Utilizing Spreadsheet Solver Methods to Determine Optimal Beam Geometry

William K. Szaroletta
Purdue University, West Lafayette, Indiana

Abstract:
The spreadsheet solver method has proven very successful as students at both upper and lower division are experiencing meaningful single-criterion optimization in addition to finding optimization applications in their other coursework. This paper will describe optimization implementation and present some sample optimization results at both levels using spreadsheet solver method to drive subsequent solid modeling and finite element analysis (FEA).

Introduction:

The author previously described in the ASEE 2002 Conference Proceedings the introduction of a suite of four optimization techniques into mechanics classes. The spreadsheet solver method has proven very successful as students at both upper and lower division are experiencing meaningful single-criterion optimization and also finding optimization applications in their other courses.

In the lower division mechanics class, a simply supported, rectangular cross-section beam with a central load was first introduced and solved using conventional analysis methods. After the students explore and understand the varying stress profile in the beam, the concept of the optimization objective function is introduced. In this beam, the objective is to produce a constant stress state on the highest stressed portions of the part. A spreadsheet solver is used to meet this objective subject to constraints for beam base and width in two modes. The first mode is a constant height beam with a varying base and the second is a constant base beam with a varying height. The students utilize the results of the spreadsheet solver for both beams to produce solid models which can be readily visualized.

In the upper division mechanics class, the students initially perform the same steps as the lower division class as a refresher. Additionally, they explore the beam shapes required for different loadings, highlighting the effect load has on the stress state within the part and required beam geometry. Subsequently, these students utilize their configure solid models for the loading cases to prepare FEA models of four basic beam types. The FEA stress states are compared with the theoretical stress states for these configurations in report and class presentations.

This paper describes the optimization introduction and presents some sample optimization results at both levels using spreadsheet solver to drive subsequent solid modeling and finite element analysis (FEA).

Analysis of Simply Supported Beam with Central Load:

In a simply supported beam with a central load, P, (see Figure 1) the reaction forces can be determined using a free body diagram (FBD) and freshman-level Statics to be of equal magnitude (e.g. P/2) and direction (opposite of P). This elementary analysis can be utilized and extended to produce the Loading, Shear and Moment diagrams shown below in Figure 2. Of interest is the Moment Diagram for this loading case which, due to the "tent" shape, produces a "tent" shape stress curve shown in Figure 4 for constant cross-section beam constructions shown in Figure 3.

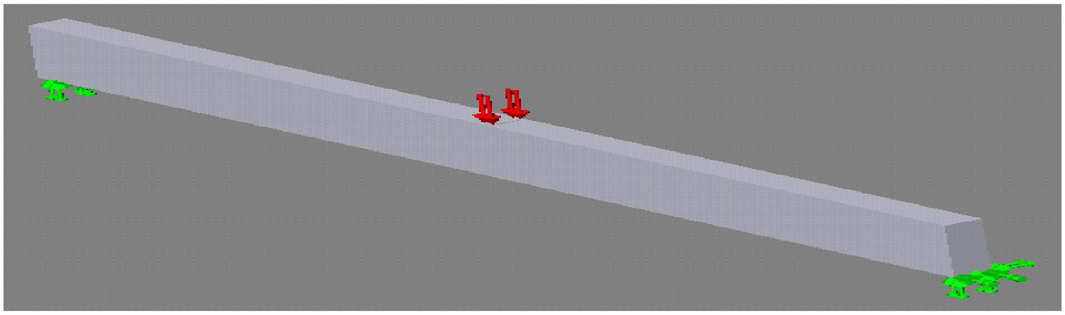
Figure 1: Simply supported beam with central load

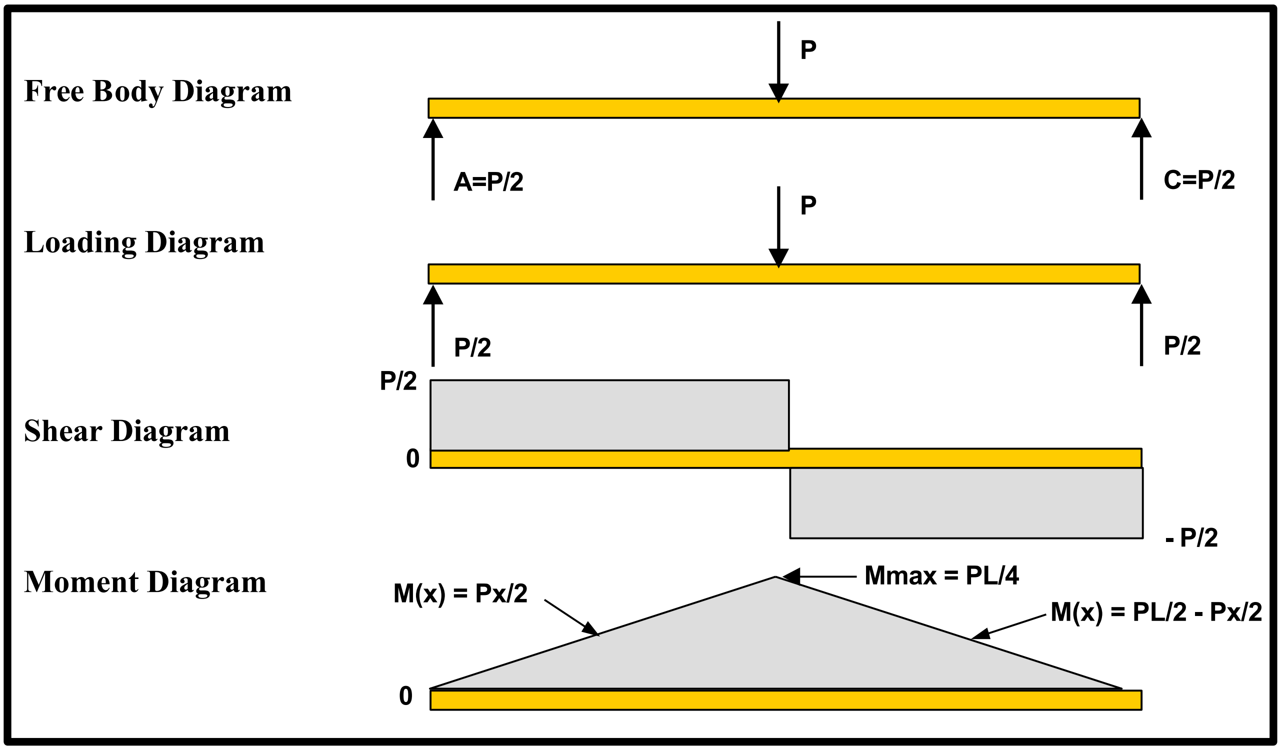

Figure 2: Free Body, Loading, Shear, and Moment diagrams for simply supported beam with central load

For a constant cross-section rectangular prismatic beam, the section modulus, Z(x), is constant along x leading to a constant flexural stress on the top surface with b(x) representing the width of the beam at any location, x and h(x) representing the height of the beam at any location, x, as shown below in Figure 3.

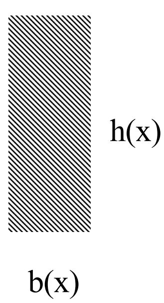

Figure 3: Rectangular cross-section of beams in this paper

For a rectangular cross-section beam, the equation for the maximum flexural stress, σ(x), which is found on the bottom surface, is shown in Eq 1 below[1,2].

$$\sigma(x) = 6 * M(x) / (b(x) * h(x)^2) \qquad \text{Eq 1}$$

A non-optimal solution through use of standard cross-section materials which are readily available, including rectangular cross-section beams, I-Beams, hollow-square beams, etc. The constant Z(x) along the x dimension leads to a σ(x) profile that mirrors M(x) with a maximum value at midspan as shown in Figure 4 below.

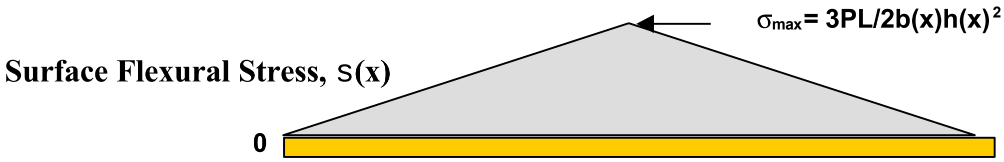

Figure 4: Simply supported beam with a surface flexural Z(x) **bending stress, s(x), that mirrors M(x) for a constant cross-section beam.**

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Constant Surface Flexural Stress as Optimization Goal:

More efficient utilization of the material (not necessary the optimum) to produce a given maximum surface stress requires some changes in the beam cross-section along the x axis. s t is clear from the above discussion that a constant-stress beam will have flexural bending stress graph that is a horizontal line as shown in Figure 5 below.

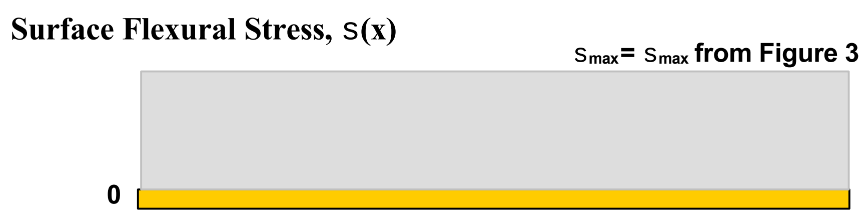

Figure 5: Simply supported beam with a constant surface flexural bending stress, s(x). Beam cross-section varies along x.

As shown previously[2], this goal produces a beam where more of the beam material is efficiently utilized and thus contributes more than the rectangular prismatic shape beam.

Beam Geometries to Achieve Constant Surface Flexural Stress:

Students were asked to take the basic rectangular prismatic beam geometry and develop the surface flexural stress magnitude for a given loading. These rectangular prismatic cross-sections essentially had a constant b(x) with h(x) constant along x but set to some value using Microsoft Excel™ Solver™ [3]. Similarly, a constant h(x) with b(x) constant along x but set to some value using Solver™ was also accomplished. The family of geometries that were explored by the students also included varying b(x) along x while holding h(x) constant and varying h(x) along x while holding b(x) constant. Subsequent to that "baseline" analysis activity, maximum surface flexural stress level goals were developed. Several groups had stress level goals of 1000 psi, 2000 psi, and 5000 psi, while other groups had 3750 psi, 5000 psi, and 8000 psi.   Each of these stress levels (e.g. five) and approaches (e.g. four) generated twenty potential solutions that are described below.

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Rectangular Prismatic Cross-Section: First Approach

First, for a rectangular prismatic beam, the required base dimension was determined using Excel Solver™ producing these stress levels while holding the height constant at 3.0". These dimensions were input into parametric solid models allowing the resulting beam volumes to be determined. The general shapes of the resulting solid models for this case are shown pictorially below in Figure 6 and a chart depicting the volume change as a function of stress level is shown in Figure 8.

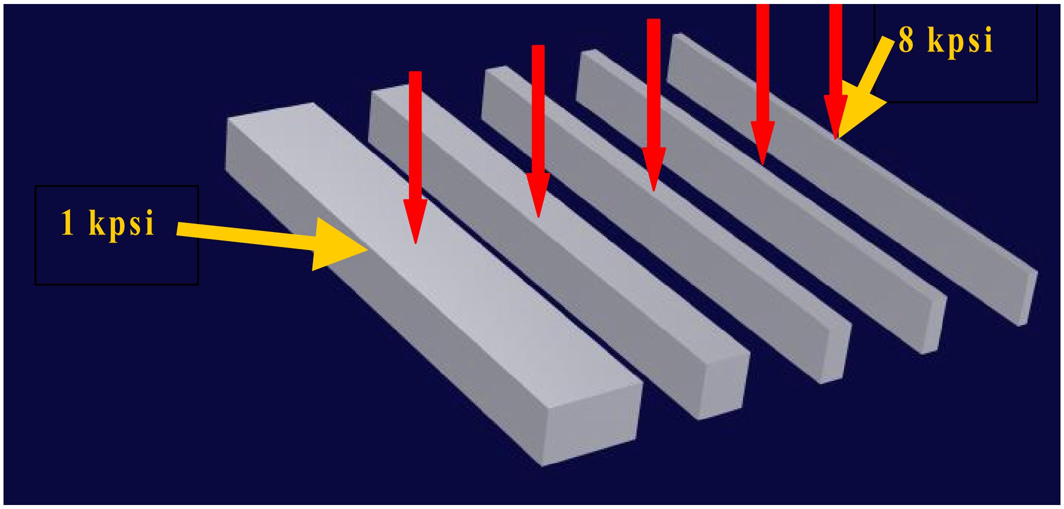

Figure 6: Rectangular Prismatic Beams to Achieve Goal Flexural Stress – Vary b(x) w/ h(x) = 3". Maximum flexural stress is obtained at midspan (range: 1 kpsi to 8 kpsi, l to r)

Second, again for a rectangular prismatic beam, the height was determined using Excel Solver™ to produce these same stress levels while holding the base constant at 1.0". These dimensions were input into parametric solid models and the volumes to be determined. The general shapes of the resulting solid models for this case are shown pictorially below in Figure 7 and a chart depicting the volume change as a function of stress level is shown in Figure 8.

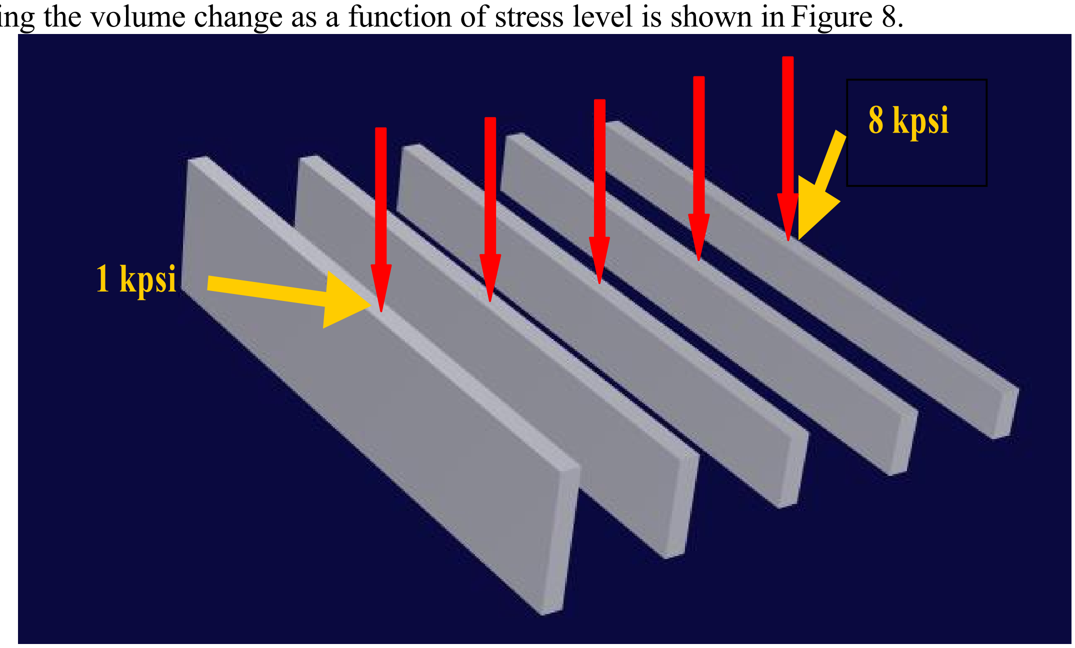

Figure 7: Rectangular Prismatic Beams to Achieve Goal Flexural Stress – Vary h(x) w/ b(x) = 1". Maximum flexural stress is obtained at midspan (range: 1 kpsi to 8 kpsi, l to r)

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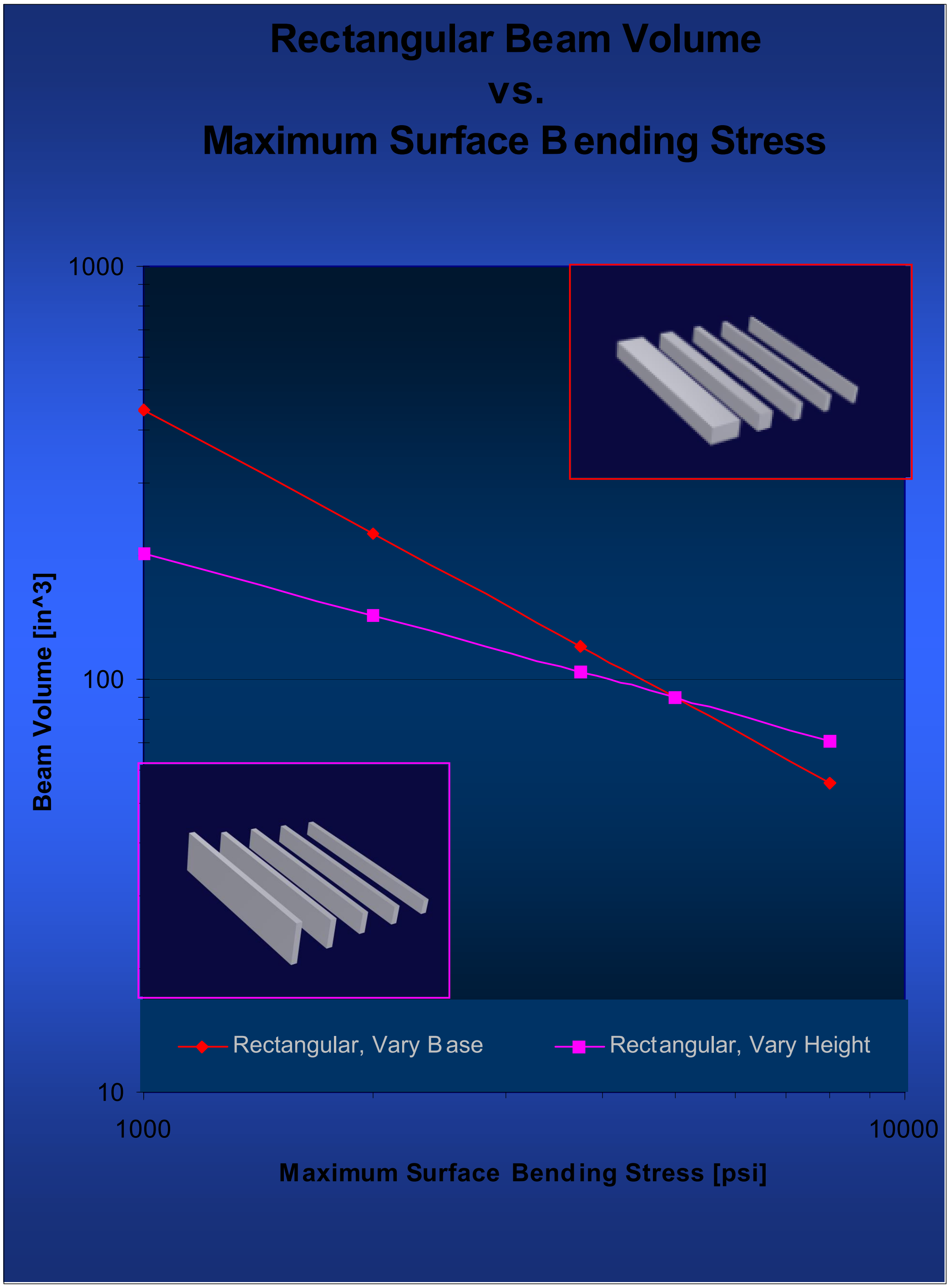


Figure 8: Results of Utilizing Solver™ on Two Rectangular Prism Cross-Section Beam Families. Note that the cross-over occurs at the baseline stress level of 5 kpsi.

Rectangular Prismatic Cross-Section: Second Approach

Next, the rectangular beam was subdivided into equal elements for which the flexural stress could be obtained using Equation 1 above. First, the height, h(x), was held constant along x equal to 3", while the base, b(x) was varied for each element to produce the desired stress utilizing Solver ™. The resulting dimensions were exported to a solid model, which enabled the volume to be determined. The general shapes of the resulting solid models for this case are shown pictorially below in Figure 9 and a chart depicting the volume change as a function of stress level is shown in Figure 11.

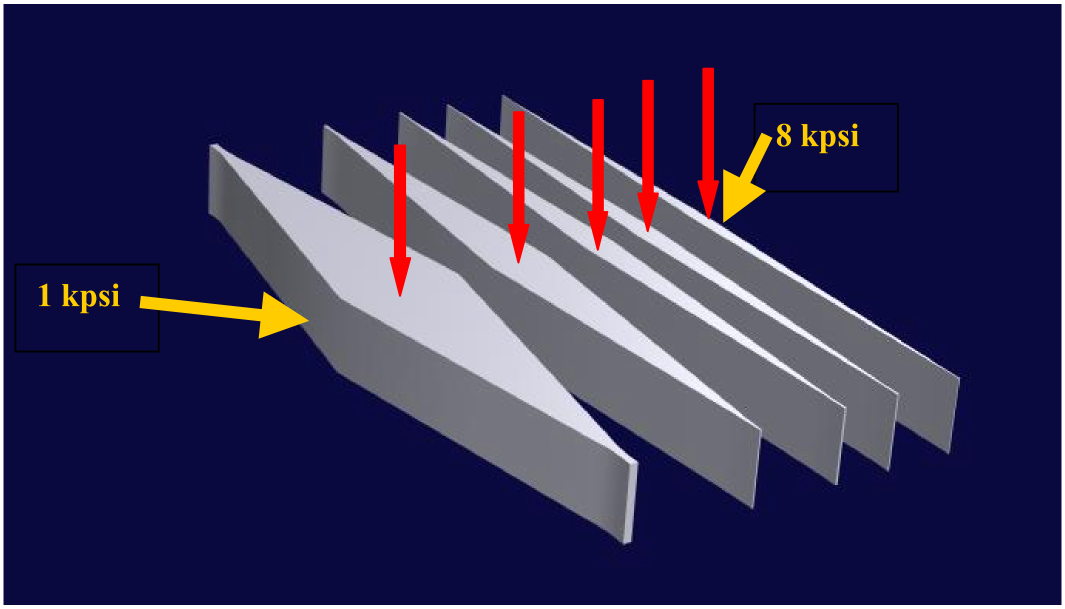


Figure 9: Rectangular Beams to Achieve Goal Flexural Stress – Vary b(x) w/ h(x) = 3". Maximum flexural stress is obtained along top surface (range: 1 kpsi to 8 kpsi, l to r)

Second, the base, b(x), was held constant along x equal to 1", while the height, h(x), was varied for each element to produce the desired stress utilizing Solver™. The general shapes of the resulting solid models for this case are shown pictorially below in Figure 10 and a chart depicting the volume change as a function of stress level is shown in Figure 11.

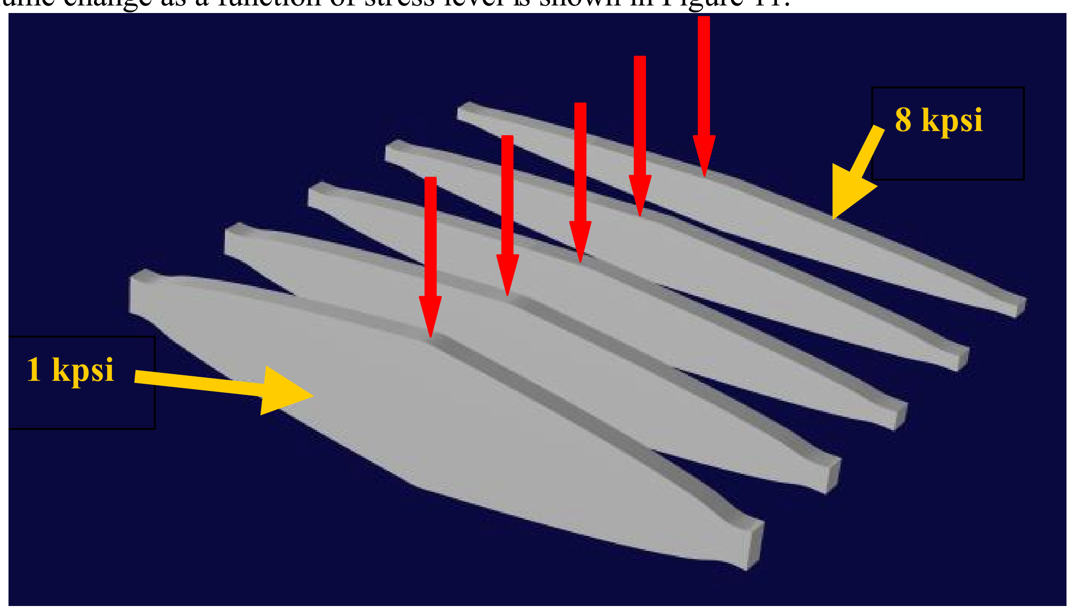


Figure 10: Rectangular Beams to Achieve Goal Flexural Stress – Vary h(x) w/ b(x) = 1". Maximum flexural stress is obtained along top surface (range: 1 kpsi to 8 kpsi, l to r)

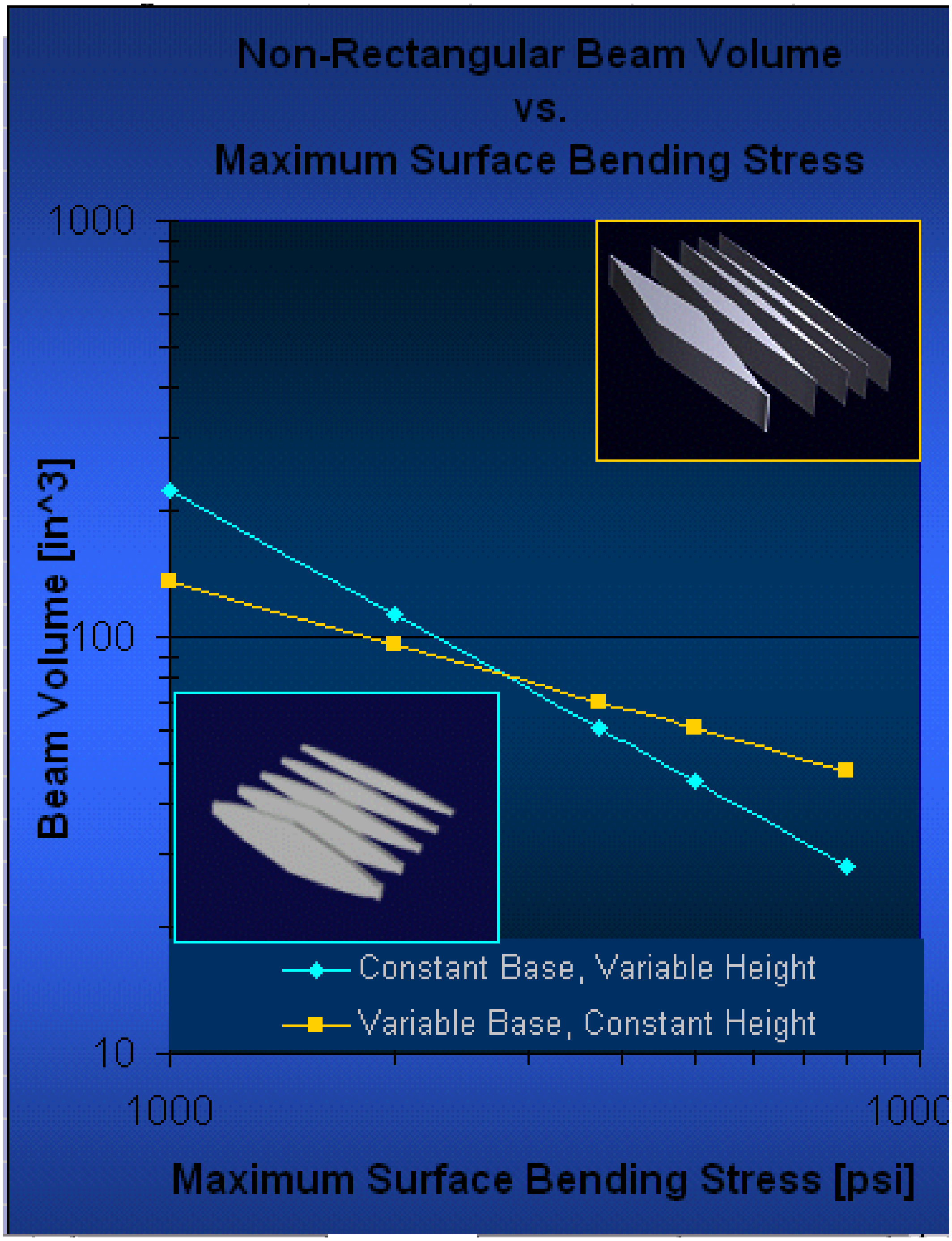


Figure 11: Results of Utilizing Solver™ on Two Rectangular Cross-Section Beam Families. Again, note the cross-over just less than 3 kpsi.

Spreadsheet Solver Software Utilized:
The spreadsheet solver method is becoming widely known and popular among Purdue University undergraduate students. For this paper, the Microsoft Excel™ add-in package Solver™ is utilized, although other packages are readily available, including MATLAB™, MathCAD™, Mathematica™, Maple™, etc. The solution for the initial (b(x) = 1" and h(x) = 3") beam is shown below in Figure 12.

Microsoft Excel - 3PB-BaseHeight-Solver

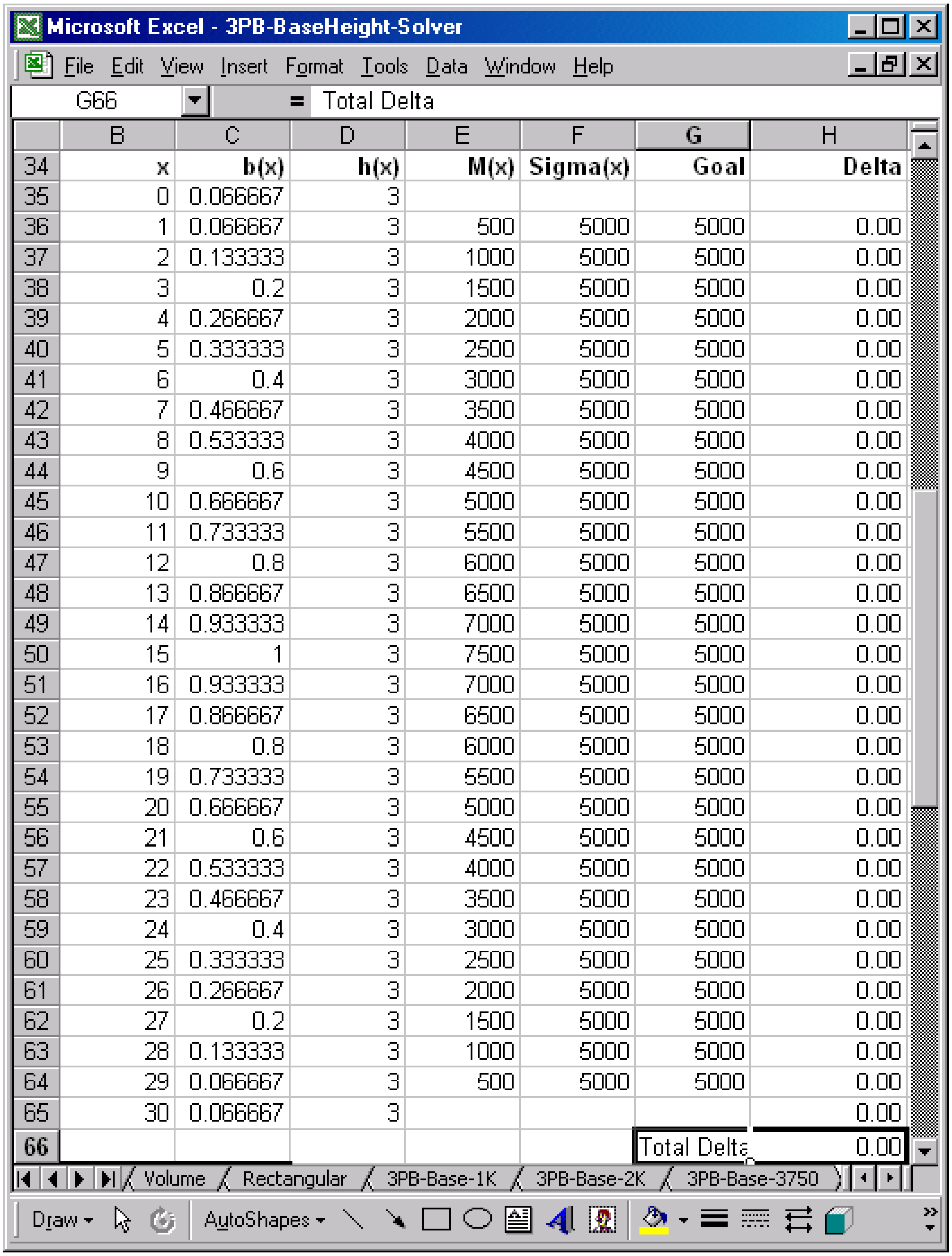
E15 = M(x)

	A	B	C	D	E	F
1	**Simply Supported Beam**					
2	Central Load, P	1000	#			
3	Support Reactions, A & C	500	#			
4	Beam Height	3	in			
5	Beam Width	1	in			
6	Beam MOI, Ixx	2.25	in^4			
7	Beam Section Modulus, Z	1.5	in^3			
8	Beam Length, L	30	in			
9	Central Moment, Mmax	7500	in-#			
10	Tensile Modulus, E	30000000	psi			
11	Bending Stress, Sigma-max	5000	psi			
12	Strain, ε-max	167	μ-in/in			
13						
14	**Bending Stress Distribution**					
15	**Z(x) = b(x)*h(x)^2/6**	**x**	**b(x)**	**h(x)**	**M(x)**	**Sigma(x)**
16		1	1	3	500	333
17		2	1	3	1000	667
18		3	1	3	1500	1000
19		4	1	3	2000	1333
20		5	1	3	2500	1667
21		6	1	3	3000	2000
22		7	1	3	3500	2333
23		8	1	3	4000	2667
24		9	1	3	4500	3000
25		10	1	3	5000	3333
26		11	1	3	5500	3667
27		12	1	3	6000	4000
28		13	1	3	6500	4333
29		14	1	3	7000	4667
30		15	1	3	7500	5000

Volume / Rectangular / 3PB-Base-1K / 3PB-Base-2K / 3PB-Base-3750 / 3PB-B

Figure 12: Initial beam analysis using Excel™.

Figure 13 below depicts a typical Excel™ output after Solver™ was utilized to determine the required element widths (b(x)) to drive the surface flexural stress to one of the goal values; in this case, 5000 psi.



	B	C	D	E	F	G	H
34	x	b(x)	h(x)	M(x)	Sigma(x)	Goal	Delta
35	0	0.066667	3				
36	1	0.066667	3	500	5000	5000	0.00
37	2	0.133333	3	1000	5000	5000	0.00
38	3	0.2	3	1500	5000	5000	0.00
39	4	0.266667	3	2000	5000	5000	0.00
40	5	0.333333	3	2500	5000	5000	0.00
41	6	0.4	3	3000	5000	5000	0.00
42	7	0.466667	3	3500	5000	5000	0.00
43	8	0.533333	3	4000	5000	5000	0.00
44	9	0.6	3	4500	5000	5000	0.00
45	10	0.666667	3	5000	5000	5000	0.00
46	11	0.733333	3	5500	5000	5000	0.00
47	12	0.8	3	6000	5000	5000	0.00
48	13	0.866667	3	6500	5000	5000	0.00
49	14	0.933333	3	7000	5000	5000	0.00
50	15	1	3	7500	5000	5000	0.00
51	16	0.933333	3	7000	5000	5000	0.00
52	17	0.866667	3	6500	5000	5000	0.00
53	18	0.8	3	6000	5000	5000	0.00
54	19	0.733333	3	5500	5000	5000	0.00
55	20	0.666667	3	5000	5000	5000	0.00
56	21	0.6	3	4500	5000	5000	0.00
57	22	0.533333	3	4000	5000	5000	0.00
58	23	0.466667	3	3500	5000	5000	0.00
59	24	0.4	3	3000	5000	5000	0.00
60	25	0.333333	3	2500	5000	5000	0.00
61	26	0.266667	3	2000	5000	5000	0.00
62	27	0.2	3	1500	5000	5000	0.00
63	28	0.133333	3	1000	5000	5000	0.00
64	29	0.066667	3	500	5000	5000	0.00
65	30	0.066667	3				0.00
66						Total Delta	0.00

Figure 13: Solver™ determined the required element widths (b(x)) to drive the surface flexural stress to a goal value. In this case, the goal was 5000 psi.

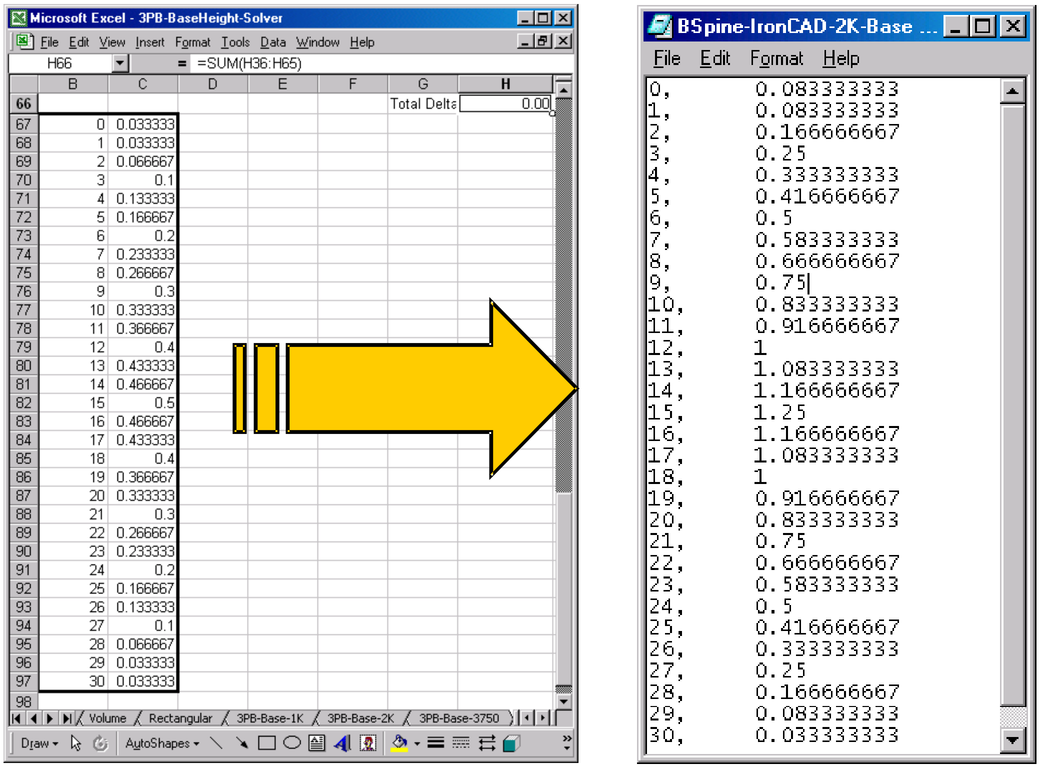


Figure 14: Solver™ determined the required beam geometry and the data that was exported from Excel™ to text editor. Solid model of beam utilized imported geometry.

CAD Software Utilized:
A number of solid modeling CAD packages would accomplish the modeling of the twenty beams presented in this paper. Construction steps are shown below for IronCAD™ [5] in Figure 15.

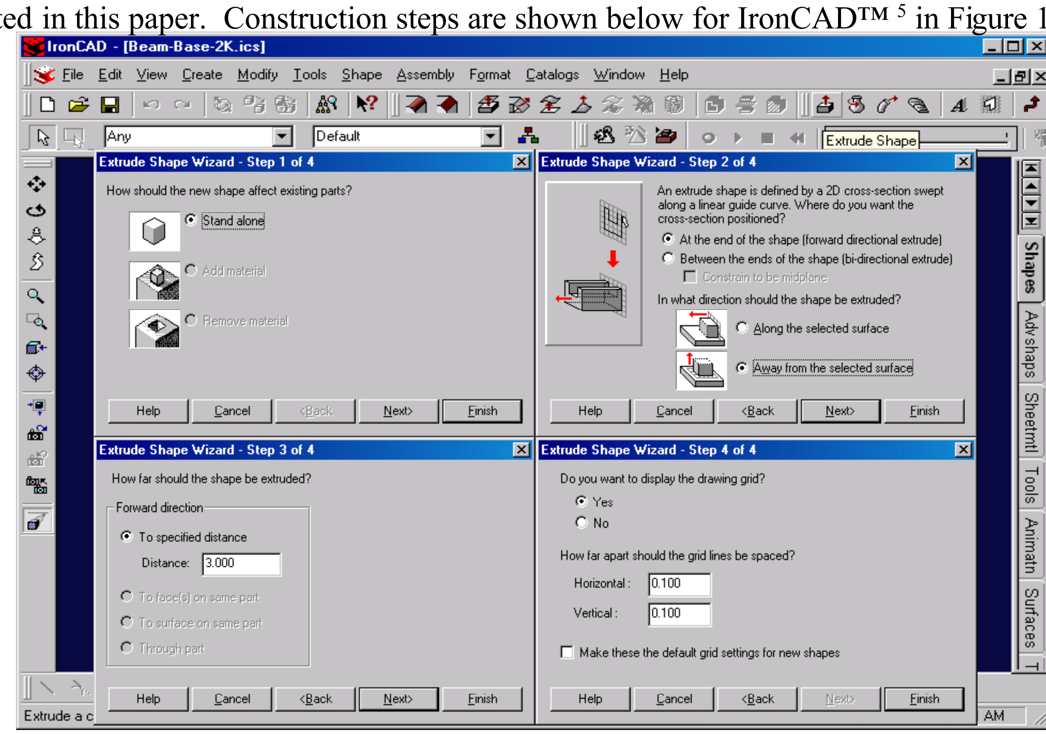


Figure 15: Part 1 -- Importing geometry from Solver™ analysis via Excel™ via text editor.

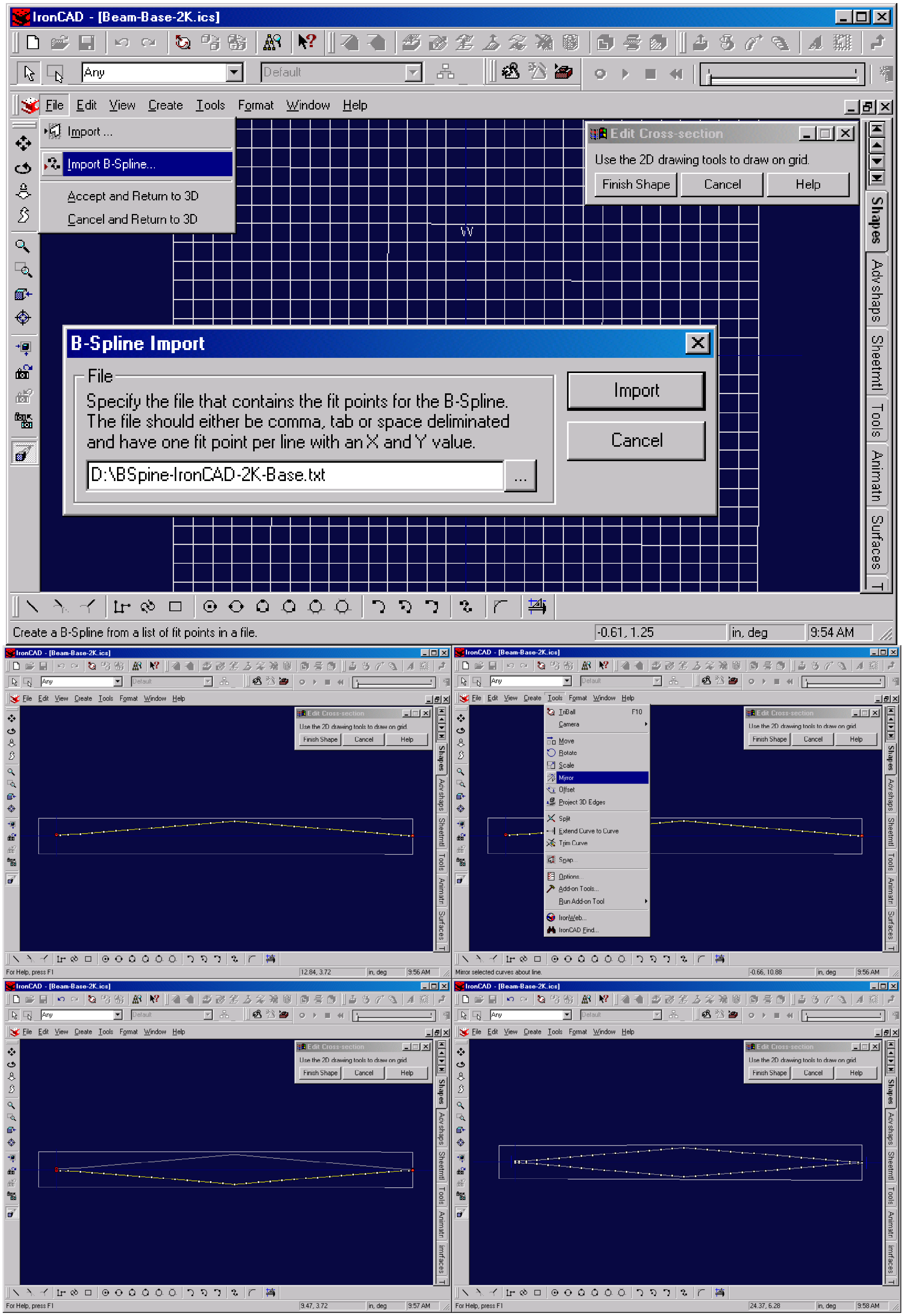


Figure 16: Part 2 -- Importing geometry from Solver™ analysis via Excel™ via text editor. Beginning construction of beam profile within IronCAD™.

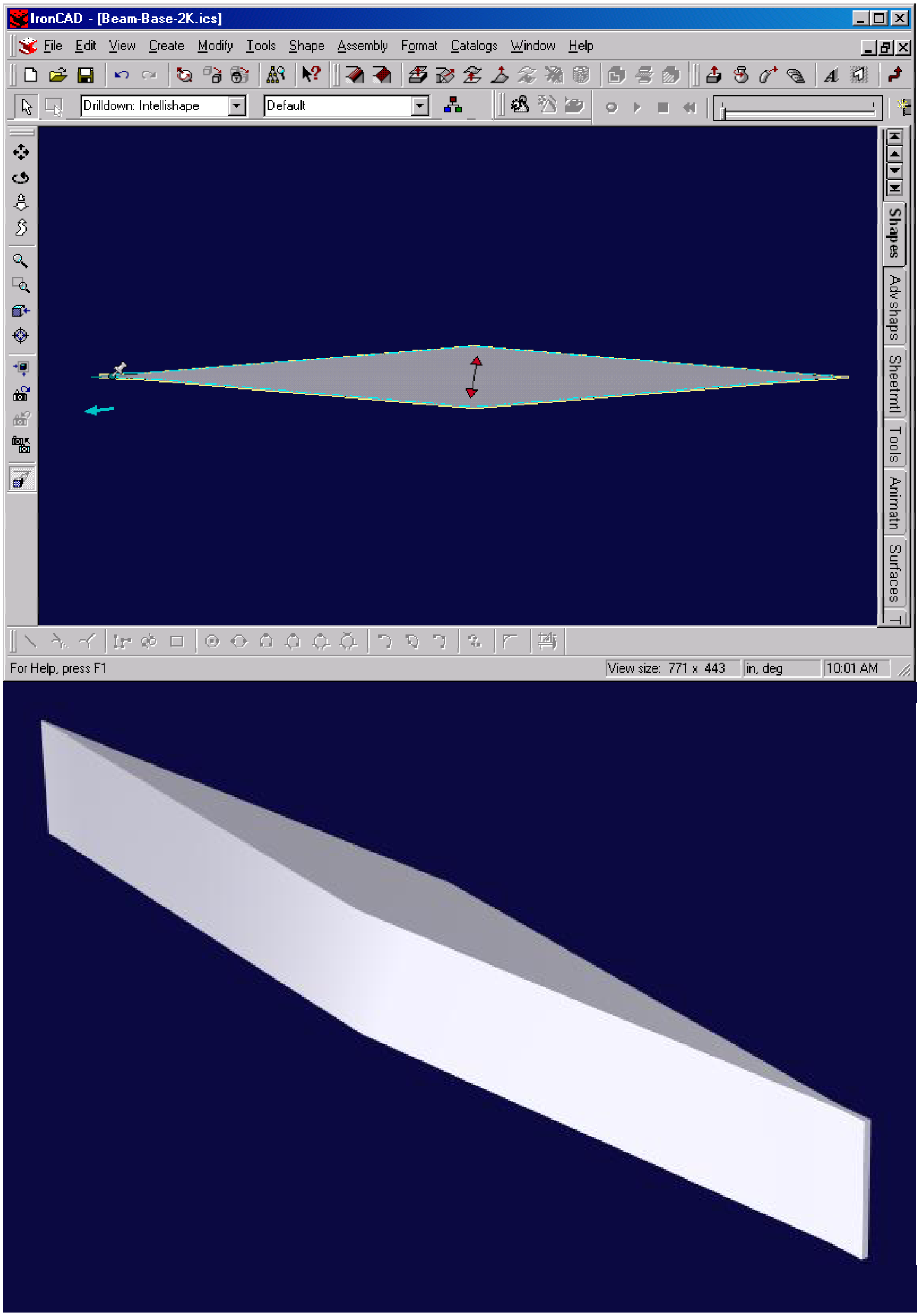


Figure 17: Part 3 -- Importing geometry from Solver™ analysis via Excel™ via text editor. Finishing construction in IronCAD™. Final part geometry for 5 kpsi stress level.

Finite Element Analysis Software Utilized:

Due to the popularity of the Finite Element Method, many FEA software packages are available to Purdue University undergraduate students including ANSYS, COSMOS/M, COSMOS DesignSTAR, Pro/Mechanica, etc.. All of the solid models in this paper were analyzed using COSMOS DesignSTAR™. For this paper, an example of COSMOS DesignSTAR™ [4] FEA analysis is shown in Figure 18 for the 5 kpsi part shown in Figure 17 above. It is informative to note the stress level on the top surface is shown in red, which matches the 5 kpsi goal for this analysis.

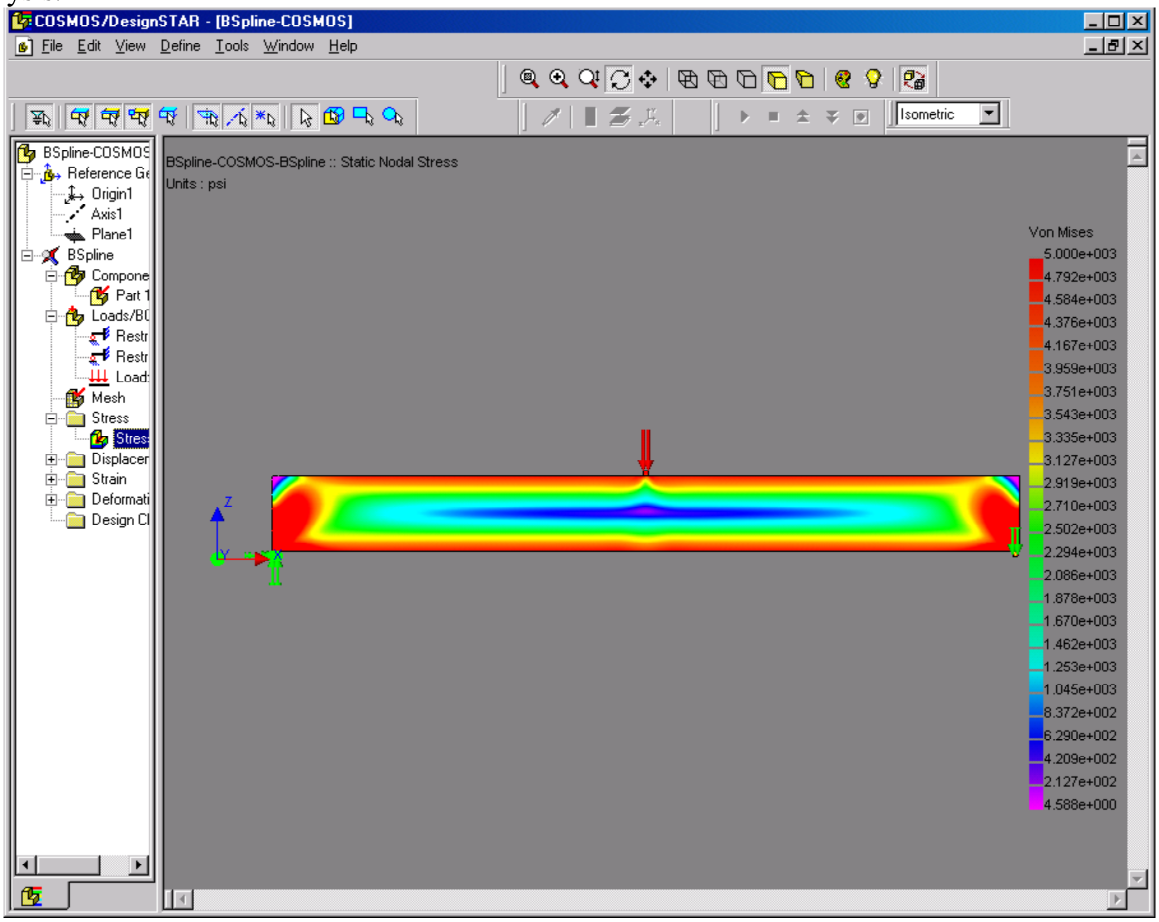


Figure 18: COSMOS DesignSTAR™ FEA study showing that Solver™ analysis produced geometry that has 5 kpsi stress level on top surface as expected.

Conclusions:

The author's previously described work into four optimization techniques into mechanics classes has resulted in a focus on Microsoft's Excel™ Solver™ for undergraduate mechanics applied optimization instruction. This paper has described both the theory and practice of optimization implementation, including spreadsheet solver, solid modeling, and finite element analysis (FEA). Mechanics students in both lower and upper divisions find the above project to be a valuable learning aid to the underlying theory of beam flexural stresses and to the software tools that are available to them.

Subsequent work in upper division mechanics classes will include prototyping and photoelastic testing of scaled versions of these twenty beams. This will enable the design analysis, FEA, and experimental analysis results to be compared and contrasted. Subsequent work in both divisions will include additionally varying the profile of the cross-section to produce higher flexural stresses at the neutral axis of the simply supported beam. Also, shear forces will be incorporated into a later project, since they were omitted from the above student work to simply the focus on both the mechanics and the spreadsheet solver method.

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Biography:

WILLIAM K. SZAROLETTA, P.E.
Professor Szaroletta is an assistant professor of mechanical engineering technology at Purdue University. A member of ASEE, he has 18 years industry experience in engineering and project management positions, with 12 awarded patents. He received his B.S. Degree in Mechanical Engineering from University of Michigan, Ann Arbor in 1977, M.S. Degree in Engineering (Product Design) from Stanford University in 1984, and a Master of Applied Mathematical Sciences Degree (Computer Science) from University of Georgia in 2000. He has 7 years university teaching experience, where his current applied research interests are rapid product design engineering, experimental mechanics laboratory automation, and applied optimization.