AC 2008-1424: WHY THE BALANCE PRINCIPLE SHOULD REPLACE THE REYNOLDS TRANSPORT THEOREM

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Why the Balance Principle Should Replace the Reynolds Transport Theorem

Abstract

The finite control volume equations for mass, momentum, and energy are important topics in introductory fluids mechanics courses for engineering students. In most contemporary U. S. textbooks these equations are derived by transformation of the corresponding equations for a control mass using the Reynolds Transport Theorem. This paper shows that there is a much simpler path to these equations: the direct application of the balance principle to a control volume. The balance principle is easier to teach, to understand, and to apply in more complex situations. It better prepares students to understand the derivation of the partial differential equations of fluid mechanics and the finite volume equations of computational fluid dynamics. For these reasons the balance principle should replace the Reynolds Transport Theorem in introductory engineering textbooks and courses in fluid mechanics.

Introduction

Everything should be made as simple as possible, but not simpler.

Albert Einstein

The equations for the conservation of mass, momentum, and energy in a finite control volume are among the main topics covered in almost all introductory fluid mechanics courses taken by engineering students in the United States. In today's most widely used textbooks, the derivation of these equations is performed by using an equation known as the Reynolds Transport Theorem (RTT). This paper will explain how this situation came to be, and will propose that a simpler, yet more powerful, approach based on an intuitively obvious balance principle should be used instead of the RTT.

A control volume is any region of space conceptually isolated from the rest of the universe in order to facilitate the solution of problems involving fluid mechanics, thermodynamics, heat transfer, or other physical phenomena. The boundary of a control volume is called its control surface. In any given situation, the specification of a control volume (a.k.a. an open system) is made by the analyst in order to best solve the problem at hand. In general, the control volume may deform or accelerate in any arbitrary manner and may contain solids as well as fluids. Learning the art of choosing an appropriate control volume is a critical objective for engineering students taking introductory courses in fluid mechanics and other transport sciences.

The control volume is an often viewed as a more convenient alternative to the control mass (a.k.a. control system or closed system) approach in which equations are written for a specified piece of matter conceptually isolated from the rest of the universe. Both the control volume and the control mass can be of finite or infinitesimal size, but in this paper attention will center on the finite case. The derivation of equations governing the matter within a finite control volume is most commonly accomplished by transforming the equations governing the control mass occupying the control volume at some instant by using an equation often called the Reynolds

Transport Theorem or simply the transport theorem. This transformation from control mass to control volume is said to be required because the fundamental laws of nature are known initially only for a control mass. This paper will show that this transformation from control mass to control volume via the RTT is an unnecessary step which can be eliminated by applying an intuitively obvious balance principle directly to the control volume.

History of the Control Volume Concept and the Reynolds Transport Theorem

Vincenti[1] divides the historical development of finite control volume analysis into four phases. In the period prior to about 1910, the control volume was used sporadically, usually in an implicit manner, in the solution of isolated problems in fluid mechanics. One of the earliest (1738) and clearest examples occurs in the solution of a liquid jet impacting a flat plate by Daniel Bernoulli in his pioneering book *Hydrodynamica*. Other notable examples cited by Vincenti are Euler's analysis of the reaction turbine in 1754, Borda's study of jet contraction in 1766, and Robert Froude's momentum theory of marine propellers in 1889. Although these applications appeared in various engineering textbooks, they were never accompanied by a systematic discussion of the control volume technique.

Vincenti calls the years from 1910 to about 1945 the period of synthesis and dissemination, and credits the German engineering scientist Ludwig Prandtl and his students with much of this work. The earliest paper cited by Vincenti is von Kármán's 1912 analysis of the vortex wake behind a bluff body. In 1913, Prandtl published a German-language handbook article on fluid mechanics which contained an explicit, systematic application of control volume analysis to a collection of problems without using an explicit control volume momentum equation.

Prandtl further developed the control volume concept in his teaching at the University of Göttingen. Prandtl's lectures were the basis of two books prepared by his assistant Oskar Tietjens and published in German in 1929 and 1931. These volumes were translated into English and published as Engineering Societies Monographs in 1934 as *Fundamentals of Hydro- and Aeromechanics* (Prandtl and Tietjens[2]) and *Applied Hydro- and Aeromechanics* (Prandtl and Tietjens[3]). Apparently for the first time in English, Prandtl and Tietjens[2] introduced a restricted form of what is now known as the Reynolds Transport Theorem (RTT) as a way of relating Newton's second law of motion for a control mass to the momentum equation for a control volume. That equation is restricted to steady (or mean-steady) flow (although it is applied to the periodic shedding of vortices in reference 2) and to fixed, nondeforming control volumes. As noted in Table 1, a similarly restricted form of the energy equation was also derived. These equations were systematically applied in Prandtl and Tietjens[2, 3] to ten problems including calculating the force on bends and vanes and the drag on various objects. Although historically significant, the present author finds that these analyses are sometimes difficult to follow. To cite one instance, the control volume shown in Figure 173 of reference 2 does not expose the force being solved for. In fairness, these perceived shortcomings may well be due to the process of transcription and translation. Vincenti points out that all of the problems in references 2 and 3 are for incompressible fluids.

Table 1. Milestones in the appearance of the Reynolds Transport Theorem in English. "General property" indicates that the control volume equation was first derived for an unspecified property which was later identified as mass, momentum, etc.

				Equation					
Author	Year	General property	Deformable control volume	Mass	Mom. (inertial)	Mom. (non-inertial)	Ang. mom.	Energy	Entropy
Prandtl & Tietjens[2]	1934a	no	no	no	yes	no	no	yes	no
Prandtl & Tietjens[3]	1934b	no	no	no	yes	no	no	no	no
Hunsaker & Rightmire[4]	1947	no	no	yes	yes	no	no	yes	no
Shapiro[5]	1953	no	no	yes	yes	no	yes*	yes	yes*
Brenkert[6]	1960	no	no	yes	yes	no	yes	yes	no
Shames[7]	1962	yes	no	yes	yes	yes	yes	yes	no
Van Wylen & Sonntag[8]	1965	no	yes	yes	yes	no	no	yes	yes
Hansen[9]	1965	yes	yes	yes	yes		no	no	no
Eshghy[10]	1966	yes	yes	yes	yes	no	yes	yes	yes
Daily & Harleman[11]	1966	no	no	yes	yes	yes	yes*	yes	no
Hansen[12]	1967	yes	yes	yes	yes	yes	yes*	yes	no

*Stated without derivation.

The years from 1945 to about 1955 are called the period of generalization and completion of control volume analysis by Vincenti, who credits engineering professors at MIT with the major role in this process in the United States. Table 1 records that the introductory fluid mechanics textbook of Hunsaker and Rightmire[4] used the Reynolds Transport Theorem concept to relate the known laws of conservation of mass, linear momentum, and energy for a control mass to the corresponding control volume equations. Interestingly, they did not use this approach when discussing angular momentum. Hunsaker and Rightmire restricted their analyses to steady flow through nonaccelerating, nondeforming control volumes. Although the equations were derived in fairly general form, they do not seem to have been used much elsewhere in the text.

In his treatise on gas dynamics, MIT professor Ascher Shapiro[5] provided very clear and careful derivations of the control volume equations for the conservation of mass, linear momentum, and energy (first law of thermodynamics) for compressible fluids using the RTT approach. He also stated without derivation, comparable forms of the angular momentum equation and the second law of thermodynamics for a control volume. Like his predecessors, Shapiro considered only nondeforming, nonaccelerating control volumes. Vincenti notes that the explicit control volume approach began to appear in thermodynamics textbooks at this time.

Spread of the RTT in U. S. Introductory Fluid Mechanics Textbooks

Vincenti labels the era after about 1955 the period of diffusion of control volume analysis in engineering education and practice. Vincenti cites few references for this period, but Table 1 summarizes the key milestones in the spread and generalization of the RTT approach in introductory fluid mechanics textbooks and related papers in the United States. The rigor and generality provided by the RTT made it ideally suited to the engineering science movement of the 1950's and 60's. In 1960, Brenkert[6] published the first introductory fluid mechanics textbook of the post-Sputnik era to use the RTT approach. The engineering science ethos then coming into vogue is exemplified in Brenkert's Preface which states that "…fundamentals are developed with a minimum of attention to practical applications…" (Would any author dare to make such a statement today?) Brenkert contained the first RTT derivation of the angular momentum equation.

In the Preface of his widely-used 1962 book, Shames[7] declared allegiance to the engineering science movement stating, "I have endeavored to write a rigorous and fundamental text on fluid mechanics to serve for a first course in science-oriented engineering programs." Shames took the RTT approach to a new level by first deriving the RTT for a general property and then specializing it to mean mass, linear momentum, angular momentum, and energy. Shames was also the first textbook to write and apply the momentum equation for noninertial control volumes.

The next development of the RTT approach was made by a group of University of Michigan professors who wrote the Series in Thermal and Transport Sciences. The first of this unified series of introductory textbooks to appear was the 1965 thermodynamics book by Van Wylen and Sonntag[8]. Although not a fluids book, Van Wylen and Sonntag deserves mention here because it introduced the deformable control volume, providing separate derivations for conservation of mass, momentum, energy, and (for the first time) the second law of thermodynamics. All derivations of the RTT up to this time were based on a picture of the geometric relationship between a moving control mass and a fixed or differently moving control volume. In 1965 Hansen[9] published a more abstract, essentially mathematical derivation of the RTT for a general property and a deforming control volume in the *Bulletin of Mechanical Engineering Education*. In the 1966 volume of the same journal Eshghy[10] achieved the same result using a refined version of the traditional geometric argument. Also in 1966, Daily and Harleman[11] published another engineering science inspired fluid mechanics book, written under a grant from the Ford Foundation. The final step in the infusion of the RTT approach into the U. S. undergraduate textbook literature was the 1967 publication of Hansen's[12] fluid mechanics volume in the Series in Thermal and Transport Sciences. Using the geometric approach rather than his mathematical derivation of 1965, Hansen[12] was the first fluid mechanics textbook to derive equations for deforming control volumes. There have been no significant advances in the use of the RTT for finite control volume analysis since Hansen's book was published.

The perceptive reader will have noticed that there is no reference to any work by Reynolds in this account of how the Reynolds Transport Theorem entered undergraduate engineering fluid mechanics education in the United States. What then is the connection between Reynolds and the RTT? In an erudite and mathematically sophisticated 1960 monograph on field theory, Truesdell and Toupin[13], derive a result equivalent to the RTT. They refer to their equation (81.3) as the "transport theorem" and attribute it to Osborne Reynolds[14]; but neither the specific equation they cite, nor any other equation that the present author can find in Reynolds[14] is equivalent to the RTT (equation 5 below). The earliest use of the name "Reynolds' transport theorem" known to the present author is in the 1962 monograph by Aris[15]. Therefore, it appears that the RTT is misnamed.

Control Volume Analysis in Contemporary U. S. Introductory Fluid Mechanics Textbooks

In the years following 1967, using the RTT to derive the mass, momentum, angular momentum, and energy equations for finite control volumes has become *de rigueur* in introductory fluid mechanics textbooks intended for engineering students other than chemical engineering majors. Table 2 shows that the RTT is used exclusively by eight contemporary books[16-23], among which are the five best selling undergraduate engineering fluid mechanics textbooks in the United States.

Table 2. Appearance of the Reynolds Transport Theorem in selected contemporary U. S. introductory fluid mechanics textbooks.

				Equation					
Author	Year	General property	Deformable control volume	Mass	Mom. (inertial)	Mom. (non-inertial)	Ang. mom.	Energy	Entropy
Street …[16]	1996	yes	no	yes	yes	no	yes	yes	no
Potter & Wiggert[17]	1997	yes	no	yes	yes	yes	yes	yes	no
Finnemore …[18]	2002	yes	no	yes	yes	no	no	yes	no
Fox …[19]	2004	yes	no	yes	yes	yes	yes	yes	yes
Crowe …[20]	2005	yes	yes	yes	yes	yes	yes	yes	no
Munson …[21]	2006	yes	yes	yes	yes	no	yes	yes	yes
Cengel & Cimbala[22]	2006	yes	yes	yes	yes	no	yes	yes	no
White[23]	2008	yes	yes	yes	yes	yes	yes	yes	no

The use of the balance principle to formulate control volume equations seems to have originated in the somewhat specialized textbooks written for chemical engineering students, whose unique curriculum often excludes them from general engineering courses. Table 3 documents the use of the balance principle in references 24-28. The popularity of the balance principle in chemical engineering education may be due to the fact that it can be applied more easily to the multiphase, multicomponent processes which are common in that profession. None of these books treats deformable control volumes.

Table 3. Use of the balance principle (BP) in introductory fluid mechanics textbooks for chemical engineers.

				Equation					
Author	Year	General property	Deformable control volume	Mass	Mom. (inertial)	Mom. (non-inertial)	Ang. mom.	Energy	Entropy
de Nevers[24]	1970	no	no	BP	BP	no	BP	BP	no
Denn[25]	1980	yes	no	BP	BP	no	no	BP	no
de Nevers[26]	1991	no	no	BP	BP	no	BP	BP	no
Wilkes[27]	1999	no	no	BP	BP	no	BP	BP	no
Darby[28]	2001	no	no	BP	BP	no	BP	BP	no

Recently a few non-chemical engineering textbooks have also adopted the balance principle, as shown in Table 4. The book by Gray[29] is oriented toward civil and environmental engineers whereas Smits[30] and Graebel[31] address a general engineering audience. Gray is the only one to apply the balance principle to a deformable control volume. It is interesting to note that Smits uses the RTT to derive the energy equation.

Table 4. Selected contemporary U. S. introductory fluid mechanics textbooks using the balance principle (BP). (RTT = Reynolds Transport Theorem)

				Equation					
Author	Year	General property	Deformable control volume	Mass	Mom. (inertial)	Mom. (non-inertial)	Ang. mom.	Energy	Entropy
Gray[29]	2000	yes	yes	BP	BP	no	no	BP	no
Smits[30]	2000	yes	no	BP	BP	no	no	RTT	no
Graebel[31]	2001	no	no	BP	BP	BP	BP	BP	no

Derivation of the Reynolds Transport Theorem

In order to better appreciate the RTT, consider the following "geometric" derivation patterned after that of Eshghy[10]. This is the basically the same approach used in all of the books in Table 2. Let B represent any scalar or vector extensive property such as mass, momentum, or energy. In Figure 1, the control mass and the control volume coincide at time t, but at t + Δt the control mass and the control volume have each moved to new positions. The numbered subregions have no overlap.

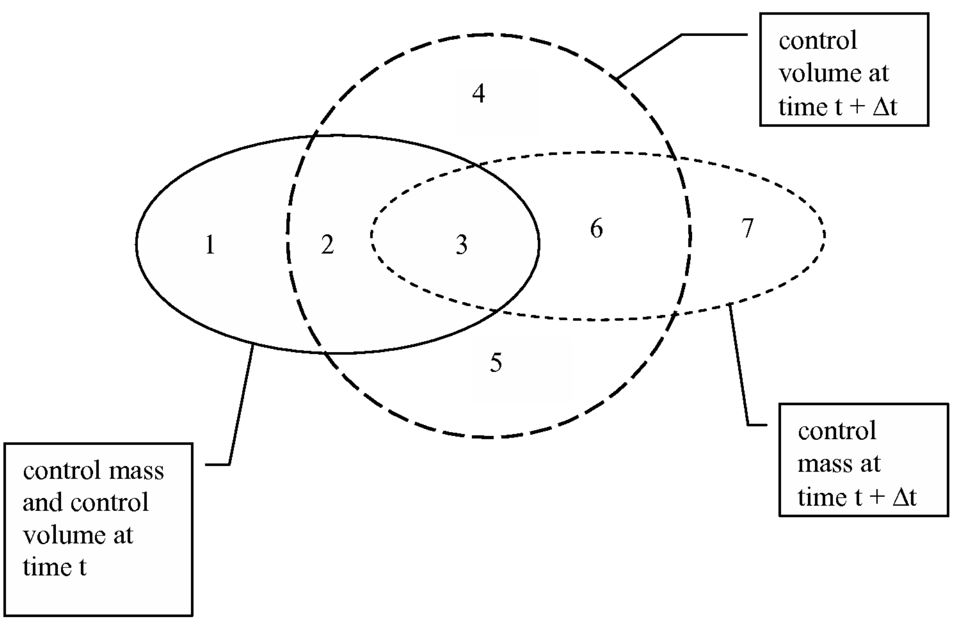

Figure 1. Sketch for the derivation of the Reynolds Transport Theorem for a moving, deforming control volume. The numbered regions are non-overlapping.

By inspection,

$$B_{CM}(t) = B_{CV}(t) = B_1(t) + B_2(t) + B_3(t) \quad (1)$$

$$B_{CM}(t + \Delta t) = B_3(t+ \Delta t) + B_6(t+ \Delta t) + B_7(t+ \Delta t) \quad (2)$$

$$B_{CV}(t + \Delta t) = B_2(t+ \Delta t) + B_3(t+ \Delta t) + B_4(t+ \Delta t) + B_5(t+ \Delta t) + B_6(t+ \Delta t) \quad (3)$$

Here CM indicates control mass and CV indicates control volume.

Now write an expression for the rate at which B_{CM} increases. The symbol lim indicates the limit as $\Delta t \to 0$.

$$\begin{aligned}
\frac{dB_{CM}}{dt} &= \lim\left\{\frac{B_{CM}(t+\Delta t) - B_{CM}(t)}{\Delta t}\right\} \\
&= \lim\left\{\frac{[B_3(t+\Delta t) + B_6(t+\Delta t) + B_7(t+\Delta t)] - B_{CV}(t)}{\Delta t}\right\} \\
&= \lim\left\{\frac{[B_2(t+\Delta t) + B_3(t+\Delta t) + B_4(t+\Delta t) + B_5(t+\Delta t) + B_6(t+\Delta t)] - B_{CV}(t)}{\Delta t}\right\} \\
&\quad - \lim\left\{\frac{B_2(t+\Delta t) + B_4(t+\Delta t) + B_5(t+\Delta t)}{\Delta t}\right\} + \lim\left\{\frac{B_7(t+\Delta t)}{\Delta t}\right\} \\
&= \lim\left\{\frac{B_{CV}(t+\Delta t) - B_{CV}(t)}{\Delta t}\right\} - \dot{B}_{in} + \dot{B}_{out}
\end{aligned}$$

$$\frac{dB_{CM}}{dt} = \frac{dB_{CV}}{dt} - \dot{B}_c \quad (4)$$

The term $\dot{B}_c = \dot{B}_{in} - \dot{B}_{out}$ represents the net amount of B convected into the control volume by the flowing fluid per time. Equation 4 is sometimes called the Reynolds Transport Theorem, but it is usually restated in the following more explicit form.

$$\frac{dB_{CM}}{dt} = \frac{d}{dt}\int_{CV} b\rho dV + \int_{CS} b\rho \mathbf{V}_r \cdot d\mathbf{A} \quad (5)$$

Here b is B per mass
- ρ is the density
- dV is a volume element
- $\mathbf{V}_r$ is the fluid velocity relative to the control surface
- d**A** is the outward directed surface area element vector.

Equation 5 is the usual form of the Reynolds Transport Theorem for a deformable control volume. Although the surface integral is taken over the entire control surface, nonzero contributions occur only where material actually crosses the control surface. To obtain the various control volume equations, one must define the appropriate intensive property b and specify dB_{CM}/dt from a knowledge of the physical laws for a control mass.

To illustrate this process, let the extensive property be linear momentum.

$$B = m\mathbf{V} \quad \Rightarrow \quad b = \mathbf{V} \tag{6}$$

Here m is the mass and $\mathbf{V}$ is the fluid velocity. Newton's second law for a control mass gives

$$\frac{dB_{CM}}{dt} = \frac{d(m\mathbf{V})}{dt} = \sum \mathbf{F}_S + \sum \mathbf{F}_B \tag{7}$$

Here $\mathbf{F}_S$ is a surface force and $\mathbf{F}_B$ is a body force.

Substitution into the RTT (equation 5) gives the momentum equation for a deformable control volume.

$$\sum \mathbf{F}_S + \sum \mathbf{F}_B = \frac{d}{dt}\int_{CV} \rho \mathbf{V}\, dV + \int_{CS} \rho \mathbf{V}\left(\mathbf{V}_r \cdot d\mathbf{A}\right) \tag{8}$$

The Balance Principle

The balance principle asserts that the rate of increase of any extensive property in a control volume is equal to the net rate of creation of B within the control volume, plus the net rate at which B crosses the control surface into the control volume. The earliest statement of the balance principle known to the present author is by none other than Osborne Reynolds[14], who counts it Axiom I in his ambitious (but incorrect) theory of the universe. The balance principle needs no derivation because it is a statement of logical necessity whose obvious truth is in stark contrast to the tedious and rather dizzying argument needed to justify the RTT. The underlying concept of the balance principle, namely the accounting of some quantity within a defined spatial region, is familiar to students from such everyday examples as finding the population of a state, the attendance at a sports arena, or the number of burger patties in a fast food restaurant. In order to apply the balance principle correctly, it is important to recognize that in general B can be transported across the control surface by nonconvective transport (i.e. without macroscopically observable motion) as well as by convection.

The equation expressing the balance principle for a control volume can be written as

$$\frac{dB_{CV}}{dt} = \dot{B}_+ + \dot{B}_c + \dot{B}_{nc} \tag{9}$$

Here $\dot{B}_+$ is the net rate of creation of B within the CV

$\dot{B}_c$ is the net rate at which B is convected into the CV

$\dot{B}_{nc}$ is the net rate of nonconvective transport of B into the CV.

No conservation laws are needed for the balance principle to be valid; it is only necessary to correctly formulate expressions for the various terms. The creation term allows one to easily incorporate physical processes such as radioactive decay as well as mathematical artifices such as point sources into the continuity equation. Although Gray[29] is the only introductory textbook to do so, writing separate terms for convective and nonconvective transport provides an important advantage over the RTT because it makes explicit the possibility of such common phenomena as heat conduction and mass diffusion. The nonconvective transport term also provides a convenient way to model phenomena such as evaporation, boundary layer suction, or seepage.

As an illustration of the use of the balance principle to formulate the basic control volume equations, reconsider the case where B represents momentum. According to Newton's second law, force is the agency that creates (or destroys) momentum. Body forces create momentum within the interior of the control volume whereas surface forces are the macroscopic manifestation of the creation of momentum at the control surface by intermolecular forces or by actual exchange of molecules. (The continual exchange of molecules across the boundary between a fluid control mass and its fluid surroundings contradicts the very concept of a control mass, although it is of no practical consequence.) With the following substitutions in the balance principle (equation 9), the momentum equation (equation 8), is readily recovered.

$$B_{CV} = \int_{CV} \rho \mathbf{V}\, dV, \quad \dot{B}_+ = \sum \mathbf{F}_B, \quad \dot{B}_c = -\int_{CS} \rho \mathbf{V}(\mathbf{V}_r \cdot d\mathbf{A}), \quad \dot{B}_{nc} = \sum \mathbf{F}_S \qquad (10)$$

Tables 3 and 4 show that no introductory fluid mechanics textbook has yet obtained the second law of thermodynamics for a control volume using the balance principle. To do so let B represent entropy. The second law of thermodynamics states that entropy can be created within the control volume by dissipative processes such as friction. Entropy can cross the control surface by convection or without bulk motion as a consequence of heat. The terms of the general balance principle applied to entropy are the following.

$$B_{CV} = \int_{CV} \rho\, s\, dV, \quad \dot{B}_+ = \int_{CV} \frac{\dot{D}}{T} dV, \quad \dot{B}_c = -\int_{CS} \rho\, s\, \mathbf{V}_r \cdot d\mathbf{A}, \quad \dot{B}_{nc} = -\int_{CS} \frac{\mathbf{q} \cdot d\mathbf{A}}{T} \qquad (11)$$

Here s is the entropy per mass, $\dot{D}$ is the dissipation rate, T is the absolute temperature, and $\mathbf{q}$ is the outward directed heat flux vector. Substitution into the balance principle (9) and rearrangement gives the desired result.

$$\frac{d}{dt} \int_{CV} \rho\, s\, dV + \int_{CS} \rho\, s\, \mathbf{V}_r \cdot d\mathbf{A} = -\int_{CS} \frac{\mathbf{q} \cdot d\mathbf{A}}{T} + \int_{CV} \frac{\dot{D}}{T} dV \qquad (12)$$

It is likewise easy to go from equation (9) to the usual forms of the finite control volume equations for mass, angular momentum, and energy. Although it is possible to apply the Reynolds Transport Theorem to an infinitesimal control volume, in most cases the differential forms of the continuity, momentum (Navier-Stokes), and energy equations are derived by applying the balance principle to an infinitesimal control volume without mentioning the balance principle by name.

Balance Principle Examples

In an introductory fluid mechanics course, the primary advantage of the balance principle over the Reynolds Transport Theorem is that it is more easily understood. For those who go on to study subjects such as heat and mass transfer, hydrology, pollutant transport[32], or electrohydrodynamics, the balance principle provides a more flexible tool, as the following examples demonstrate.

Example 1. The task is to solve for the mass of ozone in a factory building. Choose a fixed control volume that envelopes the building. Let $B_{CV} = m_O$, the total mass of ozone in the control volume. The balance equation becomes

$$\frac{dm_O}{dt} = \dot{m}_{O_+} + \dot{m}_{O_c} + \dot{m}_{O_{nc}} \qquad (13)$$

Here $\dot{m}_{O_+}$ represents the net rate of ozone creation by electric arcs, ultraviolet light, cosmic rays, radiation sources, and chemical reactions. The term $\dot{m}_{O_c}$ is the net convective inflow through open doors, windows, and ventilation intakes and discharges. The term $\dot{m}_{O_{nc}}$ accounts for nonconvective transport such as the delivery of compressed ozone tanks or by diffusion and microleaks through the building envelope. While some of these mechanisms may be negligible, the balance principle as stated in equation 9 makes it easy to identify the possibilities.

Example 2. The task is to solve for the mass of liquid water in a lake. Choose a deforming control volume that lines the bottom of the lake, coincides with the free surface and the ice-liquid interface, and cuts across any rivers entering or draining the lake. Let $B_{CV} = m_W$, the total mass of liquid water in the lake.

$$\frac{dm_W}{dt} = \dot{m}_{W_+} + \dot{m}_{W_c} + \dot{m}_{W_{nc}} \qquad (14)$$

In this case $\dot{m}_{W_+}$ includes the net creation of liquid through the melting or freezing of ice within the control volume. The $\dot{m}_{W_c}$ term represents the net inflow due to rivers or water intakes and discharges. The term $\dot{m}_{W_{nc}}$ incorporates seepage in or out through the lake bed, and evaporation, condensation, and precipitation at the free surface.

Example 3. The task is to derive an equation for the net electrical charge in an arbitrary control volume. Let $B_{CV} = Q$, the total charge in the control volume, and $b = q / \rho$, where q is the charge density (charge / volume). The balance equation for charge is

$$\frac{d}{dt}\int_{CV} q\,dV = \dot{Q}_{+} - \int_{CS} q\left(\mathbf{V}_r \cdot d\mathbf{A}\right) - \int_{CS} \mathbf{J}\cdot d\mathbf{A} \qquad (15)$$

Here $\dot{Q}_{+}$ is the net rate of charge creation due to ionization or electrochemical processes. The second term on the right is the net convection of free charge into the control volume, and the last term is the net inflow of charge due to electrical conduction currents ($\mathbf{J}$ is the outward directed current density vector [amperes/m^2]).

Conclusions

During the engineering science era of the 1950's and 60's, the Reynolds Transport Theorem (RTT) began to appear in U. S. introductory fluid mechanics textbooks as a unified methodology for deriving the finite control volume forms of the continuity, linear momentum, angular momentum, and energy equations. Today almost all American introductory fluid mechanics textbooks directed at mechanical, aerospace, civil, or general engineering students use this approach. On the other hand, most introductory fluid mechanics textbooks for chemical engineers, as well as a few other textbooks, use the balance principle instead of the RTT to reach the same results. The Reynolds Transport Theorem is mathematically correct, and mastering its derivation is a good mathematical exercise. Moreover, it is reassuring to have proof that the laws governing control masses can be translated into the laws governing the contents of control volumes. But the RTT is a difficult means to this end. Learning the RTT is not an end in itself, particularly in an introductory course. The fact that the RTT is not essential becomes obvious when one recalls that generations of engineering students learned fluid mechanics before the RTT was introduced.

Recognizing that the RTT is merely a tool, engineers should consider whether a better tool is available. Having taught and used both approaches, the present author has found that the balance principle *is* a better tool. The balance principle is much easier for teachers to explain, for students to grasp, and for analysts to apply than is the RTT because it is based on common sense instead of rather abstract mathematics. Furthermore (perhaps surprisingly), the balance principle is a better tool than the RTT for later courses which consider phenomena such as diffusion, radiation, and phase change in addition to convection because it explicitly recognizes nonconvective modes of transfer across the control surface. Finally, applying the balance principle to finite control volumes better prepares students for advanced analytical and computational studies in the fluid and transport sciences where the partial differential and finite volume equations are usually derived using the balance principle. These are important reasons why the balance principle should replace the Reynolds Transport Theorem in introductory fluid mechanics textbooks and courses for engineers.

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