow of variables and their valuations from one constraint to another. The building block nature of constraint graphs makes it easy to model complex problems involving numerous elements, simply by aggregating different constraint graphs; this adds a dimension of concreteness to an otherwise abstract mathematical system. Moreover, the process of building constraint networks embodies a conceptual planning and strategy formulation phase, a task that novices tend

to avoid consciously. Collectively, constraint graphs serve to organize and structure the mathematics of physics in such a way that more easily renders tasks of problem-solving and learning.

Further, the incorporation of this representational technique using logically networked graphs serves several additional purposes in a high school physics classroom.

- It seems to provide an alternative tool for problem-solving involving the organization of given and required elements of the problem. This approach most likely appeals to the learning styles of many students.
- The format lends itself to providing a better understanding of relationships between equations and linkages among concepts, owing to the graphical representation that is more tractable than traditional mathematics.
- The exposure to concepts of computer programming and information modeling (topics tangential to constraint graphs) is beneficial for today's students.
- And finally, we found that some students just seemed to enjoy the process more and thus performed better on assignments. One might be inclined to dismiss this final purpose, however, the departure from the ordinary toward a more novel approach may be a significant factor in making physics enjoyable rather than feared.

As we continue with a more rigorous study of these issues, we envision a variety of possible expansions regarding the use of constraint graphs in physics. A computer-based constraint graph utility might be developed enabling students to build their own templates online; students could include descriptions of linking rationale when constructing constraint graphs, or even investigate alternative representations of the graphs; students might even program their own constraint graphs in a manner similar to that discussed in Section 3.2. Given the flexible and generic nature of this constraint graph information model, it should be possible to easily incorporate these new and exciting ideas into the physics curriculum.

ACKNOWLEDGMENTS

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