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Work in Progress: Effects of Computational Aspects of Differential Equations (DE) Course Delivery on Students' Computing Experience in Engineering Instruction

Mrs. Johannah Lynn Crandall, Washington State University

Jo Crandall is a PhD candidate in Mathematics and Science Education at Washington State University. Her research interests including undergraduate engineering education, especially transferable mathematical knowledge for use in modeling software and discrete solvers. She has experience with both mathematics instruction and computing instruction in both formal and informal learning environments.

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Introduction

Recent literature and Industry 4.0 discussions [1] have highlighted the need for engineering graduates to gain computational facility in all stages of ill-posed, industry-relevant problem solving, from problem framing to understanding of and confidence in algorithm output. Chemical and mechanical engineering students grapple with both ordinary and partial differential equations in their engineering coursework using computational methods that they may not have been prepared for in their introductory differential equations (DE) course in a separate mathematics department.

Aspects of DE course delivery examined here include instructor choices of analytic and numerical methods, instructor incorporation of programming or software packages in lectures and/or assignments, and instructor use of disciplinary examples. The goal of the larger project of which this report is an initial subset is to characterize problem-solving competencies that chemical and mechanical engineering students transfer from their upper-division mathematics courses to their core engineering courses. To date, participation in the study across three universities has included 10 engineering instructors, 6 DE instructors, and 15 engineering undergraduates. Findings of the study are anticipated to inform both mathematics and engineering faculty as (1) many math programs are experiencing the need to evolve DE course delivery into an increasingly computational endeavor and (2) many engineering programs may find support for students' increased computational problem-solving through collaboration with math educators on DE course design.

Conceptual Framework

This qualitative study falls into the well-precedented category of research on transfer of learning. While *transfer* is understood as the activity of applying knowledge acquired in one setting to novel setting-specific circumstances [2], the traditional cognitive construct is augmented in this study by the harnessing of the preparation for future learning (PFL) framework [3]. PFL forefronts the activity of *knowing with* prior learning and sensitization to properties and techniques even if the learner is unable to specifically articulate their grasp of the concepts. This knowing-with process is deemed central to students' capacity to access prior mathematical and computational skill in a way which transcends their rote imitation of procedures. That is, when encountering novel problem scenarios, the extent to which students might apply prior mathematical and computational approaches is likely to be determined by their understanding, or even *feeling*, of how their skill set applies, as opposed to their memory of its prior applications. Thus, the extent to which their grasp of mathematical and computational tools survives the transit between DE instruction and later engineering application may be related to the manner of their engagement with tools in DE instruction [4],[5].

Participants, Setting, & Data Gathering

Data gathering is underway at three western U.S. four-year degree granting universities housing ABET-accredited chemical and/or mechanical engineering programs. These programs were chosen because DE figures prominently in the prerequisites of several core courses. Evidence and perspectives from mathematics instructors, engineering instructors, and engineering students who have previously taken DE are being gathered. Sources of data include course artifacts (e.g. syllabus, textbook) [6], preliminary survey, instruction observations, instructor interviews, and task-based student interviews [7]. Data analyzed in this initial report include engineering instructor and student interviews from an engineering analysis course (3 students) and a dynamical modeling course (2 students). Both are 300-level courses in which MATLAB computing is used on a regular basis to solve differential equations and the students enrolled completed DE approximately one year prior to the study.

Preliminary surveys distributed during initial class visitations are used to identify DE instructors of the current group of engineering students and catalog which mathematics and engineering courses in which the student participants are and have recently been enrolled. Interviews with DE instructors will continue as engineering instructor interviews and class observations conclude.

Student interviews consist of two question types. The first is a group of open-ended questions about the role of technology in their mathematics and engineering experience, perceptions of applicability of tools across disciplines, and perceptions of preparedness for computing in engineering. The second question type follows the posing of four differential equation problem scenarios of graduated complexity and level of engineering context. Problems were assembled with the aid of three experienced DE instructors at two of the participant universities. (The first two problems, to which initial student responses will be discussed here, are included in Table 1.)

Engineering instructor interviews pose similar, but more direct, questions about the role of technology and preparedness of students for numerical and computational methods. The four graduated-context differential equations are also shown to instructors. Instructor anticipated methods of approaching the problems is compared with student methods of approaching the problems, with special attention to mentions of the relevance of computing for solving or analyzing each problem.

Analytical Framework. An analytical framework based on themes of DE teaching and learning concern in the literature has been constructed. The prongs of the analytical framework comprise the following four dimensions:

**Analytic-numerical:* This dimension characterizes an instructor's use of analytic and numerical techniques in the solution of differential equations. Analytic methods include symbolic and algebraic manipulation to produce succinct symbolic representations and numerical methods include estimating and/or visualizing behavior on a restricted interval.

**Computational-analog:* This dimension characterizes an instructor's attitudes toward, references to, and use of technology. Computational technologies include computing activities such as the use of mathematical software packages for either identifying or visualizing solutions, and writing and/or running computer code to implement solution algorithms. Analog technologies refer primarily to the use of pencil and paper for working out solutions.

**Transmission-inquiry:* This dimension characterizes an instructor's methods of content presentation and student engagement as more transmissionist or more student-centered [8]. Modes of engagement such as active learning, problem-based learning, and collaborative learning will be classified together on the inquiry end of the instructional spectrum.

**Disciplinary examples:* This dimension characterizes the use of mathematical examples in the course structure, including: the types of disciplines represented by the examples, the extent to which examples are sourced from or align with a textbook, the extent to which examples are implemented using analytic or numerical methods, and the extent to which examples are implemented using computational or analog methods.

The **research question** asked in this subset of the larger study is: *How do these mathematical, computational, and pedagogical aspects of delivery style in DE courses affect chemical and mechanical engineering students' computing experiences in later DE applications?*

Initial Findings

This report includes initial observations from the first several weeks of the study, including instructor-anticipated and actual student problem-solving approaches using only the computational-analog, analytic-numerical, and disciplinary examples dimensions. Furthermore, while four problem situations are shown to students and instructors in interviews, only initial observations about the first two contextless problems are summarized here. Problem 1 is a first-order non-linear ODE and problem 2 is a second-order non-homogeneous ODE, both shown in Table 1. Perspectives from instructors and student participants in two courses, engineering analysis and dynamic modeling, are summarized. Students who are shown these problems have not yet been asked directly about their past DE course, but only about their general engineering and mathematics experience, to prevent artificially cuing students regarding anticipated problem structures or strategies.

Table 1

Problem 1	Problem 2
y is a function of t where $y' = 5 - 3\sqrt{y}$ and $y(0) = 2$. What will the value of y be when $t = 2$?	$y'' + y = 3\sin 2t + t\cos 2t$ What is a solution for y?

Problem 1. This is a problem used by Boyce & DiPrima [9] as an example to illustrate Euler's method. The instructor of the engineering analysis course anticipated that problem 1 would present no difficulty for students, intimating an analytic solution was within reach. The instructor of the dynamic modeling course concluded that a numerical solution would be simplest, but anticipated that this approach would likely not occur to students.

First student reactions to the problem included recognition of a differential equation (2 students), recognition of first order (1), and recognition of an initial value problem (3). Students *either* connected this course to their DE course or to experience in a course such as kinetics, dynamics, or physics, but not to both. None of the five student participants in the two courses included numerical methods of solution among their possible approaches. For problem 1, the use of physical terminology was associated with characterization of the problem by students as low difficulty.

The two students who characterized the *examples* used in their prior DE course as highly relevant and helpful explicitly identified problem 1 as a differential equation. These two students were also the only ones to use terminology and approaches that would be used by a DE instructor in approaching the problems, such as “first-order initial value problem” and outlining the process of separation of variables followed by integration, even though that precise terminology was not used. Interestingly, neither of these two students who reported the highest usefulness of DE examples used physical terminology (position, velocity, momentum, etc.) to describe what they saw in this problem.

Problem 2. For problem 2, none of the students used DE terminology such as “2nd order” or “nonhomogeneous”. Three students talked about the higher difficulty of this problem. Two students reported they would put the problem into an online equation solver, one reported they would seek out a YouTube video, one would try to simplify the equation with a trig identity, and the fifth student articulated no particular problem-solving strategy.

The engineering analysis instructor anticipated that students wouldn’t connect this problem to “the rest of their universe,” an expectation that appears to have held with the students from that course, as none of those three students either used physical terminology or referenced any course other than DE in making problem connections.

The dynamic modeling instructor once again felt a numerical solution was most appropriate for problem 2, and was the only individual in the group to explicitly mention the complicating t factor on the right-hand side. Only the two student volunteers from this course both explicitly discussed the right-hand side of the problem, one referring to its “sinusoidal pattern” and the other inferring that “something is going around a curve, or following some nonlinear path.” One of this pair of students was also the only one to make any mention of waves in making sense of the problem.

Computing. Three students reported using MATLAB for the first time this term, and all report it as challenging. The two students who came into the semester feeling prepared to code in MATLAB included one who took a MATLAB course and one who took a C course. Students who had been briefly exposed to MATLAB during their DE course did not count themselves as having prior MATLAB experience.

Two students took the C course, and while one of them found it has increased their preparedness and confidence with MATLAB, the other reported they are struggling with the differences. Interestingly, the instructor of the engineering analysis course confirmed that students who opt for the MATLAB prerequisite computing option struggle with MATLAB, while those who take the “more serious” C/C++ programming course do well in MATLAB.

The two student participants from the dynamical modeling course reported having been taught numerical methods in DE, but not having hands-on computing experience in the class. The three students from the engineering analysis course reported neither training in numerical methods nor computing experience in their DE course.

Discussion

Regardless of these problems’ suitability for analytic or numerical approaches, and regardless of students’ reported comfort level with mathematical coding, students exhibited no instinct to include numerical/computational methods in their grappling with the problems. Neither the students in the dynamic modeling class, who are seeing computational treatments of problems on nearly a daily basis, nor the students in the engineering analysis course who are seeing both the analytic and computational treatments of the same or similar problem structures within weeks of each other, expressed any instinct to treat these two problems, each of which is challenging using analytic methods, with numerical/computational methods. An early speculation on the reason for this is students’ inability to recognize that *any* problem is amenable to computational solution. Students may also associate numerical methods only with problems that have been explicitly staged for discretization and coding. Future investigation will include examining student instincts to use computational methods when problem complexity is increased even further.

Students that recalled specific DE course terminology and reported higher estimation of the usefulness of the examples in the course did not tend to think about either the first or second-order problem in terms of physical interpretations. Students who seemed at a loss for explicit mathematical terminology, and who incidentally reported less satisfaction with DE course examples, used physical interpretations such as displacement and acceleration to describe what they saw in the problems. This gives an initial impression of a difference in the cognitive tools used to understand the problem, depending on whether it is connected to a DE course experience or to a kinetics, dynamics, or physics course experience. These different perspectives of problem interpretation can be viewed in the PFL framework as potential categories of *knowing-with* that lead students to different paths of problem grappling.

Timeline for remaining data gathering is currently on track to be completed in June 2021. Analysis of anticipated and actual student work with differential equations from both mathematics and engineering perspectives is expected to provide insights into transferable competencies across a range of computational delivery styles of DE instruction.

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