Word Problems and Problems with Words: A Possible Solution

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Abstract: We began with a question: Why do our students have so much difficulty solving word problems in Math I? Another question followed: Why do our students have so much difficulty writing short (three-to-five paragraph), logical essays in English I? One possible answer: Our students approach math and English problems as if they required entirely different skills. However, aside from their expression in different languages, math and English problems require the same step-by-step analysis, also called problem solving, on the way to solutions. In this paper, we examine possible methods of teaching that analysis. We also propose a closer relationship between math and English curricula as a means of reinforcing our teaching.

Introduction

The Samuel I. Ward College of Technology at the University of Hartford offers six Engineering Technology majors: Architectural, Audio, Chemical, Computer, Electronic, and Mechanical. In addition, we teach our own math courses, from Math I, Algebra, through Math V, Differential Equations, and our own English courses, from English I, Expository Writing, through English III, Advanced Technical Communications. These courses provide an alternative to the math and Professional Writing sequences offered by the College of Arts and Sciences and offer the mathematical and writing skills relevant to the Engineering Technologies. In addition, two of our professors teach sections of the Arts and Sciences physics course, again so that Ward students are offered the relevant information.

Given the attempt to provide skills our students require at a pace at which they can master those skills, we nevertheless find that many of our freshmen struggle through both Math I and English I. They wrestle with the three-to-five-paragraph essay form, having trouble, for the most part, not with grammar and syntax, but with expressing ideas

in a logical flow that is clear to readers. And they wrestle with word problems in math, having trouble turning words into equations that they can solve. In other words, they have the basic skills of both English and math: they can turn words into sentences; they can add, subtract, multiply, divide, perform basic algebraic operations. But they must be taught to take those basic skills and apply them.

Bloom's taxonomy, summarized in Table I, offers us a convenient means of breaking down the skills we have to teach our students.

Table 1: A Summary of Bloom's Taxonomy

Level	Explanation
Knowledge	Knowing basic facts, concepts, and principles
Comprehension	Translating material from one form to another, interpreting material, estimating future trends
Application	Applying rules methods, concepts, principles in new, concrete situations
Analysis	Breaking material into component parts to understand its organizational structure
Synthesis	Putting parts together to create a new whole
Evaluation	Judging the value of material based on definite criteria

Source: *Designing and Managing MCQs: Appendix C: MCQs and Bloom's Taxonomy.* www.uct.ac.za/projects/cbe/mcqman/mcqappc.html. 12/05/2000

Two points to remember here. One: we can't expect our students to master all of these steps at once. As Wankat and Oreovicz point out in *Teaching Engineering,* moving from novice to master in any set of skills requires approximately ten years.[i] Two: learning to solve problems is learning a set of skills, just as mastering the design of buildings or the setup of a recording studio is learning a set of skills. . Solving mathematical and linguistic word problems uses all the facets of problem solving. Wankat and Oreovicz further point out that in classes students become good at routines;

that is, they learn to take basic facts and perhaps apply them in routine situations, but they don't master strategy, interpretation, and generation[ii], in Bloom's scheme, application, analysis, synthesis, and evaluation. So our students have to be taught to apply, analyze, synthesize, and evaluate, the skills that compose problem solving, and our students are going to take time to learn them.

Types of Word Problems

In our attempt to determine where the problems with solving word problems lie, we have first of all identified a hierarchy of word problems that lead the student from novice problem solver to professional. The hierarchy is based upon the difficulty in ascertaining the appropriate equation or equations to use in solving the problem and follows loosely Bloom's taxonomy, moving from comprehension to synthesis and evaluation. Here we describe each level and give an example of each.

A. **Basic**

This level is the simplest of all word problems in the sense that you perform the mechanical manipulations necessary to solve the equation, for example:

Solve for x in $3x + 2 = 8$

B. **Descriptive**

These second-level problems present the equations and data explicitly with a description of the parameters. For example:

The heat, Q, required to change the temperature of a mass, m, of a substance by an amount, ΔT, is given by the following formula:

$$Q = mc\Delta T,$$

where c is the specific heat of the substance. Determine the amount of heat required to raise the temperature of 10 g of water whose specific heat is given by 1cal/g-°C, by an amount of 12°C.

All data and the only necessary equation are clearly expressed here. In fact, the solver doesn't even have to know what the symbols mean; the description is superfluous. All the solver needs to extract from the words is that

$$m = 10 \quad c = 1 \quad \Delta T = 12$$

The solver merely plugs in the numbers and solves the equation.

C. Embedded

The third level of word problem requires the solver to construct the equation out of sentences within the problem, which is to say the equation is embedded in the wording. For example:

> John purchased two boards totaling 24 feet of lumber. One board is 3 feet longer than the other. What are the lengths of the two boards?

Solving this equation requires the solver to determine that there are two variables, name those variables, and construct the appropriate equations before solving them.

D. **Knowledge-based**

This level involves the typical problem at the end of textbook chapters in technical classes. The equations and perhaps some data are implied but not explicitly given. It is impossible to solve these problems without knowing at least some of the subject at hand as well as where to find the appropriate data and equations. For example:

> Determine the amount of heat required to raise the temperature of 10 g of water by the amount of 12ºC.

This problem is the one given in our discussion of descriptive equations, but it has now been stripped of the equation and the related information. The solver will have to find the information (usually in the accompanying chapter of the text) to construct the relevant equation with appropriate data before solving the problem.

E. Open-Ended

The most advanced type of word problem is the problem met on the job or as a thesis topic. It is typically ill formulated and lacking data. The subject area is broad. Several sources must be consulted to determine which equations are appropriate and what data must be supplied to solve it. Our goal is to prepare our students to solve this level of problem because the problems they will encounter at work, as well as in life, require this level of critical thinking.

Difficulties in Solving Word Problems

Our next step was to identify some of the common sources of difficulty our students have demonstrated in solving word problems. Here is a typical word problem

from the Math I textbook, *Basic Technical Mathematics, seventh edition.* [iii] It is another example of an embedded word problem, the lowest level of word problem that requires simultaneous analysis of both words and math.

> Two computer software programs cost $390 together. If one costs $114 more than the other, what is the cost of each?[iv]

Not, for some, a particularly difficult problem. In fact, it is generally assumed that students can extract the mathematics from the problem statement, that they know how to identify the variables, data, and embedded equations, then apply the simple-level principles from the text to solve the problem. Such word problems are typically presented as applications of the mathematical principles discussed in the text to demonstrate the relevance of those principles. The emphasis is traditionally on the mathematics being exemplified, not on the fact that each word problem is actually a multilingual (mathematics and English) expression of a real-world situation.

But for many of our students, a large question looms: *How do we translate from English to math?* It's all very well to say, "analyze the problem." But for many students, that statement is simply meaningless or even confusing. The statement implies that we should know how to analyze the problem to create the equation, that the analysis is obvious. Given that many students (and one of the authors of this paper, who was a literature major) don't know how to begin to analyze the problem, we have to define analysis in this situation to clear away the confusion.

The reasons for the confusion are many. One source of difficulty is that there are two kinds of rules in technical mathematics: manipulation rules, the rules that tell us what we can do with numbers, and knowledge rules, the rules that are based on observation of the physical world and that could change as our observations of the world are refined. The problem example given above requires only manipulation rules, but many word problems require that the solver be able to apply knowledge rules as well. For instance:

> A lead pendulum bob of mass 1 kg is released from a height of 0.25 m above a table. At its lowest point it strikes a 0.5 kg copper block, which moves of with all the momentum. What is the velocity of the copper block?[v]

The student must be able to apply the laws of conservation of energy and momentum to this two-part (preparation for the collision, then the collision itself) problem. The manipulation rules can be applied only after all the knowledge rules (the mathematical description of the physical world) have been applied. Students must learn to differentiate between manipulation rules and knowledge rules and when to apply them.

Another difficulty is that there can be two types of data in a word problem: numbers supplied in the problem statement and data implied by the statement. Our software problem tells us that the two programs together cost $390 and that one costs $114 more than the other. Our pendulum bob problem tells us the mass of the bob, its release point, and so on. But there is information implied by the words of the bob problem. "Pendulum bob," for instance, implies a swinging motion, rather than a vertical drop from a height, but two students addressing this problem missed the implication and attempted to solve the problem as if it involved a vertical drop. Students must be taught, then, to watch for and interpret the implications of phrases like "came to a stop," meaning the final velocity is zero, "started from rest" and "is released," both meaning the initial velocity is zero. Each technical field has its own language with mathematical implications that students must learn before they can solve a problem.

But more fundamental issues confront students. The problems involve words. We say that not to be facetious, but to call attention to the fact that languages involving words are not exact the way mathematical language is. Consider the following problem:

> Albert is a butcher. He wears size thirteen shoes and his collar size is 15
> ½. What does Albert weigh?

Confronted with this problem, students attempted to estimate Albert's weight because "what" can be used as a synonym for "how much." In fact, the problem is formulated specifically to invoke the confusion created by the synonym. The ambiguity created by language is a far cry from the specificity created by $2x + 2 = 8$, but the ambiguity of the word "released" in the pendulum bob problem is, in part, what caused the two students who misinterpreted the problem to do so. (By the way, the answer to the question of what Albert weighs is that Albert weighs meat.)

Another source of confusion is that a word used to mean one thing in ordinary English can mean something quite different when used in mathematics and technical

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fields. One of our students for whom English is not her native language struggled in Math I until she realized that "difference" in math means subtraction.

We also find that solving word problems is a multi-intelligent activity in the sense presented by Howard Gardner in *Frames of Mind*[vi] and subsequent books. Solving word problems requires at least the use of linguistic and logical-mathematical intelligences, and since technical people typically need to see a picture of a problem or create one to solve a problem, spatial intelligence, too. Most of our students are typically underdeveloped in linguistic and logical-mathematical intelligences and strongest in kinesthetic, that is, bodily and physical, and perhaps spatial, visual, intelligence. They are hands-on types for whom abstraction—mathematics is abstraction of patterns from the physical world, after all—can be difficult. They have to learn strategies for translating a problem into terms they understand.

One such strategy is the Rivard Methodology for solving embedded word problems, shown in Figure 1. This approach, developed by Glenn Rivard at Ward College for Math I students, specifically addresses the issue of analysis by providing a set of steps for breaking down the problem and translating it into mathematical language. It recognizes that word problems are problems stated in words, first English words, then mathematical words, numbers and symbols, put together in sentences, equations, following strict grammatical, algebraic, rules so that the sentences or equations mean something. It teaches the student the rules for constructing equations, too, so that they are acquiring a set of tools that they can apply generally in any mathematical situation.

Figure 1: The Rivard Methodology for Solving Mathematical Word Problems

Step 1: **Identify the comparison.**

The problem expresses two different points in space or time, for instance, left and right, before and after, coming and going, mixed and unmixed. In other words, the problem expresses some process of change. Because this first step is the origin of the fundamental equation or equations in the mathematical translation, you must be sure to get this step right.

- Write this comparison down

Step 2: **Morph the verbs.**

- Continue by rewriting the sentences. Watch for conjunctions ("and," "while," "but," and so on) and separate the phrases at the point of the conjunction.

- Rewrite as required so each sentence is expressed with a form of the verb "to be." In mathematics, "to be" is represented by an equal sign.

- Identify the question(s).

Step 3: **Morph the nouns.**

- Give each noun a unique symbolic name of one letter and replace all references to the same noun with that symbolic name. In mathematics, these symbols that you have created are the variables in the equation. In the process of creating the variables, you are identifying and removing synonyms and adjectives from the original problem because they are generally irrelevant.

- Watch for plural terms referring to your nouns and replace them with your symbolic names.

Step 4: **Add implied relations.**

The first three steps involved English only, no mathematics. At this step, you actually start translating from standard English to a minimal sentence that can easily be translated into mathematics.

- Examine the nouns, now symbols, and add statements that express the relations between them and write these relations as simple sentences using forms of the verb "to be."

Note that these relationships may not be clearly expressed in the problem, but may be related to the comparison you identified at the beginning of this process. For example, if you are mixing two alloys of steel (iron and carbon), the comparison is between mixed and unmixed states, so the relation is that the amount of carbon is constant before and after mixing.

Sometimes, the relation involves a "whole equals the sum of its parts" statement. For example, if 50 ft^2 of floor will be covered by tiles that are each 0.85 ft^2, then the number of tiles multiplied by the area of each tile must equal the total floor area.

If you are lucky, statements in your list clearly express a relationship. For example, the statement "It takes Mary 20 minutes longer to driver to her mother's house when

the kids are in the car" morphs to "T_2 is T_1 plus 20."

From your work with the problem, you should know now which field of study, if any, is represented in the problem. Each field has its own fundamental knowledge and principles that you may have to apply to the problem in the form of an equation that expresses the relations between the nouns, for example, Ohm's Law in electronics, Fourier's Law in heat transfer, interest formulae in finance, and Murray's Law in circulatory physiology. You may have to find such equations externally to the problem you are working on.

Step 5: **Translate to mathematics.**

You now have three categories of statements: the definitions of the variables, the relationships between or among the variables, and the variables themselves. The second category, the relationships between or among the variables, is the source of the equations, and the questions suggest which equations might be applicable from a particular field of study.

- Now translate the verbs to equal signs and the conjunctions to their mathematical counterparts. "T_2 is T_1 plus 20" becomes "$T_2 = T_1 + 20$," an equation you know how to solve.

Figure 2 offers an example of a problem analyzed using Rivard's Methodology.

Figure 2: An example of Rivard's Methodology

Problem: Ten 6.0V and 12V batteries have a total voltage of 84V. How many of each type are there?

Step 1: Identify the comparison.
   Batteries separate and combined.

Step 2: Morph the verbs.
   There *are* ten batteries.
   Some *are* 6V batteries.
   Some *are* 12V batteries.
   The total voltage *is* 84V.
   How many 6V batteries *are* there?
   How many 12V batteries *are* there?

Step 3: Morph the nouns.
   Let V_6 be the 6V batteries and V_{12} be the 12V batteries, so
   The sum of V_6 and V_{12} *is* 10.

There *are* V_6 6V batteries.
There *are* V_{12} 12V batteries.
The total voltage *is* 84V
What is V_6?
What is V_{12}?

Step 4: Add implied relations
The total voltage (of batteries combined in series) *is* equal to the sum of their separate voltages.
Regrouping the statements, we have

Definitions:	There are V_6 6V batteries.
	There are V_{12} 12V batteries.
Relations:	The sum of V_6 and V_{12} is 10.
	The total voltage is 84V.
	The total voltage is equal to the sum of their separate voltages.
Questions:	What is V_6?
	What is V_{12}?

Step 5: Translate to mathematics.
$V_6 + V_{12} = 10$
$V_6 (6) + V_{12} (12) = 84$
Find V_6 and V_{12}.

These steps may seem cumbersome to people whose strength lies in logical-mathematical intelligence, but for students who do not know where to begin solving a word problem, the Rivard Methodology is necessary as a means of analysis. The hope is that eventually the methodology will become unnecessary.

Other Approaches to Solving Word Problems in Mathematics

The Rivard Methodology is not the only possible means of teaching students to analyze a word problem. It seems to work well for people who have strong linguistic intelligence. For students whose strength is kinesthetic or spatial, other methods have to be taught. It is possible, for instance, to teach spatially strong students to diagram a problem to analyze it. The kinesthetically strong students may model the problem to analyze it.

Addressing the various strengths and weaknesses of our students is crucial. Many of our students consider using the calculator to be "doing math." They state explicitly that they have never understood how to "do math" involving equations and particularly word problems. A significant number of students report actually never having learned the multiplication tables, a situation repeated at other colleges where we have spoken to colleagues. Consequently, using alternative pedagogical techniques to bring these

students "up to speed" is critical. Each semester in which their deficiencies and supposed inability to work with word problems are not addressed puts them another semester behind.

Solving Word Problems and Problems with Words in the Writing Class

In addition to identifying a hierarchy of word problems in mathematics, we have identified a hierarchy of writing problems as well. As in math, the hierarchy of writing problems loosely follows Bloom's Taxonomy, moving from knowledge and comprehension to synthesis and evaluation, based on the increasing difficulty of providing information in logical and clear language. Note that, although writing and mathematics are different languages, they require the same problem-solving skills, writing problems fall into four, rather than five, categories.

A. Mechanical

This level involves true/false and multiple-choice problems. You must mechanically recognize a correct answer, for example

The author of "The Road Not Taken" is

   a. Carl Sandburg
   b. Edgar Lee Masters
   c. Edward Tufte
   d. None of the above

True or false: "Shooting an Elephant" by George Orwell is an example of a narrative essay.

No writing is required in response to these questions, but you must understand the question and respond.

B. Embedded

Second-level problems require a recitation of fact in response to a question. You must generally respond with a complete sentence, the form of which is suggested by the question. For example,

Q: Who is the author of "The Road Not Taken?"

You should frame your answer as "Robert Frost is the author of 'The Road Not Taken.'"

Note that fill-in-the-blank questions are a simpler form of embedded problem that require you to understand the form of the answer. In other words, you must

recognize that in the question "_____ is the author of 'The Road Not Taken," the required answer is a specific proper noun.

C. Knowledge-Based

A knowledge-based writing problem requires you to take facts not given in the problem statement and arrange them logically in response to a question. The response may require you to consider the facts in a new way, but that way may be suggested by the question. For example,

Q: What was new in Western Europe in the sixteenth century?

To respond to this question, you must arrange the facts detailing religious and mercantile changes, explorations, science, and so forth in a clear and logical progression that can stand on its own. More experienced or thoughtful writers may also abstract ideas from the facts and summarize the progression by synthesizing it and evaluating it, telling readers why what was new in Western Europe in the sixteenth century is important to us.

Lab reports, incident reports, and the like are knowledge-based writing problems.

D. Open-Ended

The open-ended writing problem is the problem met as a thesis topic or on the job. Typically, it is ill-formulated, lacking facts, perhaps not even suggesting a possible direction. You must narrow the topic and seek information before even attempting to write. The written response must be logical and end in a synthesis and evaluation. Business plans and grant requests are open-ended writing problems.

Difficulties in Solving Writing Problems

The obvious difficulties in solving writing problems lie, first, in difficulties with the language itself. Grammar is supposed to offer a guide to constructing clear sentences and paragraphs, but the English language offers a multiplicity of vocabulary and grammatical form that can bewilder and confound even native-born speakers. Nonnative speakers often are dazed and confused by basic forms like articles or progressive verb tenses, not to mention the astonishing range of vocabulary that offers synonyms for "thin" like "lean," "slim," "skinny," and "emaciated." Only one of those choices is a

complimentary term ("slim"), one is an insult ("skinny"), one is used more often to refer to meat ("lean"), and one is probably a medical report ("emaciated"). The language can start to feel like a barrier instead of a means of communication (an important consideration in this era of globalization). Reviewing grammar and working on vocabulary are important for writing, but it is also important to acknowledge that difficulties with the language will interfere with solving math word problems, too. Remember the anecdote about the student who didn't understand that "difference" referred to subtraction. We must take pains not simply to ensure that our students understand that the language gets used in math class, too, but also to remind ourselves of that fact, too.

A second layer of problems arises in the need to arrange sentences logically. And because logic is a strong component of analysis, whether of a math problem or a writing assignment, exercises to strengthen logical skills can be particularly useful. It is also important to be explicit about problem-solving skills. For example, brainteasers like the question of what Albert weighs provide fun for the students, but unless such problems are placed in the context of logic-building, the students have no idea why they're bothering with the exercise.

So, starting in our freshman expository writing class, we have begun to talk specifically about levels of thinking using Bloom's taxonomy, about multiple intelligences and areas of individual strengths and weaknesses. We are finding exercises that can address both writing skills and problem solving. For example, we assign a brainteaser as a writing exercise. The writers must not only solve the problem, but they must also write about how they solved the problem. They must describe the process they used to derive an answer. Such exercises can be used in upper level writing classes, as well, because it is important to continue the teaching of problem solving skills throughout our students' formal education.

For example, in Advanced Technical Communications, we organize the students into groups and require them to invent a patentable object or improvement to an object and create a business proposal and presentation convincing a corporation to manufacture and sell their invention. The project addresses problem solving at all levels of Bloom's taxonomy and requires students to draw on the knowledge and multiple intelligences of

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their group members, as well. Such a project can confirm that students have begun to learn and master problem-solving techniques, but also can indicate where we need to do a better job teaching those techniques in lower level classes.

Conclusion

As a result of our question, why can't our students write short essays or solve word problems, we have begun to examine the assumptions we bring to our teaching. We realize that to say "analyze the problem" is ineffective; we must teach various means of analysis, including the Rivard Methodology, for which we must thank our esteemed colleague Glenn Rivard (who also participated in discussions of problems in teaching Math I and many other insightful discussions about teaching). We must also teach problem solving as a set of skills that students need along with the technical skills of their majors. By regarding our students as kinesthetically and spatially intelligent, we can respect their strengths, understand their weaknesses and offer them information and methods of working that address both their weaknesses and their strengths.

In addition, we know that the Rivard Methodology and brainteaser exercises are only two possibilities for teaching students to solve problems in writing and in math. These two methods focus largely on comprehension and analysis, not on synthesis or evaluation, in Bloom's taxonomy; they are merely a beginning for our students' formal education in problem-solving skills. We must continue to develop methods for teaching problem-solving skills at the levels we are currently covering and at the synthesis and evaluation levels, because the ability to solve problems at all levels can be crucial to our students' mastery of their chosen technical field. We must also develop problem-solving methods that students whose strengths are not linguistic or logical-mathematical can use to master problem solving because the set of problem-solving skills is a complement to technical skills and provides students with ability to be life-long learners.

Endnotes

[i] Wankat, Phillip C. and Oreovicz, Frank S. *Teaching Engineering*, New York: McGraw-Hill, Inc., p. 68. This text is no longer in print, but is available at unitflops.ecn.purdue.edu/ChE/News/Books.
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[vi] Gardner, Howard. *Frames of Mind: The Theory of Multiple Intelligence* (New York: BasicBooks, 1983).

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