Why Math Matters: Demonstrating the Relevance of Mathematics in ECE Education

Dr. Yajing Liu, Colorado State University

Yajing Liu is a PhD student in the Department of Electrical and Computer Engineering at Colorado State University. She received the BS degree in Mathematics from Shaanxi Normal Univeristy, Xi'an, Shaanxi, China, in 2010.

Prof. Ali Pezeshki, Colorado State University

Ali Pezeshki received the BSc and MSc degrees in electrical engineering from University of Tehran, Tehran, Iran, in 1999 and 2001, respectively. He earned his PhD degree in electrical engineering at Colorado State University in 2004. In 2005, he was a postdoctoral research associate with the Electrical and Computer Engineering Department at Colorado State University. From January 2006 to August 2008, he was a postdoctoral research associate with The Program in Applied and Computational Mathematics at Princeton University. In August 2008, he joined the faculty of Colorado State University, where he is now an Associate Professor in the Department of Electrical and Computer Engineering, and the Department of Mathematics. His research interests are in statistical signal processing, coding theory, applied harmonic analysis, and bioimaging.

Dr. Sourajeet Roy, Colorado State University

Sourajeet Roy received the B.Tech. degree from Sikkim Manipal University, Gangtok, India, in 2006, and the M.E.Sc. and Ph.D. degrees from Western University, London, ON, Canada, in 2009 and 2013, respectively, all in electrical engineering.

Dr. Roy currently serves as an Assistant Professor with the Department of Electrical and Computer Engineering, Colorado State University, Fort Collins, CO, USA. His current research interests include modeling and simulation of high speed circuits, signal and power integrity analysis of electronic packages, and uncertainty quantification of microwave/ RF circuits.

Dr. Roy is a recipient of the Vice-Chancellors Gold Medal at the undergraduate level in 2006, the Queen Elizabeth II Graduate Scholarship in Science and Technology in 2012, and the Ontario Graduate Scholarship in 2012. He currently serves as the reviewer for IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, IEEEE TRANSACTIONS ON COMPONENTS, PACKAGING AND MANUFACTURING TECHNOLOGY, IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY and IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS I: REGULAR PAPERS. He also currently serves as the guest associate editor for IEEE TRANSACTIONS ON COMPONENTS, PACKAGING AND MANUFACTURING TECHNOLOGY. His student has won the Best Poster Paper Award at the 23rd IEEE Conference on Electrical Performance of Electronic Packaging and Systems (EPEPS) in 2014.

Prof. Branislav M. Notaros, Colorado State University

Branislav M. Notaros is Professor in the Department of Electrical and Computer Engineering at Colorado State University, where he also is Director of Electromagnetics Laboratory. He received a Ph.D. in electrical engineering from the University of Belgrade, Yugoslavia, in 1995. His research publications in computational and applied electromagnetics include more than 150 journal and conference papers. He is the author of textbooks Electromagnetics (2010) and MATLAB-Based Electromagnetics (2013), both with Pearson Prentice Hall. Prof. Notaros served as General Chair of FEM2012, Colorado, USA, and as Guest Editor of the Special Issue on Finite Elements for Microwave Engineering, in Electromagnetics, 2014. He was the recipient of the 1999 Institution of Electrical Engineers (IEE) Marconi Premium, 2005 Institute of Electrical and Electronics Engineers (IEEE) MTT-S Microwave Prize, 2005 UMass Dartmouth Scholar of the Year Award, 2012 Colorado State University System Board of Governors Excellence in Undergraduate Teaching Award, 2012 IEEE Region 5 Outstanding Engineering Educator Award, 2014 Carnegie Foundation for the Advancement of Teaching Colorado Professor of the Year Award, 2015 American Society for

Engineering Education ECE Distinguished Educator Award, 2015 IEEE Undergraduate Teaching Award, and many other research and teaching awards.

Prof. Tom Chen, Colorado State University

Tom Chen received his Ph.D. from the University of Edinburgh. After spending 4 years with Philips Semiconductors in Europe, he joined the Department of Electrical & Computer Engineering at Colorado State University. Prof. Chen published more than 180 journal and conference papers in the areas of analog and digital VLSI design and CAD for VLSI design. Prof. Chen served as the General Chair of 2015 IEEE Midwest Symposium on Circuits and Systems, and as the Guest Editor of IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems Special Issue on Design Quality and Design Closure: Present Issues and Future Trend", 2005. He also served as the Guest Editor of the Microelectronics Journal on Quality Electronic Design, 2005. His research interests include VLSI circuit and system design, CAD methodology for VLSI design, and bioelectronics.

Prof. Anthony A. Maciejewski, Colorado State University

Anthony A. Maciejewski received the BS, MS, and PhD degrees in electrical engineering from Ohio State University, Columbus in 1982, 1984, and 1987, respectively. From 1988 to 2001, he was a professor of electrical and computer engineering at Purdue University, West Lafayette. He is currently a professor and head of the Department of Electrical and Computer Engineering at Colorado State University. He is a fellow of IEEE. A complete vita is available at: http://www.engr.colostate.edu/ ˜aam.

Why Math Matters: Demonstrating the Relevance of Mathematics in ECE Education

Yajing Liu, Ali Pezeshki, Sourajeet Roy, Branislav M. Notaros,
Tom Chen, Anthony A. Maciejewski
Department of Electrical & Computer Engineering
Colorado State University

Abstract

This paper describes a team's efforts to transform the educational experience for electrical and computer engineering (ECE) students by demonstrating the importance of mathematics and the power of mathematical thinking. Math is a mainstay in ECE education. A holistic understanding of ECE concepts is highly dependent on students' grasp of key topics in mathematics, yet students are often intimidated by the mathematics required for the major and struggle to see why math matters. The dismal consequence is that students lack motivation and confidence, and eventually abandon the major.

In 2015, the ECE department received a five-year RED (Revolutionizing Engineering Departments) grant from the National Science Foundation to revamp the current engineering educational system. Performing research that leads to scalable and sustainable change in engineering education, the department is, in effect, throwing away courses to help students learn more meaningfully and effectively. The new pedagogical and organizational model treats the undergraduate degree as a complex system, giving consideration to the interconnectivity and integration of fundamental concepts across the program. This holistic approach emphasizes knowledge integration and weaves key threads (creativity, foundations, and professionalism) throughout the curriculum, stitching together and reinforcing relevant themes from the freshman to senior years.

With the goal of showing students why math is critical to becoming a successful engineer, this paper focuses on the foundations thread of the RED project. Even though the foundations thread encompasses topics in both math and science, this paper shines a light on mathematics and the role it plays in retention. The paper highlights the importance of the foundations "thread champion," a newly assigned role held by a faculty member who teaches subjects in the technical core of the ECE junior year and holds a joint faculty appointment in the Department of Mathematics. Working in concert with a Graduate Teaching Fellow, the thread champion and fellow faculty are garnering interest and building motivation by showing students that almost every calculation they perform is critical to solving real-world engineering problems. Whether putting a problem into the context of an engineering application such as the smart phone, or highlighting the fascinating history of mathematics, this paper explains the process the team is using to demonstrate why math

matters – now and in the future. The team will know they are successful when students begin to view courses such as calculus as a building block for deeper learning instead of a subject to endure and disregard.

INTRODUCTION

In 2015, the Electrical and Computer Engineering (ECE) department at Colorado State University (CSU) received a five-year RED (Revolutionizing Engineering Departments) grant from the National Science Foundation to revamp the current engineering educational system[1,2]. Performing research that leads to scalable and sustainable change in engineering education, the department is, in effect, throwing away courses to help students learn more meaningfully and effectively. The new pedagogical and organizational model treats the undergraduate degree as a complex system, giving consideration to the interconnectivity and integration of fundamental concepts across the program. This holistic approach emphasizes knowledge integration and weaves key threads throughout the curriculum, stitching together and reinforcing relevant themes from the freshman to senior years as illustrated in Figure 1.

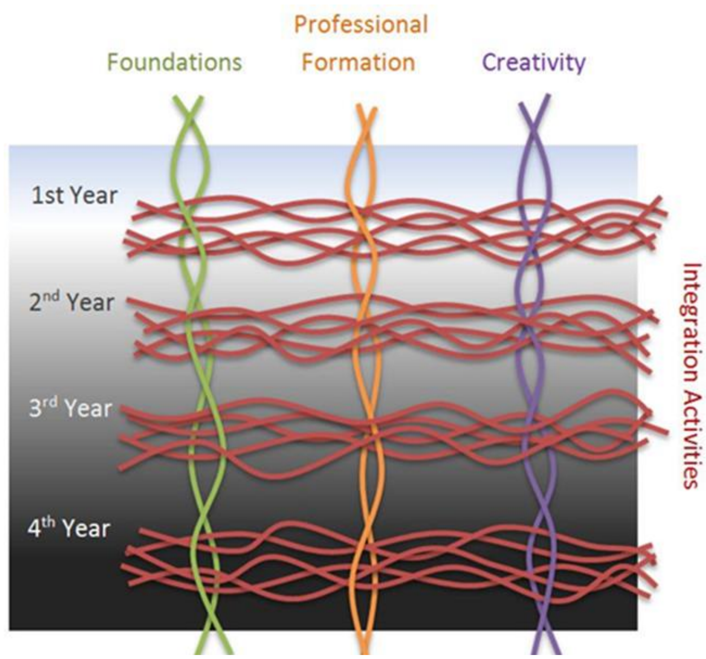

Figure 1: Pedagogical and organizational structure of ECE curriculum in the RED project

Technical courses in the middle two years represent significant challenges to students enrolled in the ECE program. The amount of content covered increase significantly over time, and many students find it difficult to grasp the concepts because they are abstract and mathematically intense. Further, students often do not see the connections between technical courses and how the material fits into the big picture. As a result, they lose confidence and motivation. To address these issues, our new pedagogical model builds on the concept of *nanocourses*[3,4], and emphasizes the integration of knowledge – a learning model well-grounded in education pedagogy and supported by research[5]. Our faculty have been working in teams to dissect, rearrange, and synchronize fundamental course concepts into cohesive and self-contained *Learning Studio Modules (LSMs)* that culminate in *Knowledge Integration (KI)* activities at key junctures in the sophomore and junior years of the ECE program. By properly aligning LSMs from different core competency areas (circuits, electronics, linear systems, and electromagnetics), a set of KI activities are created to illustrate how the topics fit together to solve real-world engineering problems. Instead of teaching courses in silos, we are now working collaboratively to show how concepts connect across topics and to professional practice. In addition, key faculty members have been assigned new roles as *integration specialists* and *thread champions*. Integration specialists work with fellow faculty to synthesize content and identify touch points for knowledge integration over the undergraduate program, taking the form of horizontal threads that illustrate how fundamental concepts from different core competency areas are interrelated. Thread champions devise content, tools, and assessment methods for weaving knowledge threads in *foundation* (math and science), *creativity* (research, design, and optimization tools), and *professionalism* (communications, cultural adaptability, ethics, leadership, and teamwork) throughout the curriculum. The reader is referred to[1,2,6] for a detailed description of the new pedagogical and organizational structure of our RED project, and to[7] for our preliminary work on the professionalism thread.

This paper focuses on the foundation thread of the RED project and describes our team's efforts to transform the educational experience for our ECE students by demonstrating the importance of mathematics and the power of mathematical thinking. The foundation thread encompasses both math and science, but the focus of this paper is on mathematics. A holistic understanding of ECE concepts is highly dependent on students' grasp of key topics in mathematics[8], and yet students are often intimidated by the wide range of fundamental math concepts required for the major and struggle to see why math matters. This paper describes the process our team is using to demonstrate the relevance of mathematics in ECE. We are at the early stages of this process and this paper is intended to simply provide an illustration of how the foundation thread is being implemented. But we hope that by continuing this effort we can develop mathematical thinking in our students and evolve their view of mathematics from a subject to endure to an apparatus for deep learning.

FOUNDATION THREAD

Gaining competency in the technical core of ECE requires command of a wide range of fundamental math concepts, including complex variables; sinusoidal and exponential functions; limits; continuity; differentiability; line, surface, and volume integrals; partial derivatives; gradients; curls; Laplacians; Taylor series expansions; polynomial factorizations; and vector representations in Cartesian, polar, and spherical coordinates. Many of our ECE students find it difficult to grasp the core concepts because they are often mathematically intense and seem abstract. A relatively large fraction of freshman students fail in calculus courses, fall behind in their program, and eventually lose interest in the major all together, because they cannot relate calculus to their perception of engineering. Many of the students who pass the calculus sequence look at it as an unnecessary hurdle that the major has subjected them to. As the students progress to the junior year, which is the most mathematically intense year of the ECE undergraduate program at CSU, they are faced with the reality that they need mastery of the calculus sequence more than ever. The junior students often complain that they do not remember or have lost their command of calculus, because it has been two years since they took the sequence, or because they did not pay much attention to it in the first place due to a lack of perceived value.

The foundation thread of the RED project is intended to address these issues. The thread is spearheaded by an ECE faculty member who not only teaches core ECE courses, but also holds a joint appointment in the Department of Mathematics and is familiar with their curriculum and teaching practices. An understanding of the teaching culture and curricula of both departments facilitates communication and collaborative efforts. The thread champion is assisted by a Graduate Teaching Fellow (GTF), an ECE Ph.D. student with a strong background in mathematics. Together, they design and deliver content that is intended to show the students how mathematics can be used to develop intuition about the fundamental concepts covered in the LSMs and KI activities. Our efforts so far have been on encouraging mathematical thinking and reinforcing mathematical knowledge at the junior year. Efforts for the freshman and sophomore years are currently underway.

Junior Year: Foundation Modules

The technical core in the junior year consists of the Linear Systems sequence (ECE311-2), the Electronics Principles sequence (ECE331-2), and the Electromagnetics sequence (ECE341-2). As part of the RED project, we have divided each of these courses into five LSMs, where each LSM covers a set of anchoring concepts, and have aligned the coverage of the LSMs to enable KI activities across courses. A list of the LSMs for the first semester of the junior year is provided in Table 1. More information about our approach to LSMs and KI activities can be found in [1,2,6].

The foundation thread champion and GTF together have dissected and rearranged the calculus sequence into several small modules, each of which covers the fundamental math concepts required in one or two LSMs. Table 2 shows the list of foundation modules and their corresponding LSMs for the first semester of the Junior year. Each foundation module is covered in one or two lectures

Table 1: LSMs for the first semester of the Junior year

	ECE 311	ECE 331	ECE 341
LSM 1	Transient and complex exponential signals	Fundamental semiconductor physics	Electrostatic field in free space
LSM 2	Linear time-invariant systems	Diodes, diode models, and applications	Electrostatic field materials media
LSM 3	Spectrum analysis of continuous-time signals	Large signal analysis for BJTs and FETs	Steady electric currents
LSM 4	Spectrum analysis of discrete-time signals	Small signal analysis for BJTs and FETs	Magnetostatic field
LSM 5	Frequency response of LTI systems and sampling	OPAMP networks	Low-frequency electromagnetic field

Table 2: Foundations modules aligned with LSMs

Foundation Module	Learning Studio Module
Complex-valued functions; Cartesian and polar representations; Magnitude and phase; Steinmetz's phasor analysis	ECE 311: LSM 1
Vectors; Magnitude and direction; Cartesian, cylindrical, and spherical coordinate systems	ECE 341: LSM 1,2
Line, surface, and volume integrals; Circulation and flux; Gradient; Directional derivative; Divergence; Laplacian	ECE 341: LSM 1,2
Running sums of sequences; Geometric series; Integration of complex exponentials and sinusoids	ECE 311: LSM 2
Nonlinear equations; Numerical methods for solving nonlinear equations; Newton-Raphson method	ECE 331: LSM 2,3
Complex exponential functions; Eigenfunctions; Orthogonality; Sinc functions	ECE 311: LSM 3
Complex exponential sequences; Eigensequences; Dirichlet kernels	ECE 311: LSM 4
Taylor series; First-order approximations; Linearization; Approximation error	ECE 331: LSM 4
Cross product of vectors; Triple products of vectors; Curl; Stokes' theorem; Laplacian of a vector	ECE 341: LSM 4,5
Complex-valued functions; Magnitude and phase spectra; Interpolation	ECE 311: LSM 5

by the GTF during the same weeks that the corresponding LSMs are taught by the technical core faculty. The examples and homework problems assigned in these modules are designed to resemble the type of problems that the students encounter during the corresponding LSMs. Attending these lectures and working the assigned homework problems earns students extra credit in the respective courses and serves as an incentive for students to spend the extra time. This approach enables delivery of the relevant mathematics at the time it is in fact needed and makes the connection between mathematics and ECE more evident. This is in contrast to the common practice of front-loading of mathematics, where the relevant mathematics for each course is reviewed in the first few lectures of the course, which the students often find to be dry.

To better describe how the math foundations thread improves student learning, we give two examples in this section.

Example 1: A major source of confusion and frustration for ECE students is complex numbers and complex-valued functions. They learn complex numbers in the calculus sequence early on, but they find the topic abstract and do not see its relevance to ECE or other practical problems. A common question from our junior students is "why do we use complex-valued signals in linear systems, electric circuits, and electronics when every signal that can be generated in a lab or by an electronic device is real-valued?" We answer this question in our first foundation module during the same week that the students are learning about continuous-time and discrete-time complex exponentials in LSM1 of Linear Systems (ECE311).

The module starts with presenting a history of complex numbers[9]. The story begins in approximately 60 AD when first references to square roots of negative numbers appear in solving problems in geometry. The formal development of complex number theory starts in 16th century out of a need for rooting polynomials. The symbol $i = \sqrt{-1}$ is introduced in early 17th century. Almost 100 years later, Leonard Euler presents all the solutions to $x^n - 1 = 0$, introducing the now well-known Euler's identity $e^{i\theta} = \cos(\theta) + i\sin(\theta)$, and eventually the fundamental theorem of algebra is established. The students find the history lesson fascinating, because they now realize that complex numbers were conceived for solving practical and fundamental problems.

The module then discusses how "complex-valued thinking" can simplify the analysis of electric circuits. The students are already familiar with modeling electric circuits as ordinary differential equations from their sophomore year electric circuits course. We present examples to show them that finding the solutions of an ordinary differential equation in response to trigonometric functions is often cumbersome, requiring command of trigonometric identities and a great deal of manipulations. But this is exactly what they need to do for analyzing the response of electric circuits to AC sources. We then show that the same problem becomes surprisingly simple once the trigonometric functions are replaced by their complex-valued versions from Euler's identity, following Steinmetz's phasor analysis.[10] This gives the students a real appreciation for complex numbers and the value that this way of thinking brings to solving ECE problems. After this value is established, we review the rules of manipulations of complex numbers and complex-valued functions.

Example 2: Consider the circuit in Figure 2. Such circuits are encountered when we introduce diode models in LSM 2 of Electronic Principles (ECE331).

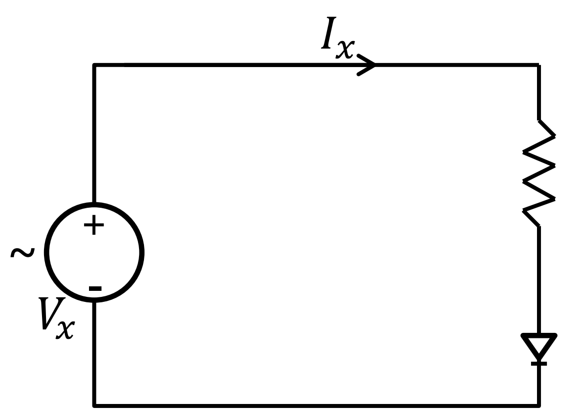

Figure 2: Circuit diagram for example 2

Suppose we wish to calculate the current through the diode, I_x, for a given value of DC input voltage V_x , give a resistance R in series. Using Kirchhoff's voltage law, one would need to solve the nonlinear equation

$$V_x = I_x R + 0.026\ln(\frac{I_x}{I_0}). \tag{1}$$

To solve this nonlinear equation, the students need to use an iterative method, e.g., the Newton-Raphson method. The problem is that students often do not remember such methods from their math courses. To address this issue, the GTF is giving a foundation lecture on iterative methods for solving nonlinear equations during the same week that diode models are covered in ECE 331, and the students specifically apply these methods to examples of the type in (1), where the nonlinear equation involves equating an affine function of x to an exponential function of x. Later on, when small signal analysis is discussed in LSM4 of ECE 331 for linearizing diode models, a math foundation lecture on Taylor series and linearization is delivered by the GTF with specific examples revolving around linear approximations to exponential functions, as encountered in the diode's voltage-current equation. Through these lectures the GTF is able to illustrate how the knowledge of mathematics facilitates learning in ECE 331.

Freshman Year: ECE-Specific Recitation Sessions

At the Freshman year, the ECE students take the calculus sequence offered by our math department. The calculus lecture sessions are accompanied by weekly recitation sessions where students work a series of problems under the guidance of a graduate teaching assistant. We are working with the calculus faculty coordinators in the math department to create specific recitation sessions for ECE students. The foundation thread champion and GTF, in concert with the recitation graduate teaching assistant, will design and introduce ECE-related math problems that are aligned with the weekly recitation schedule for calculus.

The idea is to show the students that the calculations that they are assigned have practical meanings. When they integrate a sinusoidal or exponential signal they are in effect calculating the voltage across a capacitor for a sinusoidal or exponential current pattern. Or, when they calculate a surface integral of a vector field they are in fact calculating a flux which eventually ties to Gauss's law in the junior year. They may not understand how to mathematically describe a capacitor or what flux means, but we can excite them and arouse their curiosity about the calculations by demonstrating the functionality of a capacitor or the effect of flux using a Van de Graaff generator. This approach is in essence similar to the approach taken in[11], where the authors advocate for introducing simple engineering problems during calculus lectures to motivate the students. A Mathematics Applications Inventory (MAI) test, such as the one developed at Cornell University[12], may be used in assessing the success of our effort at the freshman level.

CONCLUDING REMARKS

We have described the process that our team is using to demonstrate the relevance of mathematics in ECE education as part of the NSF RED award to our department. We are at the early stages of this process and this paper is intended to simply provide an illustration of how the foundation thread is being implemented. Our efforts so far have been focused on encouraging mathematical thinking at the junior year, but we plan to extend our work to the freshman and sophomore years as well. Our approach has been to dissect and rearrange the calculus sequence into small modules and to deliver each module at the right time, as the math becomes necessary to develop intuition in an ECE LSM. Our hope is that this approach makes the connection between mathematics and ECE more evident and compels our students to view mathematics as a powerful tool for deeper learning in ECE.

ACKNOWLEDGEMENT

This work is supported by the NSF, IUSE/Professional Formation of Engineers: Revolutionizing Engineering Departments (RED), Grant EEC-1519438. Any opinions expressed in this paper are those of the authors and are not those of the NSF.

References

[1] Chen, T., & Maciejewski, A. A., & Notaros, B. M., & Pezeshki, A., & Reese, M. D. (2016, June), *Mastering the Core Competencies of Electrical Engineering through Knowledge Integration* Paper presented at 2016 ASEE Annual Conference & Exposition, New Orleans, Louisiana. 10.18260/p.25683

[2] Maciejewski, A. A., & Chen, T. W., & Byrne, Z. S., & de Miranda, M. A., &, Sample McMeeking, L. B., &, Notaros, B. M., &, Pezeshki, A., & Roy S., & Leland, A. M., & Reese, M. D., & Rosales, A. H., & Siller, T. J., & Toftness, R. F., & Notaros, O., *A Holistic Approach to Transforming Undergraduate Electrical Engineering Education* Paper to apper in IEEE Access.

[3] Bentley, A. M., & Artavanis-Tsakonas, S. & Stanford, J. S. (2008) *Nanocourses: A Short Course Format as an Educational Tool in a Biological Sciences Graduate Curriculum* Life Sciences Education, Vol.7, pp. 175-183.

[4] Gutlerner, J. L., & Van Vactor, D., (May 2013) *Catalyzing Curriculum Evolution in Graduate Science Education* Cell, Vol.153, pp. 731-736.

[5] Borgford-Parnell, J., & Deibel, K., & Atman, C. J., (2010) *From engineering design research to engineering pedagogy: Bringing research results directly to the students* Int. J. of Engineering Education, 26 (4), pp.748-759.

[6] Chen, T., & Notaros, B. M., & Pezeshki, A., & Roy, S., & Maciejewski, A. A., & Reese, M. D. *WIP: Knowledge Integration to Understand Why* Paper submitted to the 2017 ASEE Annual Conference & Exposition, Columbus, Ohio.

[7] Rosales, A. H., & Leland, A. M., & Notaros, O., & Toftness, R. F., & Siller, T. J., & De Miranda, M. A., & Cook, A., & Reese, M. D., & Byrne, Z., & Weston, J. W., & Maciejewski, A. A. (2016, June), *Preliminary Work on Weaving Professionalism Throughout the Engineering Curriculum* Paper presented at 2016 ASEE Annual Conference & Exposition, New Orleans, Louisiana. 10.18260/p.25946

[8] Bischof, G., & Zwölfer, A., & Rubeša, D. (2015, June), *Correlation Between Engineering Students' Performance in Mathematics and Academic Success* Paper presented at 2015 ASEE Annual Conference & Exposition, Seattle, Washington. 10.18260/p.23749

[9] Green, D. R. (Jun. 1976) *The Historical Development of Complex Numbers,* The Mathematical Gazette, Vol. 60, No. 412, pp. 99-107.

[10] Araújo, A. E. A., & Tonidandel, D. A. V. (June 2013) *Steinmetz and the Concept of Phasor: A Forgotten Story* Journal of Control, Automation, and Electrical Systems. 10.1007/s40313-013-0030-5

[11] Allen, M. R., & Wilson, D. A. (2013, June), *Making Mathematics Relevant to Engineering Students* Paper presented at 2013 ASEE Annual Conference & Exposition, Atlanta, Georgia. https://peer.asee.org/22267

[12] Terrell, M., & Terrell, R., & Schneider, L. (Jun. 2010), *Assessing Engineering Students' Ability To Use The Mathematics They Have Learned* Paper presented at 2010 Annual Conference & Exposition, Louisville, Kentucky. https://peer.asee.org/16109