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WIP: A visual and intuitive approach to teaching first order systems to Mechanical Engineering students

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WIP: A visual and intuitive approach to teaching first order systems to Mechanical Engineering students

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Abstract

First Order Differential Equations is a topic prevalent in mathematics and several engineering classes. Mechanical engineering specifically is a field where understanding first order systems is crucial. It is a cornerstone of topics including dynamic systems, vibrations, and fluid dynamics. Despite this, many students struggle with conceptual understanding of this subject. The equations and mathematics can be overwhelming and frustrating, in part because it is often hard to visualize first order systems.

Today's students are exposed to many distractions. If students feel bored or frustrated with a lecture, oftentimes they will browse the internet on their laptops or pull out their phones to entertain themselves with social media (Facebook, Instagram, etc.) or games. They learn differently, more intuitively, experiencing short attention spans. They expect the material and presentation methods to be clear, visual, and intuitive.

The goal of this paper is to help instructors explain, and students understand, the fundamental concept of First Order Differential Equations in an intuitive and example-based approach by simplifying the introduction to the topic to something that is clear and easy to intuitively comprehend. To accomplish this, the paper starts with a visual background into first order systems and an explanation of exponential growth vs. exponential decay. It then transitions into: Mechanics examples which are chosen to cover multiple different mechanical engineering topics, such as shock absorbers (vibrations), acceleration rates of different vehicle types (dynamics), and toilet mechanisms (mechanical feedback). Next, the paper moves into thermodynamic examples, such as time constants of different stove types and the cooling rate of a hot coffee cup. Finally, the paper relates the topic to examples from other STEM disciplines, such as charging a cell phone (electrical Engineering), measuring change in pressure between two connected vessels (physics), carbon dating using half-life measurement (chemistry), and DC motors transfer function (electro-mechanical).

The point of this approach is to provide students with visual and intuitive examples that relate textbook explanations to real life scenarios. We believe that when using these intuitive examples, students tend to better understand the topic of first order systems. This paper is a *work in progress.* The presented information is meant to be supplemental in nature and *not to replace existing textbooks,* or other teaching and learning methodologies. *The contents of this work have been shared with students in a remote (Zoom-based) classroom setting and assessed* following the lecture using an anonymous questionnaire. The initial results, based on 40 responses, indicate that this teaching method is effective in helping students comprehend the basic idea behind the concept of First Order Differential Equations. This approach to teaching and learning has been

tested in the past for topics in Statics (explaining center of gravity), Statistics (explaining normal distribution), Calculus (explaining integration and explaining derivation by chain, product, and quotient rules), Thermodynamics (explaining entropy), Differential Equations, Control Systems, Digital Signal Processing, Newton's Laws of Motion, and Computer Algorithms. In all of these cases, students found this approach to be very effective for learning, and they highly praised the intuitive and engaging examples.

1. Introduction

Most mathematics textbooks are loaded with mathematical formulas and explanations with little focus on conceptual understanding. Textbooks focusing on differential equations are no different. This method is useful because it is written in a precise manner, but at the same time students may become frustrated with the material as they do not intuitively grab some of the concepts and miss the "aha" moment – a moment of sudden insight or understanding. Today's student is looking for a connection to the real world. For this reason, it is important for instructors to modify their approach to teaching by first introducing the concept in a clear way that is easy to understand, and only later focus on the textbook math.

The focus of this paper is to provide a set of supplementary material that can help instructors to teach and students to comprehend, a specific mathematics concept, namely First Order Differential Equations. It is a collection of examples, some known and some unknown, that teachers and learners can choose from. It is not intended for teachers to teach every example in this work, but rather to select the ones they feel would be most effective for their learning group. By introducing this concept in visual and intuitive ways, the material becomes more meaningful for the student, particularly when they can relate it to real-life. This paper is a *work in progress, as more assessments and results are forthcoming.* Our research approach was to collect visual and intuitive examples and present them to students who will then provide feedback regarding the effectiveness of the approach. This feedback was meant to guide future iterations of the work and provide direction for additional assessments that follow.

Assessment: The contents of this work have been shared with students in a remote (Zoom-based) classroom setting and assessed following the lecture using an anonymous questionnaire. The initial results, based on 40 responses, are detailed in Appendix D, and indicate that this teaching method is effective in helping students comprehend the basic idea behind the concept of First Order Differential Equations. Students generally felt that understanding the concept of First Order Differential Equations was important, as shown in their responses to question 1. They also felt that learning differential equations through methods such as visual examples (question 2), hands-on activities (question 3), and in-class exercises (question 4) was important, while the general opinions on learning through methods such as traditional presentations (question 7) and reading the relevant textbook material (question 8) were more mixed. Soon this work will be presented to a larger group of students learning First Order Differential Equations, and their feedback will be assessed using more rigorous formative and summative assessments.

An extra credit assignment was given to the same students to come up with their own real-life examples of First Order Differential Equations, including an explanation of why it qualifies as a First Order Differential Equation. Students were required to describe 15 examples from different disciplines including engineering, mathematics, physics, chemistry, biology, and medicine. Students' feedback on this assignment indicated that it helped them comprehend First Order Differential Equations in a more thorough way.

As mentioned earlier, the main contribution of this work is to provide students with visual and intuitive examples that relate textbook explanations to real life scenarios. Instructors can pick and choose examples to supplement their textbook-based teaching to help students in comprehending the material. This paper, which is an expansion of the work in [1], starts with a brief mathematical background of first order systems – explaining exponential equations and the time constant using graphs. It then moves into real-life examples of first order systems in different STEM areas focusing on mechanics and thermodynamics, and covering other topics such as electromechanical devices, electrical engineering, physics, and chemistry. These examples briefly discuss the relevant equations, solutions, graphs, and time constants where applicable without allowing them to become the focus of the work. In addition, some non-STEM examples are included in an appendix. It is important for students to gain knowledge through their exposure to different areas of expertise and life experiences, and, in fact, ABET accreditation requires it. For example, one of the ABET outcomes specifically states, "An ability to acquire and apply new knowledge as needed, using appropriate learning strategies." For this reason, we have included more than just mechanical engineering examples. The results from the recently performed survey shows that these examples are helping students learn. Beyond just a collection of examples, this work asserts that learning through visual and intuitive methods is more effective than traditional presentations and book learning, specifically for introductory purposes.

Extensive work has been done attempting to find more effective ways of teaching foundational STEM courses. Trying to find out why so many students struggle with mathematics and what can be done about it shows that there is no one single approach [2]. One study of visualization in a freshman Chemistry course showed results that, "suggest that visualization skills do facilitate concept learning, but they do not generalize to higher education in the sciences" [3], which shows that visual approaches are helpful for the introduction of new concepts. Studies have also found that visualization improves student motivation in computer science [4], mathematics [5] and in collaborative online learning environments [6]. Intuitive and engaging approaches have been used to introduce topics such as center of gravity [7], the normal distribution [8], control systems [9], and entropy [10] with positive results.

Mathematics and a Visual Background of First Order Systems

Though the mathematics of these concepts are well known, we felt that it was important to include a summary for the purpose of a more complete work. A differential equation includes both a function and at least one derivative of that function. If only the first derivative is included in the equation, it is considered a First Order Differential Equation. Simply put, the solution to this type of equation depends on how fast the value of that solution is changing. An example of a First Order Different Equation is:

$$ax(t) = \frac{dx(t)}{dt}$$

The solution to a linear first order DE is an exponential function of the form:

$$x(t) = ce^{at}$$

Where: c – constant
a - constant

This solution, the exponential function, has two general shapes, depending on whether "a" has a positive or negative value. The general shape of an exponential function is shown in Figure 1. The left figure visualizes a case where the constant "a" is positive, the middle figure shows the case where "a" is negative, and the right figure show response to a unit step.

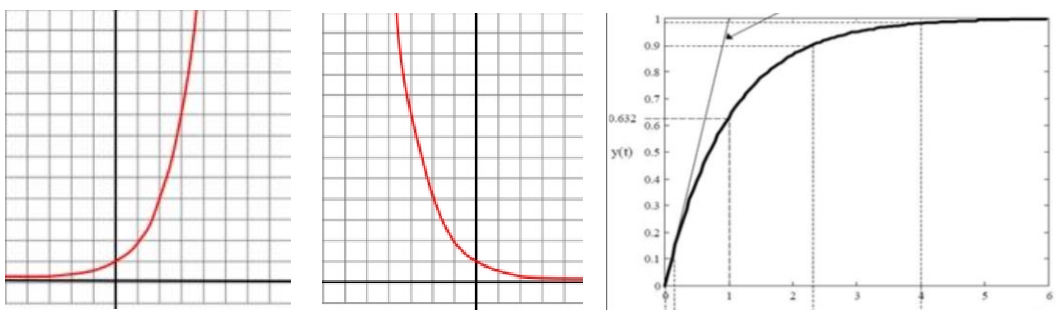

Figure 1: General exponential function with positive exponent (left), negative exponent (middle), and unit step response for τ=1 (right)

The time constant, generally refers to with the symbol τ, is a characteristic of the response of first order systems. After one time constant, an exponential response has reached ~63.2% of the way from the initial condition to its final value.

2. Mechanical Examples

Three Vehicles

This example focuses on the time it takes different types of vehicles to accelerate to a specified speed. First, consider a typical sedan. If accelerating from a stop with constant pressure on the gas pedal, is the speed linear, exponential, or a step function?

The answer is exponential. As the vehicle's speed increases, the acceleration goes down. If this doesn't seem immediately intuitive, consider the force felt when accelerating from a stop. If you "floor it," you are immediately thrown back into your seat. As speed increases, this force holding you in your seat becomes less and less. After a few seconds you can once again lean forward.

Now consider three different vehicles: a sports car, a minivan, and a moving truck. Intuitively, it makes sense that these vehicles accelerate at different rates. As shown in the graph of Figure 2, the sports car reaches top speed much faster than the minivan. Likewise, the minivan reaches top speed faster than the moving truck.

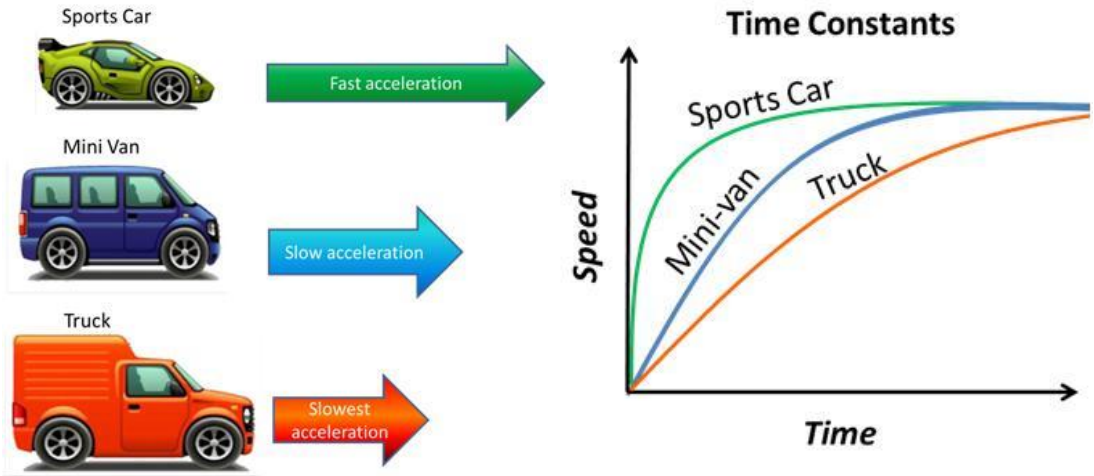

Figure 2: Different types of vehicles have different acceleration rates

The different speeds of the vehicles represent First Order Differential Equations.

Toilet Mechanism

The toilet flush system is a good mechanical example of a first order system. Figure 3 shows a detailed layout of a typical toilet mechanism.

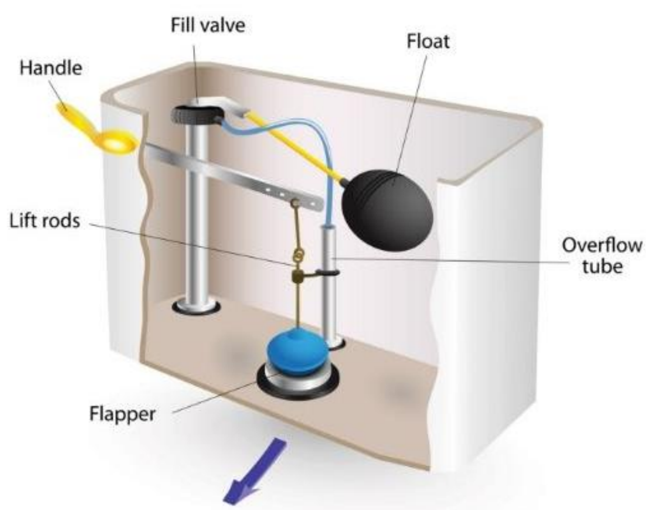

Figure 3: Flushing mechanism for a typical toilet, courtesy of fastplumbers.net.au

When the user flushed a toilet, the flapper opens, allowing water to drain from the tank. Once the water level is sufficiently low, the flapper shuts once more. Simultaneously, the float lowers along with the water level. As the float lowers it causes the fill valve to open, sending water into the tank. Once the flapper valve has shut, the water level begins to rise as more and more water flows into the tank. This causes the float to rise once more, gradually shutting the fill valve. This is shown in Figure 4.

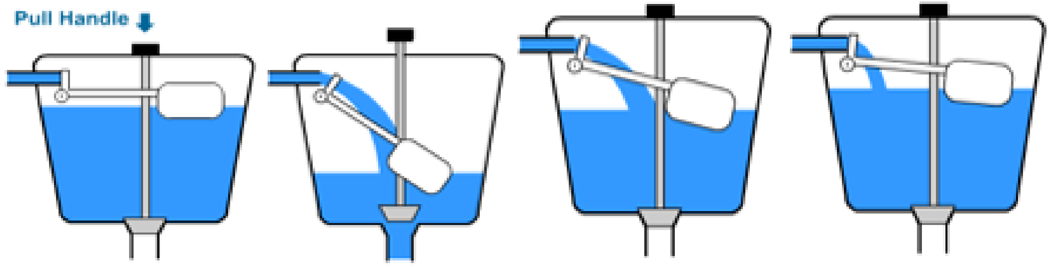

Figure 4: The float opens or shuts the fill valve based on water level

The fill valve's position is not simply open or shut. As the water level (and thus the float) rises, the fill valve begins shutting which causes less water to flow into the tank. Since the rate at which water enters the tank changes based on the amount of water currently in the tank, this mechanical system is a good approximation of a first order system.

Shock Absorber

Shock absorbers, like the one shown in Figure 5, are a specific type of damper. Dampers are devices that absorb energy to damp or reduce impulses. In automobiles, shock absorbers are paired with springs, and the system acts to reduce the felt effect of bumps and vibrations, which results in a smoother ride for the passenger as well as improved lifespan of various components.

Figure 5: Shock absorber

Many shock absorbers work with a hydraulic fluid. As the shock absorber compresses, the fluid is pressurized and flows through a small orifice. The pressure of the fluid limits the rate of contraction of the absorber, and the pressure bleeds down as the fluid flows.

Generally, a shock absorbing system containing spring, damper, and mass system acts as a second order system and can be modelled with the equation [11]:

$$kx + c\dot{x} = m\ddot{x}$$

Where: k – spring stiffness
c – damping
m – mass
x – displacement
$\dot{x}$ – velocity
$\ddot{x}$ – acceleration

A shock absorbing system can be considered a first order system: if the mass is very small compared to the spring stiffness and damping, we can consider just the spring and damper working in tandem:

$$kx + c\dot{x} = 0$$

3. Thermodynamics Examples

Stove Types

Figure 6: Gas (left) and electric (right) stoves

Consider two different stove types – gas and electric – as shown in Figure 6. Gas stoves operate by using a spark to ignite the gas, which creates a flame. When this occurs, the gas stove is immediately giving off the full extent of its heat energy. This can be plotted as a theoretical step function, like the left side of Figure 7. In real life, it takes a few milliseconds for this to take place, but for our purposes it can be considered effectively instantaneous. On the other hand, electric stoves operate using electrical current which heats up coils. This heating of the coils takes time, which means that unlike the gas stove, electric stoves do not immediately give off the full extent of heat energy. This is shown on the right side of Figure 7, and this curve approximates a first order function.

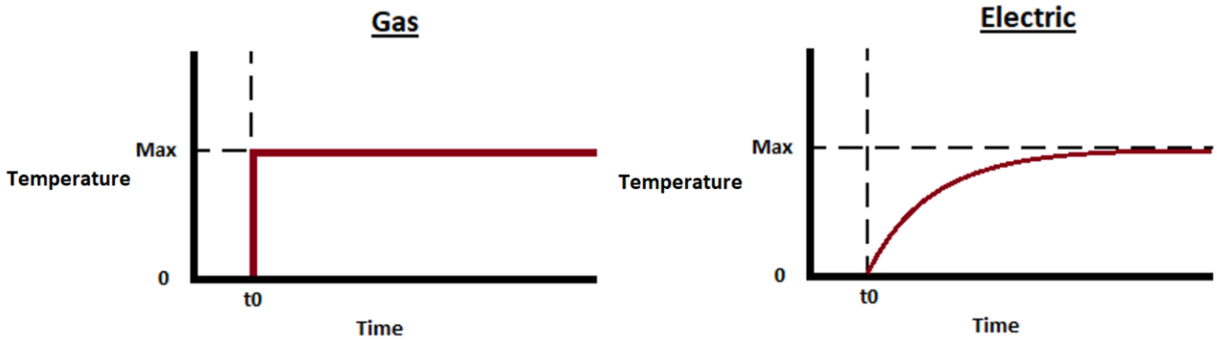

Figure 7: Gas stoves act as a step function, while electric stoves act as a first order function.

The same concept holds true when the stove is turned off as well. For the gas burner, the flame extinguishes immediately as the gas is cut off. The energy given off by the burner itself drops to zero as a step function, as shown on the left side of Figure 8. On the other hand, when the electric stove is turned off the current flow stops but the coils still have energy that must dissipate. Thus, the electric stove still exhibits the characteristics of a first order function, as shown on the right side of Figure 8.

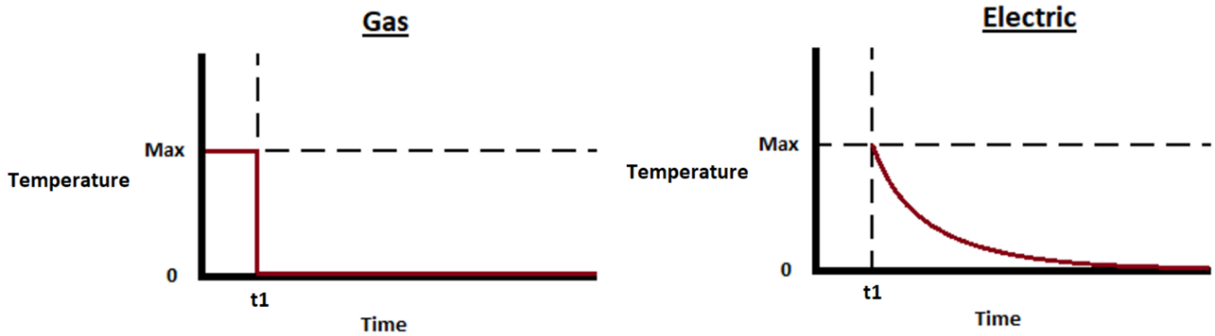

Figure 8: Gas stoves act as a step function, while electric stoves act as a first order function.

Coffee Cooling

Energy transfer because of temperature differences takes place in an exponential manner. The greater the difference in temperature, the greater the rate of heat transfer. This is experienced by millions of people daily as they wait for their fresh coffee to cool. When you first pour the coffee, it is much too hot to drink, but after just a couple minutes it reaches an ideal temperature. As you continue drinking the coffee, you may notice that it remains within an enjoyable temperature range for quite some time before getting too cold.

An in-class experiment was performed where the temperature of a hot cup of coffee was measured every minute until reaching approximately room temperature. Figure 9 shows how the temperature of a cup of coffee changes over time. The green dots of Figure 9 represent data points while the orange line is the trendline. This experiment is easy to perform during class or as an assignment.

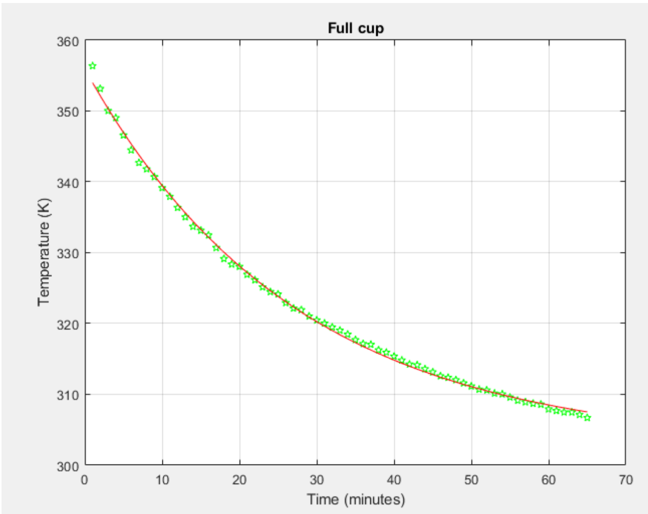

Figure 9: Experimental results of temperature measurements of a coffee cup. Data points in green, curve fit in orange.

Through visual analysis, the trendline of the data in Figure 9 shows an exponential function. The cooling of a cup of coffee represents a first order system. This is because the rate of temperature change depends on the current temperature difference – the hotter the coffee is, relative to room temperature, the faster it will cool. This is why coffee remains in the enjoyable temperature range for a long time.

The time constant of the cooling rate of Figure 9 can be easily determined through two methods. The first method is to find the time required to go from the initial temperature to ~63.2% of the way to equilibrium. The second is to follow the tangent line from the graph at time zero until it intersects with the x-axis.

The following is a brain teaser related to coffee colling (Figure 10): One day, two brothers had a dispute. The first brother, Joe, claimed that coffee stays hotter if one pours cold creamer ten minutes after initially pouring the coffee. The second brother, Moe, claimed that it would stay hotter after 10 minutes if cold creamer is added right away. Which brother is correct?

Figure 10: Joe and Moe's dispute, visualized.

Moe is correct – the coffee stays hotter if cold creamer is added right away. In the beginning, Joe's coffee is hotter than Moe's because it has no creamer. This means that Joe's coffee will cool more rapidly. Since Moe puts creamer in his coffee right away, his coffee starts closer to room temperature which results in a lower rate of cooling. After 10 minutes, when Joe adds creamer to his coffee, his drink's temperature will drop to a lower temperature than Moe's. The graph in Figure 11 shows how both coffees cool and zooms in to emphasize the temperature drop after 10 minutes [12].

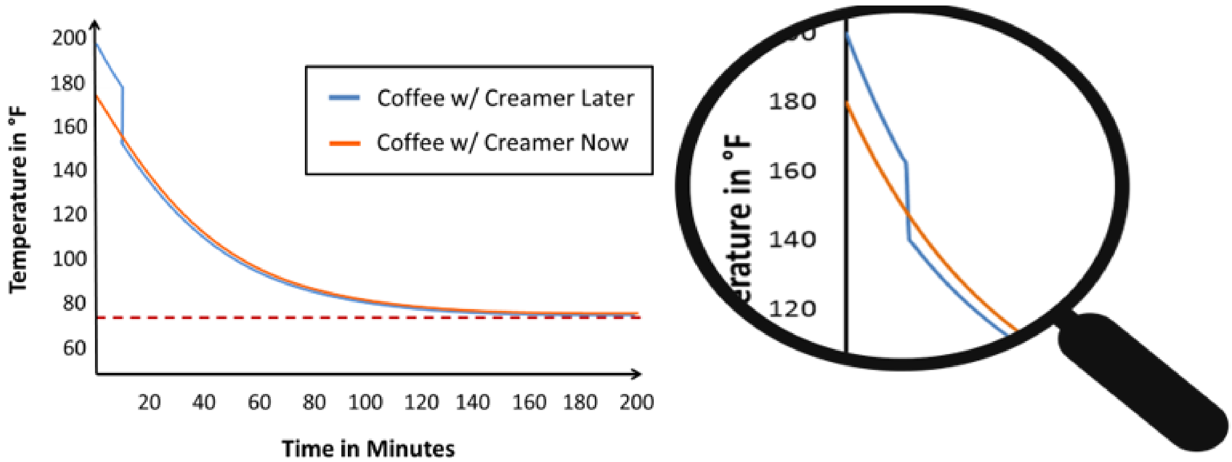

Figure 11: Visualization of coffee cooling based on when creamer is added.

Ice Packs

Ice packs are commonly used to keep food cold in lunchboxes and coolers. There are many kinds of ice packs, but two of the more common ones are singular packs (shaped like a brick) and packs of several small cubes. These ice packs are shown in Figure 12. When choosing between these two types of ice packs, which is more effective?

Figure 12: Two common types of ice packs

Because the ratio of surface area to total area is higher for the small cubes than for the brick pack, the rate of heat transfer is greater for those small cubes. This makes the answer straightforward: if you need to have food colder sooner and for a shorter period, use the cubes. If you want the food to stay colder for a longer period, and don't mind that it takes longer to get cold, use the brick pack. The chart in Figure 13 shows the rates of change for the two different pack types.

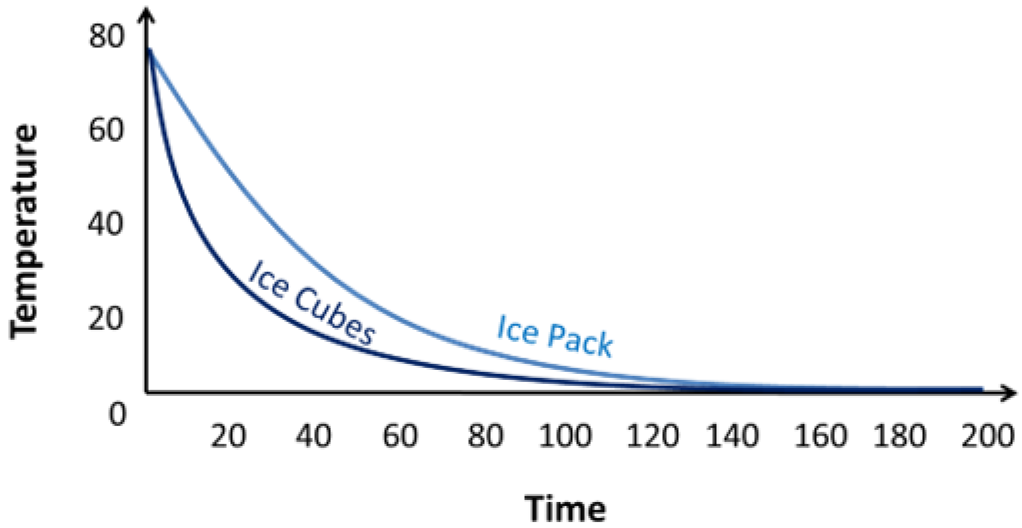

Figure 13: Depiction of temperature change for the two different types of ice packs.

4. **Electromechanical Example – DC Motor**

DC motors are common engineering examples of first order systems. DC motors have electrical and mechanical time constants. Since the mechanical time constant is longer, it is dominant and the system can be approximated to have only one time constant.

The graph shown in Figure 14 shows a system with a step input V_a and a DC motor's resulting output (angular velocity - ω). Eventually, ω becomes a constant as steady state operation is reached, but in transient responses it is a first order system.

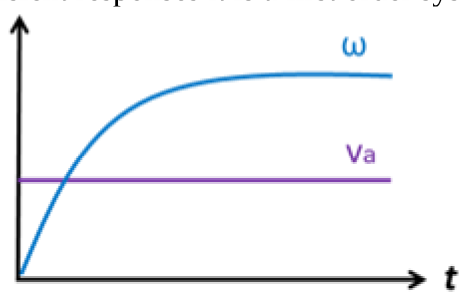

Figure 14: Voltage represents a step function, while ω is a DC Motor's response to that step.

5. **Other Examples**

RC and RL Circuits – Cell Phone

Resistor-capacitor (RC) and resistor-inductor (RL) circuits are both analyzed using differential equations. Capacitors store energy, and while this energy is being built up, the capacitor resists current flow. The equation for voltage across a capacitor is given as:

$$i(t) = C\frac{dv}{dt}$$

Where: i(t) – current as a function of time
C – capacitance
v – voltage
t – time

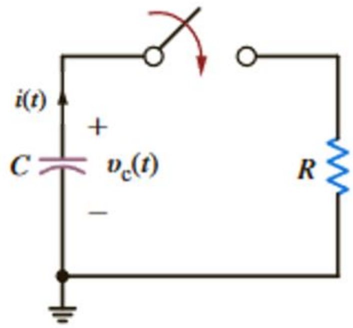

Figure 15: An RC circuit where the switch is closed at time t=0. Courtesy of [13]

For an RC circuit like the one in Figure 15, when the switch is closed Kirchoff's current law can be written as:

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

Where: R - resistance

Since the voltage through the circuit is proportional to the rate of change of the voltage, it is a first order system. The time constant for an RC circuit is given as $\tau = RC$.

Inductors store energy in a magnetic field. When a current initially flows in an RL circuit, the magnetic field builds up, crossing the coiled wire and creating current flow which opposes the current from the power source. This limits the rate at which current flow builds up in the entire circuit. Similarly, when current stops flowing through the circuit, the magnetic field is diminished, and the magnetic field's contraction crosses the coiled wire, creates some residual current in the circuit. The equation for voltage across an inductor is given as:

$$v(t) = L\frac{di(t)}{dt}$$

Where: L – inductance

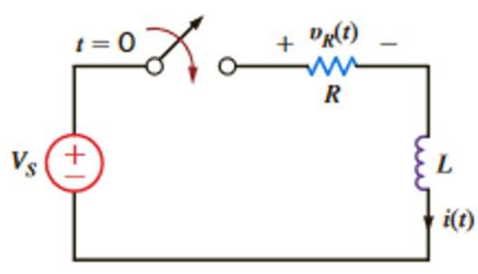

Figure 16: An RL circuit where the switch is closed at time t=0. Courtesy of [13]

For an RL circuit like the one in Figure 16, when the switch is closed Kirchoff's voltage law can be written as:

$$i(t)R + L\frac{di(t)}{dt} = V_s$$

Where: V_s – source voltage

Since the current through the circuit depends on the rate of change of the current, it is also a first order system. The time constant for an RL circuit is given as L/R.

A real-life example of an RC circuit is in a cell phone. Have you ever noticed that when your cell phone's battery is low, the phone seems to charge faster? As the battery gets more and more charged, the charging rate also seems to slow. It's not just an illusion or a sense of impatience – the rate at which your phone charges depends on how charged it already is.

As the phone charges, the battery's voltage increases. This means the difference in voltage between the outlet and the battery decreases. Since the difference in voltage goes down, the current flow also decreases which results in a slower change in the voltage difference going forward. Since the rate of charging changes based on the amount charged, this charging phenomenon represents a real world first order system. Let's take a look at a graphical representation of this process.

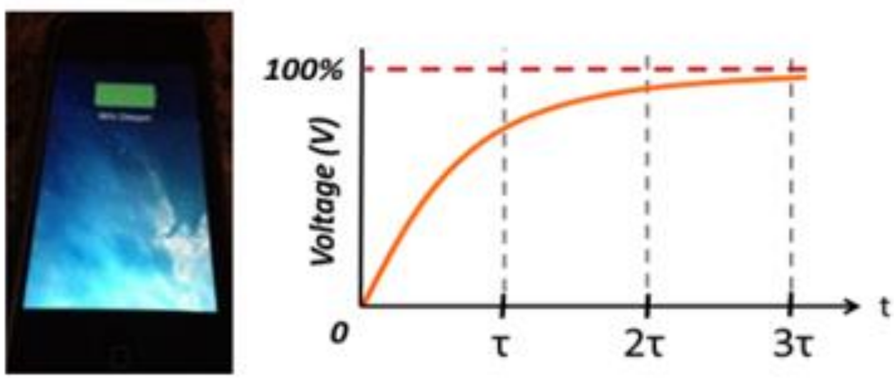

Figure 17: Charging a cell phone – a first order system

In Figure 17, we can see that the slope of the graph is initially very steep – that is, voltage initially increases very quickly. As time goes on, the slope of the line gradually decreases – the line starts to level off.

This is an easy experiment to perform at home. Plug your phone in when the battery is at 5% and time how long it takes to charge to 20%. Then, when your phone is at 85%, plug it in and time how long it takes your phone to charge to 100%. Which one takes longer?

Diffusion

Diffusion is essentially the movement of molecules from a region of higher concentration to a region of lower concentration. One example, shown in Figure 18, is a drop of dye or food coloring into a cup of water. Initially, the dye is dropped into one specific spot, but as time passes it spreads until it is uniformly distributed throughout the water.

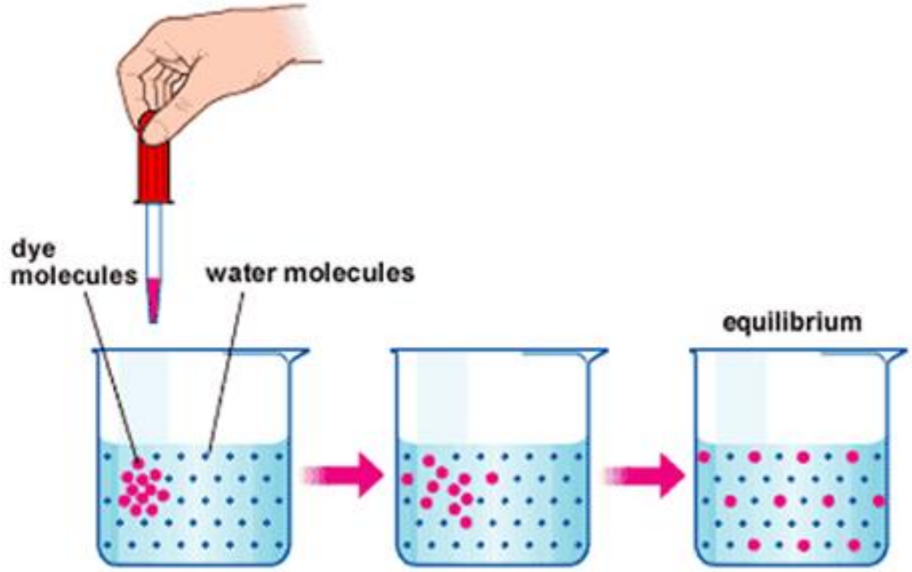

Figure 18: Adding dye to a cup of water

Another example of diffusion is the movement of gas between regions of different pressure. Imagine two sides of a tank separated by a divider. Side "A" is filled with a specific gas. Side "B" is a theoretically perfect vacuum. Once a portion of the divider is removed, the molecules on side A will tend to move towards the area of lower concentration (side B). Over a period of time, the concentration difference between the two sides gets smaller and smaller, and eventually both sides reach equilibrium. This process is shown in Figure 19.

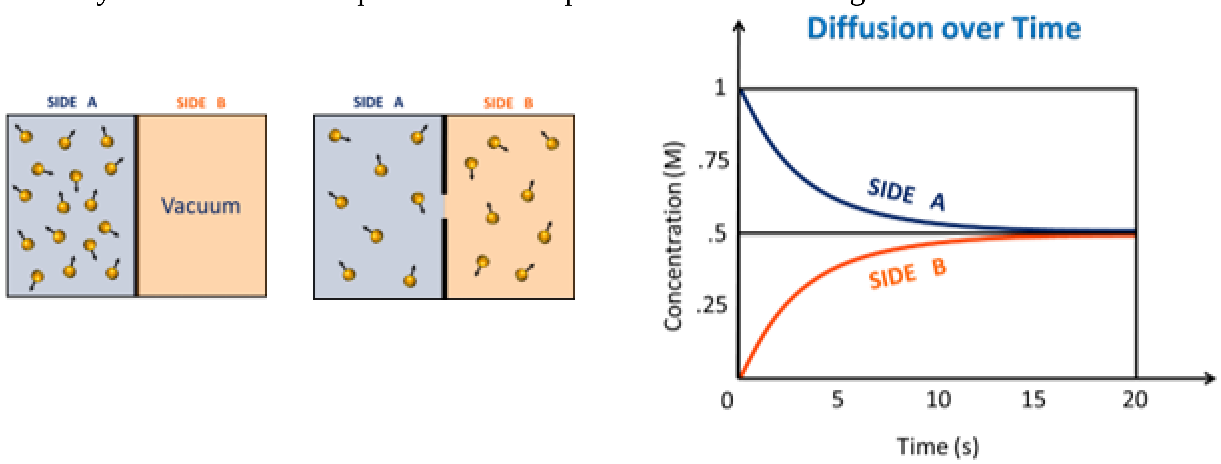

Figure 19: Diffusion of molecules in a tank

In all diffusion situations, the rate of diffusion goes down as the system gets closer and closer to equilibrium (see Figure 19, right). For this reason, diffusion approximates a first order system.

Bucket with Hole

Picture a tank with a hole at the base, like in Figure 20.

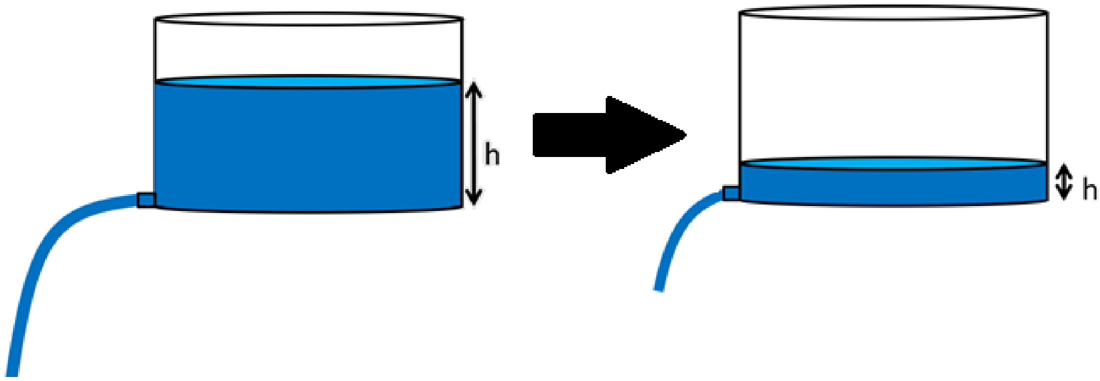

Figure 20: More water flows from the hole when there is more water in the tank

Since the pressure felt at the hole is based on the height of the water in the tank, the flow through the hole decreases as the water level in the tank goes down. This is an example of a non-linear first order differential equation. The flow of the water through the hole is given by the following equation, derived in [14]:

$$a * \sqrt{2 * g * h} = A * \frac{dh}{dt}$$

Where: a – cross-sectional area of the hole
g – acceleration due to gravity
h – height of water in the tank
A – cross-sectional area of the tank

Half-life

The concept of a half-life is used to measure the decay rate of radioactive substances. Specifically, half-life (t½) refers to the amount of time that is required for half of a given amount of radioactive substance to decay. While obviously important in the field of Nuclear Physics, half-life also has important implications In Biology and Chemistry with regard to medical uses.

One crucial use for half-life determination is in carbon dating, which is a method of determining the age of an object. Carbon-14, which has a half-life of about 5700 years, is naturally present in organic compounds such as plants and animals. When fossilized remains are found, the amount of Carbon-14 remaining in the sample can be used to determine when the animal died.

Since half-life refers to the amount of time required for half of a radioactive substance to decay, the decay is not linear. After one half-life, half of the substance remains. After two half-lives, half of that half remains (one quarter of the original amount). This is shown in Figure 21 in table form, and in Figure 22 in graphical form.

Number of Half-lives	Fraction of substance remaining
0	1
1	0.5
2	0.25
3	0.125
4	0.0625

Figure 21: Number of Half-lives vs. fraction of sample remaining

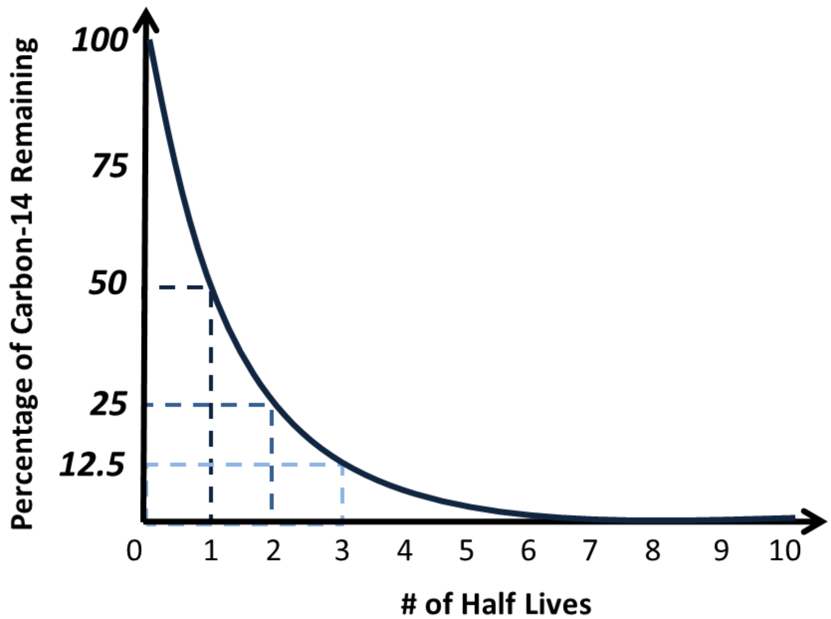

Figure 22: Number of half-lives vs. percentage of sample remaining

6. Acknowledgements

Special thanks to Professor Moshe Barak for great discussions as well as providing us with images of the toilet mechanism. We also thank Mr. Juan D. Yepes for his help with the anonymous questionnaire.

7. Conclusion

The explanations in this paper were chosen to introduce the concept of First Order Differential Equations in visual and intuitive ways. The paper is not comprehensive and does not attempt to replace current teachings and textbooks. The explanations intentionally do not focus on numerical solutions or heavy mathematics to avoid intimidating students, but rather focus on the basic understanding of the concept itself. We hope that those who teach differential equations will use some of these examples as a supplement to their teaching, and that students find this resource helpful in understanding a concept that is widely regarded as difficult. The initial assessment of this work as described in this paper is an indicator that visualization of the concept of First Order Differential Equations is highly desired by students.

8. References

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Appendix A – Survey Results

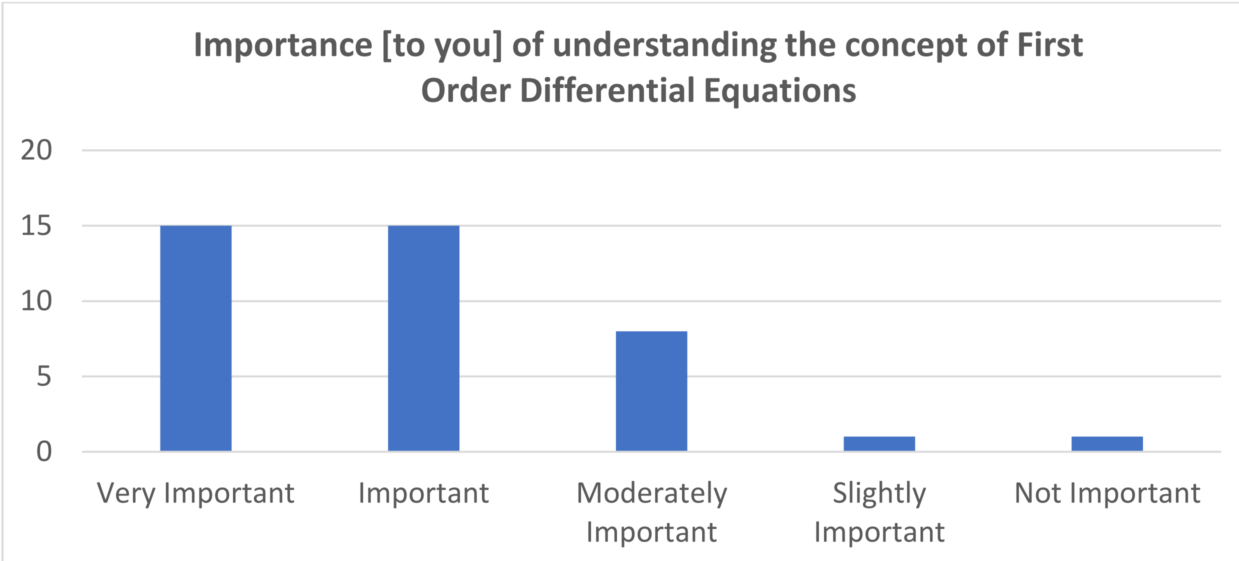

Figure A.1: Student feedback on importance of understanding First Order DE concepts.

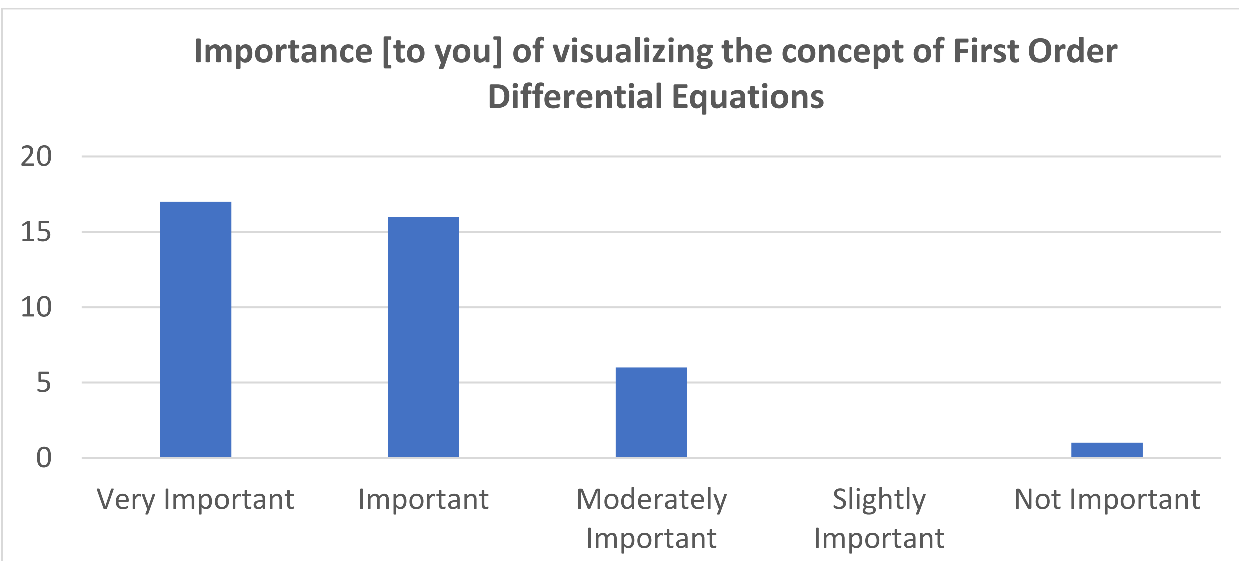

Figure A.2: Student feedback on importance of visualizing the concept of First Order DE's

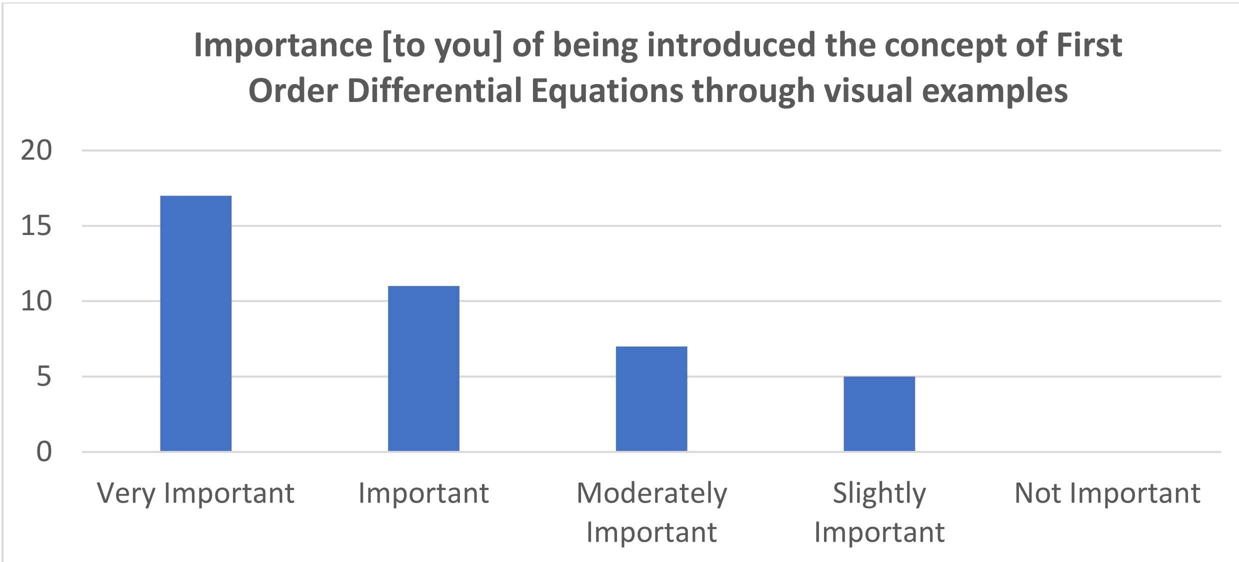

Figure A.3: Student feedback on importance of being introduced to First Order DE's through visual examples

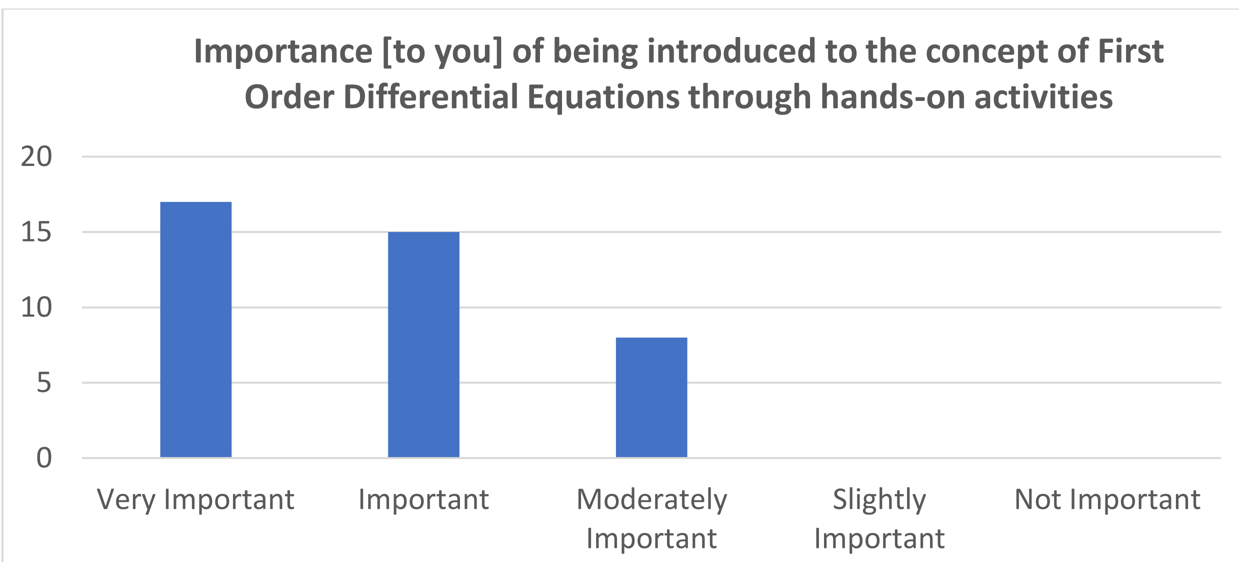

Figure A.4: Student feedback on importance of being introduced to First Order DE's through hands-on activities

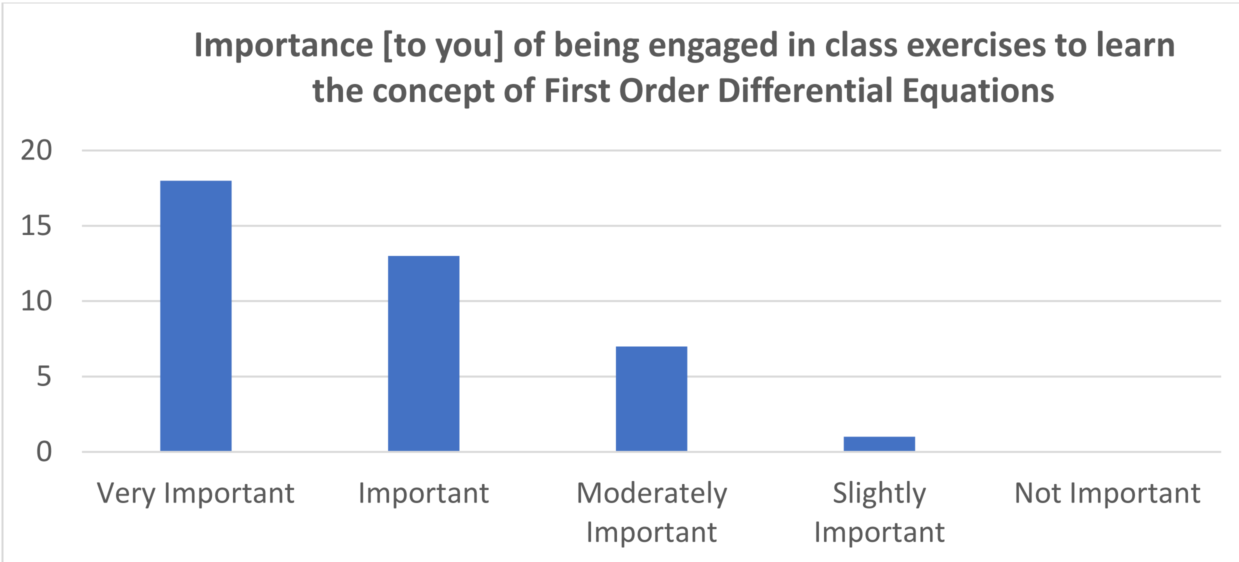

Figure A.5: Student feedback on importance of being engaged in class exercises when learning First Order DE's

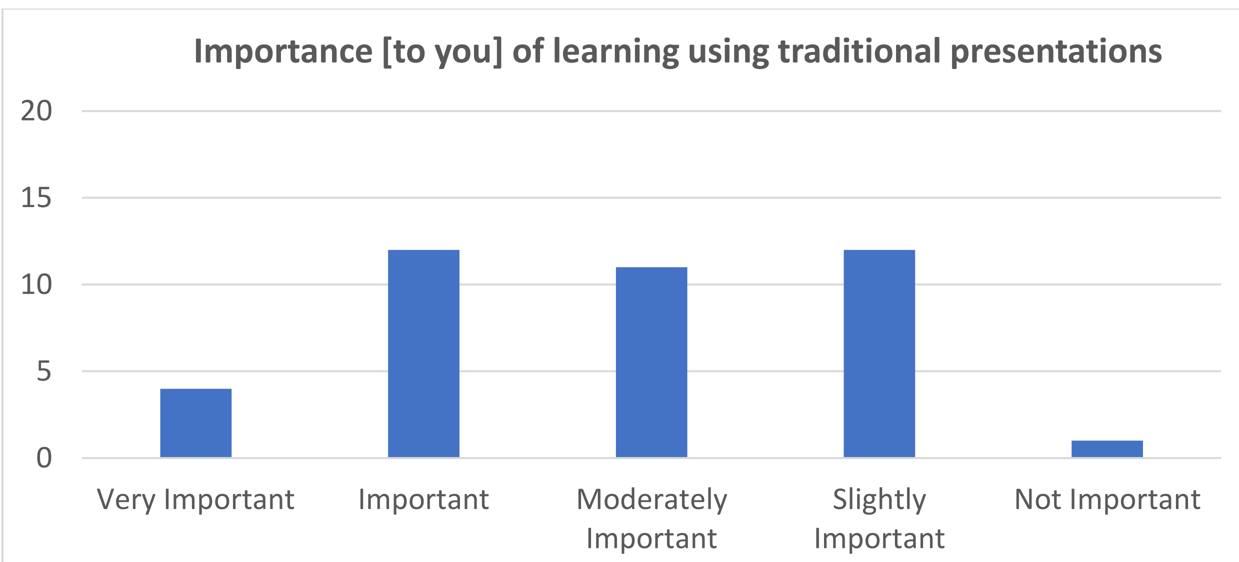

Figure A.6: Student feedback on importance of learning through traditional presentations

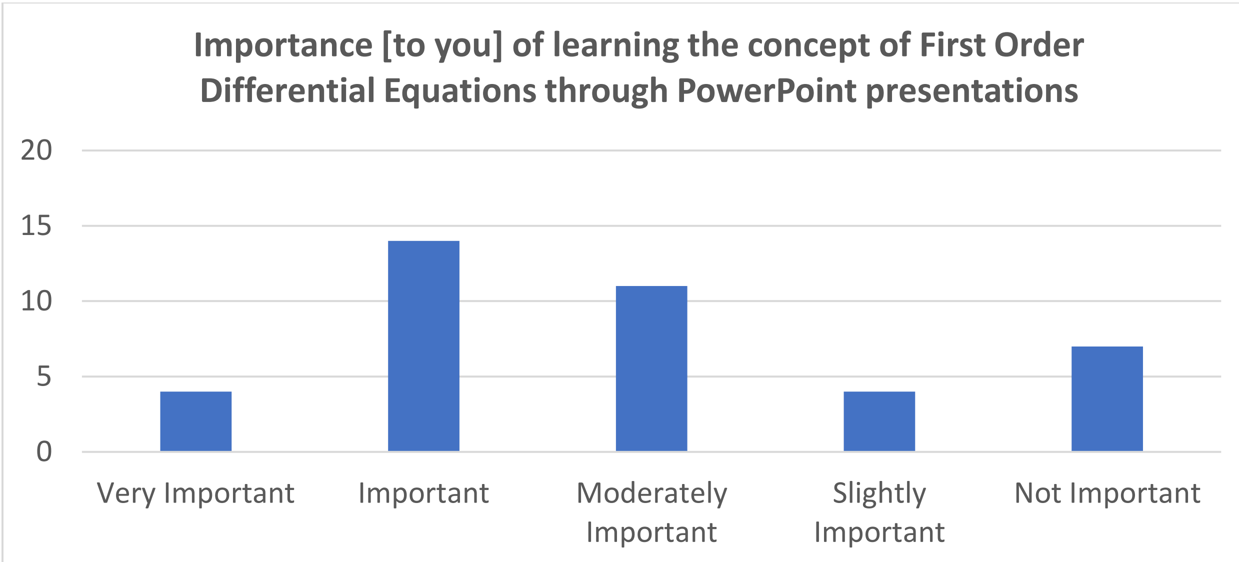

Figure A.7: Student feedback on importance of learning First Order DE's through PowerPoint presentations

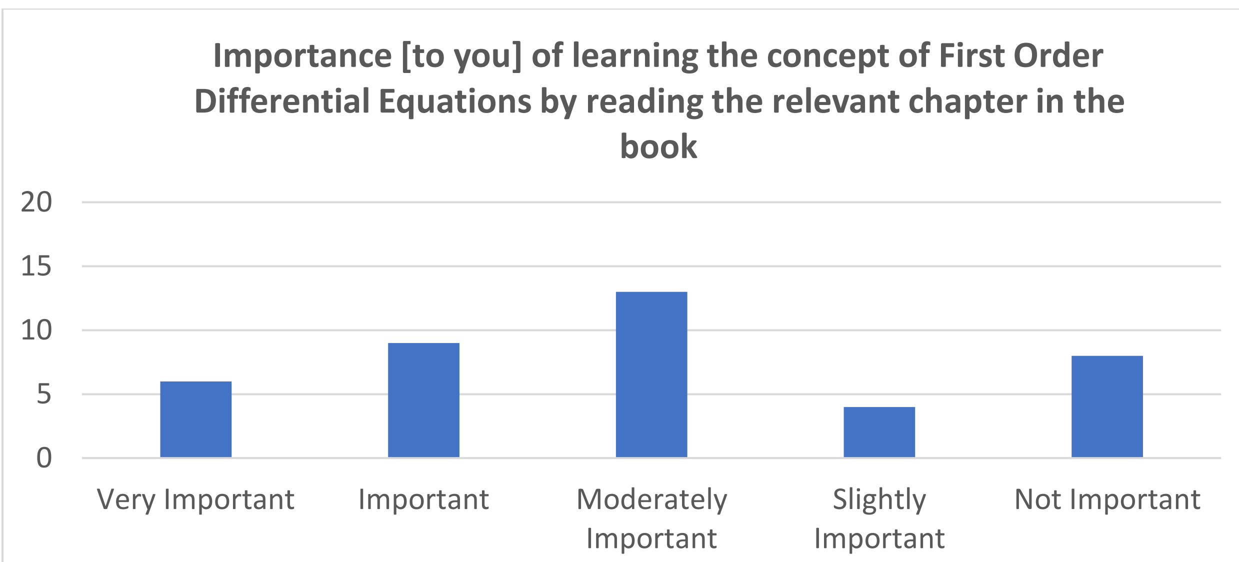

Figure A.8: Student feedback on importance of learning First Order DE's by reading the textbook

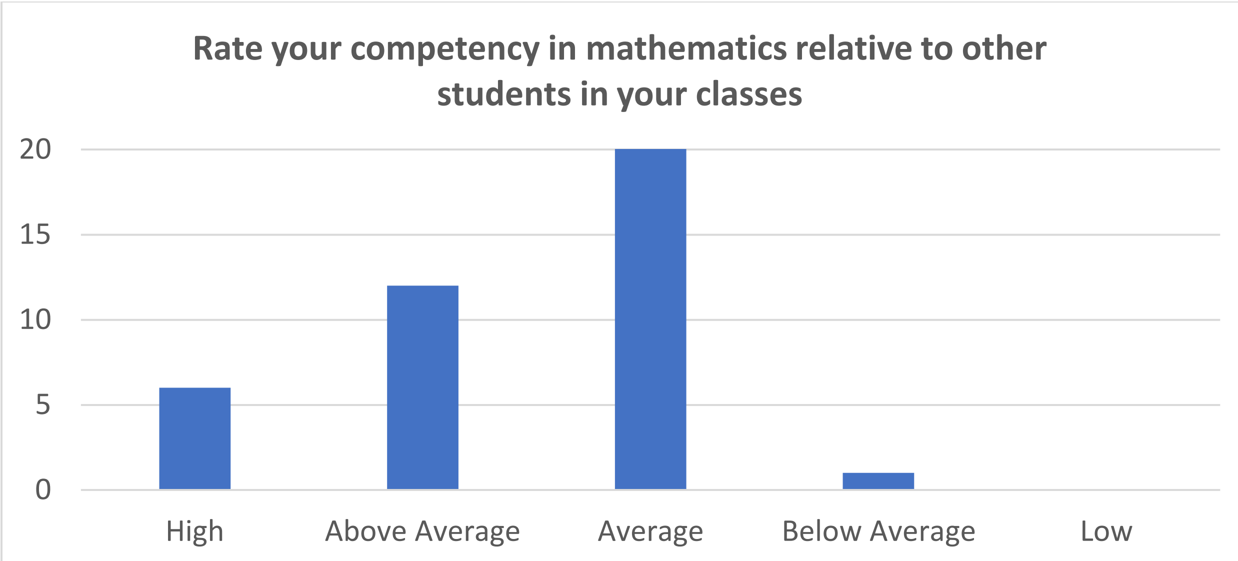

Figure A.9: Student self-assessment of mathematics competency

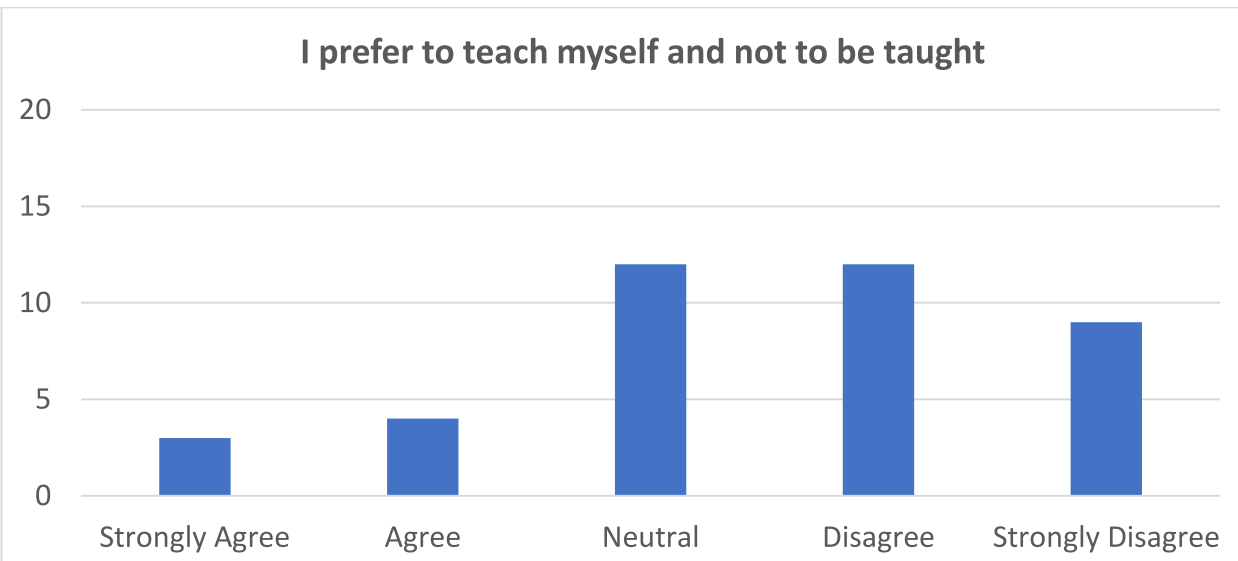

Figure A.10: Student opinions on self-learning